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***Guidelines for Evaluating and Expressing  
the Uncertainty of NIST Measurement Results***

***Barry N. Taylor and Chris E. Kuyatt***

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<sup>1</sup>At Boulder, CO 80303.

<sup>2</sup>Some elements at Boulder, CO 80303.

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# ***Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results***

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January 1993



**U.S. Department of Commerce**  
Barbara Hackman Franklin, Secretary

**Technology Administration**  
Robert M. White, Under Secretary for Technology

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# FOREWORD

Results of measurements and conclusions derived from them constitute much of the technical information produced by NIST. It is generally agreed that the usefulness of measurement results, and thus much of the information that we provide as an institution, is to a large extent determined by the quality of the statements of uncertainty that accompany them. For example, only if quantitative and thoroughly documented statements of uncertainty accompany the results of NIST calibrations can the users of our calibration services establish their level of traceability to the U.S. standards of measurement maintained at NIST.

Although the vast majority of NIST measurement results are accompanied by quantitative statements of uncertainty, there has never been a uniform approach at NIST to the expression of uncertainty. The use of a single approach within the Institute rather than many different approaches would ensure the consistency of our outputs, thereby simplifying their interpretation.

To address this issue, in July 1992 I appointed a NIST Ad Hoc Committee on Uncertainty Statements and charged it with recommending to me a NIST policy on this important topic. The members of the Committee were:

D. C. Cranmer  
Materials Science and Engineering Laboratory

K. R. Eberhardt  
Computing and Applied Mathematics Laboratory

R. M. Judish  
Electronics and Electrical Engineering Laboratory

R. A. Kamper  
Office of the Director, NIST/Boulder Laboratories

C. E. Kuyatt  
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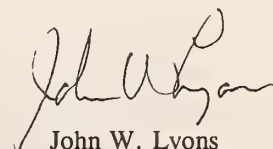
R. L. Watters  
Chemical Science and Technology Laboratory

This action was motivated in part by the emerging international consensus on the approach to expressing uncertainty in measurement recommended by the International Committee for Weights and Measures (CIPM). The movement toward the international adoption of the CIPM approach for expressing uncertainty is driven to a large extent by the global economy and marketplace; its worldwide use will allow measurements performed in different countries and in sectors as diverse as science, engineering, commerce, industry, and regulation to be more easily understood, interpreted, and compared.

At my request, the Ad Hoc Committee carefully reviewed the needs of NIST customers regarding statements of uncertainty and the compatibility of those needs with the CIPM approach. It concluded that the CIPM approach could be used to provide quantitative expressions of measurement uncertainty that would satisfy our customers' requirements. The Ad Hoc Committee then recommended to me a specific policy for the implementation of that approach at NIST. I enthusiastically accepted its recommendation and the policy has been incorporated in the NIST Administrative Manual. (It is also included in this Technical Note as Appendix C.)

To assist the NIST staff in putting the policy into practice, two members of the Ad Hoc Committee prepared this Technical Note. I believe that it provides a helpful discussion of the CIPM approach and, with its aid, that the NIST policy can be implemented without excessive difficulty. Further, I believe that because NIST statements of uncertainty resulting from the policy will be uniform among themselves and consistent with current international practice, the policy will help our customers increase their competitiveness in the national and international marketplaces.

January 1993



John W. Lyons  
Director,

National Institute of Standards and Technology



# GUIDELINES FOR EVALUATING AND EXPRESSING THE UNCERTAINTY OF NIST MEASUREMENT RESULTS

## 1. Introduction

1.1 In October 1992, a new policy on expressing measurement uncertainty was instituted at NIST. This policy is set forth in "Statements of Uncertainty Associated With Measurement Results," Appendix E, NIST Technical Communications Program, Subchapter 4.09 of the Administrative Manual (reproduced as Appendix C of these *Guidelines*).

1.2 The new NIST policy is based on the approach to expressing uncertainty in measurement recommended by the CIPM<sup>1</sup> in 1981 [1] and the elaboration of that approach given in the *Guide to the Expression of Uncertainty in Measurement* (hereafter called the *Guide*), which is being prepared by individuals nominated by the BIPM, IEC, ISO, or OIML [2].<sup>1</sup> The CIPM approach is founded on Recommendation INC-1 (1980) of the Working Group on the Statement of Uncertainties [3]. This group was convened in 1980 by the BIPM as a consequence of a 1978 request by the CIPM that the BIPM study the question of reaching an international consensus on expressing uncertainty in measurement. The request was initiated by then CIPM member and NBS Director E. Ambler. A 1986 request by the CIPM to ISO asking it to develop a broadly applicable guidance document based on Recommendation INC-1 (1980) led to the development of the *Guide*. It is at present the most complete reference on the general application of the CIPM approach to expressing measurement uncertainty, and its development is giving further impetus to the worldwide adoption of that approach.

1.3 Although the *Guide* represents the current international view of how to express uncertainty in measurement based on the CIPM approach, it is a rather lengthy document. We have therefore prepared this Technical Note with the goal of succinctly presenting, in the context of the new NIST policy,

those aspects of the *Guide* that will be of most use to the NIST staff in implementing that policy. We have also included some suggestions that are not contained in the *Guide* or policy but which we believe are useful. *However, none of the guidance given in this Technical Note is to be interpreted as NIST policy unless it is directly quoted from the policy itself.* Such cases will be clearly indicated in the text.

1.4 The guidance given in this Technical Note is intended to be applicable to most, if not all, NIST measurement results, including results associated with

- international comparisons of measurement standards,
- basic research,
- applied research and engineering,
- calibrating client measurement standards,
- certifying standard reference materials, and
- generating standard reference data.

Since the *Guide* itself is intended to be applicable to similar kinds of measurement results, it may be consulted for additional details. Classic expositions of the statistical evaluation of measurement processes are given in references [4-7].

## 2. Classification of Components of Uncertainty

2.1 In general, the result of a measurement is only an approximation or estimate of the value of the specific quantity subject to measurement, that is, the **measurand**, and thus the result is complete only when accompanied by a quantitative statement of its uncertainty.

2.2 The uncertainty of the result of a measurement generally consists of several components which, in the CIPM approach, may be grouped into two categories according to the method used to estimate their numerical values:

- A. those which are evaluated by statistical methods,
- B. those which are evaluated by other means.

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<sup>1</sup>CIPM: International Committee for Weights and Measures; BIPM: International Bureau of Weights and Measures; IEC: International Electrotechnical Commission; ISO: International Organization for Standardization; OIML: International Organization of Legal Metrology.



2.3 There is not always a simple correspondence between the classification of uncertainty components into categories A and B and the commonly used classification of uncertainty components as “random” and “systematic.” The nature of an uncertainty component is conditioned by the use made of the corresponding quantity, that is, on how that quantity appears in the mathematical model that describes the measurement process. When the corresponding quantity is used in a different way, a “random” component may become a “systematic” component and vice versa. Thus the terms “random uncertainty” and “systematic uncertainty” can be misleading when generally applied. An alternative nomenclature that might be used is

“component of uncertainty arising from a random effect,”

“component of uncertainty arising from a systematic effect,”

where a random effect is one that gives rise to a possible random error in the *current measurement process* and a systematic effect is one that gives rise to a possible systematic error in the current measurement process. In principle, an uncertainty component arising from a systematic effect may in some cases be evaluated by method A while in other cases by method B (see subsection 2.2), as may be an uncertainty component arising from a random effect.

NOTE - The difference between error and uncertainty should always be borne in mind. For example, the result of a measurement after correction (see subsection 5.2) can unknowably be very close to the unknown value of the measurand, and thus have negligible error, even though it may have a large uncertainty (see the *Guide* [2]).

2.4 Basic to the CIPM approach is representing each component of uncertainty that contributes to the uncertainty of a measurement result by an estimated standard deviation, termed **standard uncertainty** with suggested symbol  $u_i$ , and equal to the positive square root of the estimated variance  $u_i^2$ .

2.5 It follows from subsections 2.2 and 2.4 that an uncertainty component in category A is represented by a statistically estimated standard deviation  $s_i$ , equal to the positive square root of the statistically estimated variance  $s_i^2$ , and the associated number of degrees of freedom  $\nu_i$ . For such a component the standard uncertainty is  $u_i = s_i$ .

The evaluation of uncertainty by the statistical analysis of series of observations is termed a **Type A evaluation (of uncertainty)**.

2.6 In a similar manner, an uncertainty component in category B is represented by a quantity  $u_j$ , which may be considered an approximation to the corresponding standard deviation; it is equal to the positive square root of  $u_j^2$ , which may be considered an approximation to the corresponding variance and which is obtained from an assumed probability distribution based on all the available information (see section 4). Since the quantity  $u_j^2$  is treated like a variance and  $u_j$  like a standard deviation, for such a component the standard uncertainty is simply  $u_j$ .

The evaluation of uncertainty by means other than the statistical analysis of series of observations is termed a **Type B evaluation (of uncertainty)**.

2.7 Correlations between components (of either category) are characterized by estimated covariances [see Appendix A, Eq. (A-3)] or estimated correlation coefficients.

### 3. Type A Evaluation of Standard Uncertainty

A Type A evaluation of standard uncertainty may be based on any valid statistical method for treating data. Examples are calculating the standard deviation of the mean of a series of independent observations [see Appendix A, Eq. (A-5)]; using the method of least squares to fit a curve to data in order to estimate the parameters of the curve and their standard deviations; and carrying out an analysis of variance (ANOVA) in order to identify and quantify random effects in certain kinds of measurements. If the measurement situation is especially complicated, one should consider obtaining the guidance of a statistician. The NIST staff can consult and collaborate in the development of statistical experiment designs, analysis of data, and other aspects of the evaluation of measurements with the Statistical Engineering Division, Computing and Applied Mathematics Laboratory. Inasmuch as this Technical Note does not attempt to give detailed statistical techniques for carrying out Type A evaluations, references [4-7], and reference [8] in which a general approach to quality control of measurement systems is set forth, should be consulted for basic principles and additional references.



## 4. Type B Evaluation of Standard Uncertainty

4.1 A Type B evaluation of standard uncertainty is usually based on scientific judgment using all the relevant information available, which may include

- previous measurement data,
- experience with, or general knowledge of, the behavior and property of relevant materials and instruments,
- manufacturer’s specifications,
- data provided in calibration and other reports, and
- uncertainties assigned to reference data taken from handbooks.

Some examples of Type B evaluations are given in subsections 4.2 to 4.6.

4.2 Convert a quoted uncertainty that is a stated multiple of an estimated standard deviation to a standard uncertainty by dividing the quoted uncertainty by the multiplier.

4.3 Convert a quoted uncertainty that defines a “confidence interval” having a stated level of confidence (see subsection 5.5), such as 95 or 99 percent, to a standard uncertainty by treating the quoted uncertainty as if a normal distribution had been used to calculate it (unless otherwise indicated) and dividing it by the appropriate factor for such a distribution. These factors are 1.960 and 2.576 for the two levels of confidence given (see also the last line of Table B.1 of Appendix B).

4.4 Model the quantity in question by a normal distribution and estimate lower and upper limits  $a_-$  and  $a_+$  such that the best estimated value of the quantity is  $(a_+ + a_-)/2$  (i.e., the center of the limits) and there is 1 chance out of 2 (i.e., a 50 percent probability) that the value of the quantity lies in the interval  $a_-$  to  $a_+$ . Then  $u_j \approx 1.48a$ , where  $a = (a_+ - a_-)/2$  is the half-width of the interval.

4.5 Model the quantity in question by a normal distribution and estimate lower and upper limits  $a_-$  and  $a_+$  such that the best estimated value of the quantity is  $(a_+ + a_-)/2$  and there is about a 2 out of 3 chance (i.e., a 67 percent probability) that the value of the quantity lies in the interval  $a_-$  to  $a_+$ . Then  $u_j \approx a$ , where  $a = (a_+ - a_-)/2$ .

4.6 Estimate lower and upper limits  $a_-$  and  $a_+$  for the value of the quantity in question such that the probability that

the value lies in the interval  $a_-$  to  $a_+$  is, for all practical purposes, 100 percent. Provided that there is no contradictory information, treat the quantity as if it is equally probable for its value to lie anywhere within the interval  $a_-$  to  $a_+$ ; that is, model it by a uniform or rectangular probability distribution. The best estimate of the value of the quantity is then  $(a_+ + a_-)/2$  with  $u_j = a/\sqrt{3}$ , where  $a = (a_+ - a_-)/2$ .

If the distribution used to model the quantity is triangular rather than rectangular, then  $u_j = a/\sqrt{6}$ .

If the quantity in question is modeled by a normal distribution as in subsections 4.4 and 4.5, there are no finite limits that will contain 100 percent of its possible values. However, plus and minus 3 standard deviations about the mean of a normal distribution corresponds to 99.73 percent limits. Thus, if the limits  $a_-$  and  $a_+$  of a normally distributed quantity with mean  $(a_+ + a_-)/2$  are considered to contain “almost all” of the possible values of the quantity, that is, approximately 99.73 percent of them, then  $u_j \approx a/3$ , where  $a = (a_+ - a_-)/2$ .

The rectangular distribution is a reasonable default model in the absence of any other information. But if it is known that values of the quantity in question near the center of the limits are more likely than values close to the limits, a triangular or a normal distribution may be a better model.

4.7 Because the reliability of evaluations of components of uncertainty depends on the quality of the information available, it is recommended that all parameters upon which the measurand depends be varied to the fullest extent practicable so that the evaluations are based as much as possible on observed data. Whenever feasible, the use of empirical models of the measurement process founded on long-term quantitative data, and the use of check standards and control charts that can indicate if a measurement process is under statistical control, should be part of the effort to obtain reliable evaluations of components of uncertainty [8]. Type A evaluations of uncertainty based on limited data are not necessarily more reliable than Type B evaluations.

## 5. Combined Standard Uncertainty

5.1 The combined standard uncertainty of a measurement result, suggested symbol  $u_c$ , is taken to represent the estimated standard deviation of the result. It is obtained by

combining the individual standard uncertainties  $u_i$  (and covariances as appropriate), whether arising from a Type A evaluation or a Type B evaluation, using the usual method for combining standard deviations. This method, which is summarized in Appendix A [Eq. (A-3)], is often called the *law of propagation of uncertainty* and in common parlance the “root-sum-of-squares” (square root of the sum-of-the-squares) or “RSS” method of combining uncertainty components estimated as standard deviations.

NOTE – The NIST policy also allows the use of established and documented methods equivalent to the “RSS” method, such as the numerically based “bootstrap” (see Appendix C).

**5.2** It is assumed that an estimated correction (or correction factor) is applied to compensate for each recognized systematic effect that significantly influences the measurement result and that every effort has been made to identify such effects. The relevant uncertainty to associate with each recognized systematic effect is then the standard uncertainty of the applied correction. The correction may be either positive, negative, or zero, and its standard uncertainty may in some cases be obtained from a Type A evaluation while in other cases by a Type B evaluation.

#### NOTES

1 The uncertainty of an estimated correction applied to a measurement result to compensate for a systematic effect is not the systematic error in the measurement result due to the effect. Rather, it is a measure of the uncertainty of the result due to incomplete knowledge of the value of the correction. The terms “error” and “uncertainty” should not be confused (see also the note of subsection 2.3).

2 Although it is strongly recommended that corrections be applied for all recognized significant systematic effects, in some cases it may not be practical because of limited resources. Nevertheless, the expression of uncertainty in such cases should conform with these guidelines to the fullest possible extent (see the *Guide* [2]).

**5.3** The combined standard uncertainty  $u_c$  is a widely employed measure of uncertainty. The NIST policy on expressing uncertainty states that (see Appendix C):

Commonly,  $u_c$  is used for reporting results of determinations of fundamental constants, fundamental metrological research, and international comparisons of realizations of SI units.

Expressing the uncertainty of NIST’s primary cesium frequency standard as an estimated standard deviation is an example of the use of  $u_c$  in fundamental metrological research. It should also be noted that in a 1986

recommendation [9], the CIPM requested that what is now termed combined standard uncertainty  $u_c$  be used “by all participants in giving the results of all international comparisons or other work done under the auspices of the CIPM and Comités Consultatifs.”

**5.4** In many practical measurement situations, the probability distribution characterized by the measurement result  $y$  and its combined standard uncertainty  $u_c(y)$  is approximately normal (Gaussian). When this is the case and  $u_c(y)$  itself has negligible uncertainty (see Appendix B),  $u_c(y)$  defines an interval  $y - u_c(y)$  to  $y + u_c(y)$  about the measurement result  $y$  within which the value of the measurand  $Y$  estimated by  $y$  can be asserted to lie with a level of confidence of approximately 68 percent. That is, it may be asserted with an approximate level of confidence of 68 percent that  $y - u_c(y) \leq Y \leq y + u_c(y)$ , which is commonly written as  $Y = y \pm u_c(y)$ .

The probability distribution characterized by the measurement result and its combined standard uncertainty is approximately normal when the conditions of the Central Limit Theorem are met. This is the case, often encountered in practice, when the estimate  $y$  of the measurand  $Y$  is not determined directly but is obtained from the estimated values of a significant number of other quantities [see Appendix A, Eq. (A-1)] describable by well-behaved probability distributions, such as the normal and rectangular distributions; the standard uncertainties of the estimates of these quantities contribute comparable amounts to the combined standard uncertainty  $u_c(y)$  of the measurement result  $y$ ; and the linear approximation implied by Eq. (A-3) in Appendix A is adequate.

NOTE – If  $u_c(y)$  has non-negligible uncertainty, the level of confidence will differ from 68 percent. The procedure given in Appendix B has been proposed as a simple expedient for approximating the level of confidence in these cases.

**5.5** The term “confidence interval” has a specific definition in statistics and is only applicable to intervals based on  $u_c$  when certain conditions are met, including that all components of uncertainty that contribute to  $u_c$  be obtained from Type A evaluations. Thus, in these guidelines, an interval based on  $u_c$  is viewed as encompassing a fraction  $p$  of the probability distribution characterized by the measurement result and its combined standard uncertainty, and  $p$  is the *coverage probability* or *level of confidence* of the interval.



## 6. Expanded Uncertainty

6.1 Although the combined standard uncertainty  $u_c$  is used to express the uncertainty of many NIST measurement results, for some commercial, industrial, and regulatory applications of NIST results (e.g., when health and safety are concerned), what is often required is a measure of uncertainty that defines an interval about the measurement result  $y$  within which the value of the measurand  $Y$  can be confidently asserted to lie. The measure of uncertainty intended to meet this requirement is termed **expanded uncertainty**, suggested symbol  $U$ , and is obtained by multiplying  $u_c(y)$  by a **coverage factor**, suggested symbol  $k$ . Thus  $U = ku_c(y)$  and it can be confidently asserted that  $y - U \leq Y \leq y + U$ , which is commonly written as  $Y = y \pm U$ .

It is to be understood that subsection 5.5 also applies to the interval defined by expanded uncertainty  $U$ .

6.2 In general, the value of the coverage factor  $k$  is chosen on the basis of the desired level of confidence to be associated with the interval defined by  $U = ku_c$ . Typically,  $k$  is in the range 2 to 3. When the normal distribution applies and  $u_c$  has negligible uncertainty (see subsection 5.4),  $U = 2u_c$  (i.e.,  $k = 2$ ) defines an interval having a level of confidence of approximately 95 percent, and  $U = 3u_c$  (i.e.,  $k = 3$ ) defines an interval having a level of confidence greater than 99 percent.

NOTE – For a quantity  $z$  described by a normal distribution with expectation  $\mu_z$  and standard deviation  $\sigma$ , the interval  $\mu_z \pm k\sigma$  encompasses 68.27, 90, 95.45, 99, and 99.73 percent of the distribution for  $k = 1, k = 1.645, k = 2, k = 2.576,$  and  $k = 3$ , respectively (see the last line of Table B.1 of Appendix B).

6.3 Ideally, one would like to be able to choose a specific value of  $k$  that produces an interval corresponding to a well-defined level of confidence  $p$ , such as 95 or 99 percent; equivalently, for a given value of  $k$ , one would like to be able to state unequivocally the level of confidence associated with that interval. This is difficult to do in practice because it requires knowing in considerable detail the probability distribution of each quantity upon which the measurand depends and combining those distributions to obtain the distribution of the measurand.

NOTE – The more thorough the investigation of the possible existence of non-trivial systematic effects and the more complete the data upon

which the estimates of the corrections for such effects are based, the closer one can get to this ideal (see subsections 4.7 and 5.2).

6.4 The CIPM approach does not specify how the relation between  $k$  and  $p$  is to be established. The *Guide* [2] and Dietrich [10] give an approximate solution to this problem (see Appendix B); it is possible to implement others which also approximate the result of combining the probability distributions assumed for each quantity upon which the measurand depends, for example, solutions based on numerical methods.

6.5 In light of the discussion of subsections 6.1-6.4, and in keeping with the practice adopted by other national standards laboratories and several metrological organizations, the stated NIST policy is (see Appendix C):

Use expanded uncertainty  $U$  to report the results of all NIST measurements other than those for which  $u_c$  has traditionally been employed. To be consistent with current international practice, the value of  $k$  to be used at NIST for calculating  $U$  is, by convention,  $k = 2$ . Values of  $k$  other than 2 are only to be used for specific applications dictated by established and documented requirements.

An example of the use of a value of  $k$  other than 2 is taking  $k$  equal to a  $t$ -factor obtained from the  $t$ -distribution when  $u_c$  has low degrees of freedom in order to meet the dictated requirement of providing a value of  $U = ku_c$  that defines an interval having a level of confidence close to 95 percent. (See Appendix B for a discussion of how a value of  $k$  that produces such a value of  $U$  might be approximated.)

6.6 The NIST policy provides for exceptions as follows (see Appendix C):

It is understood that any valid statistical method that is technically justified under the existing circumstances may be used to determine the equivalent of  $u_i$ ,  $u_c$ , or  $U$ . Further, it is recognized that international, national, or contractual agreements to which NIST is a party may occasionally require deviation from NIST policy. In both cases, the report of uncertainty must document what was done and why.

## 7. Reporting Uncertainty

7.1 The stated NIST policy regarding reporting uncertainty



is (see Appendix C):

Report  $U$  together with the coverage factor  $k$  used to obtain it, or report  $u_c$ .

When reporting a measurement result and its uncertainty, include the following information in the report itself or by referring to a published document:

- A list of all components of standard uncertainty, together with their degrees of freedom where appropriate, and the resulting value of  $u_c$ . The components should be identified according to the method used to estimate their numerical values:
  - A. those which are evaluated by statistical methods,
  - B. those which are evaluated by other means.
- A detailed description of how each component of standard uncertainty was evaluated.
- A description of how  $k$  was chosen when  $k$  is not taken equal to 2.

It is often desirable to provide a probability interpretation, such as a level of confidence, for the interval defined by  $U$  or  $u_c$ . When this is done, the basis for such a statement must be given.

7.2 The NIST requirement that a full description of what was done be given is in keeping with the generally accepted view that when reporting a measurement result and its uncertainty, it is preferable to err on the side of providing too much information rather than too little. However, when such details are provided to the users of NIST measurement results by referring to published documents, which is often the case when such results are given in calibration and test reports and certificates, it is imperative that the referenced documents be kept up-to-date so that they are consistent with the measurement process in current use.

7.3 The last paragraph of the NIST policy on reporting uncertainty (see subsection 7.1 above) refers to the desirability of providing a probability interpretation, such as a level of confidence, for the interval defined by  $U$  or  $u_c$ . The following examples show how this might be done when the numerical result of a measurement and its assigned uncertainty is reported, assuming that the published detailed description of the measurement provides a sound basis for

the statements made. (In each of the three cases, the quantity whose value is being reported is assumed to be a nominal 100 g standard of mass  $m_s$ .)

$m_s = (100.021\ 47 \pm 0.000\ 70)$  g, where the  $\pm$  uncertainty is an expanded uncertainty  $U = ku_c$ , with  $U$  determined from a combined standard uncertainty (i.e., estimated standard deviation)  $u_c = 0.35$  mg and a coverage factor  $k = 2$ . Since it can be assumed that the possible estimated values of the standard are approximately normally distributed with approximate standard deviation  $u_c$ , the unknown value of the standard can be asserted to lie in the interval defined by  $U$  with a level of confidence of approximately 95 percent.

$m_s = (100.021\ 47 \pm 0.000\ 79)$  g, where the  $\pm$  uncertainty is an expanded uncertainty  $U = ku_c$ , with  $U$  determined from a combined standard uncertainty (i.e., estimated standard deviation)  $u_c = 0.35$  mg and a coverage factor  $k = 2.26$  based on the  $t$ -distribution for  $\nu = 9$  degrees of freedom, and defines an interval within which the unknown value of the standard can be asserted to lie with a level of confidence of approximately 95 percent.

$m_s = 100.021\ 47$  g with a combined standard uncertainty (i.e., estimated standard deviation) of  $u_c = 0.35$  mg. Since it can be assumed that the possible estimated values of the standard are approximately normally distributed with approximate standard deviation  $u_c$ , the unknown value of the standard can be asserted to lie in the interval  $m_s \pm u_c$  with a level of confidence of approximately 68 percent.

When providing such probability interpretations of the intervals defined by  $U$  and  $u_c$ , subsection 5.5 should be recalled. In this regard, the interval defined by  $U$  in the second example might be a conventional confidence interval (at least approximately) if all the components of uncertainty are obtained from Type A evaluations.

7.4 Some users of NIST measurement results may automatically interpret  $U = 2u_c$  and  $u_c$  as quantities that define intervals having levels of confidence corresponding to those of a normal distribution, namely, 95 percent and 68 percent, respectively. Thus, when reporting either  $U = 2u_c$  or  $u_c$ , if it is known that the interval which  $U = 2u_c$  or  $u_c$  defines has a level of confidence that differs significantly from 95 percent or 68 percent, it should be so stated as an

aid to the users of the measurement result. In keeping with the NIST policy quoted in subsection 6.5, when the measure of uncertainty is expanded uncertainty  $U$ , one may use a value of  $k$  that does lead to a value of  $U$  that defines an interval having a level of confidence of 95 percent if such a value of  $U$  is necessary for a specific application dictated by an established and documented requirement.

7.5 In general, it is not possible to know in detail all of the uses to which a particular NIST measurement result will be put. Thus, it is usually inappropriate to include in the uncertainty reported for a NIST result any component that arises from a NIST assessment of how the result might be employed; the quoted uncertainty should normally be the actual uncertainty obtained at NIST.

7.6 It follows from subsection 7.5 that for standards sent by customers to NIST for calibration, the quoted uncertainty should not normally include estimates of the uncertainties that may be introduced by the return of the standard to the customer's laboratory or by its use there as a reference standard for other measurements. Such uncertainties are due, for example, to effects arising from transportation of the standard to the customer's laboratory, including mechanical damage; the passage of time; and differences between the environmental conditions at the customer's laboratory and at NIST. A caution may be added to the reported uncertainty if any such effects are likely to be significant and an additional uncertainty for them may be estimated and quoted. If, for the convenience of the customer, this additional uncertainty is combined with the uncertainty obtained at NIST, a clear statement should be included explaining that this has been done.

Such considerations are also relevant to the uncertainties assigned to certified devices and materials sold by NIST. However, well-justified, normal NIST practices, such as including a component of uncertainty to account for the instability of the device or material when it is known to be significant, are clearly necessary if the assigned uncertainties are to be meaningful.

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## Appendix A

### Law of Propagation of Uncertainty

A.1 In many cases a measurand  $Y$  is not measured directly, but is determined from  $N$  other quantities  $X_1, X_2, \dots, X_N$  through a functional relation  $f$ :

$$Y = f(X_1, X_2, \dots, X_N). \quad (\text{A-1})$$

Included among the quantities  $X_i$  are corrections (or correction factors) as described in subsection 5.2, as well as quantities that take into account other sources of variability, such as different observers, instruments, samples, laboratories, and times at which observations are made (e.g., different days). Thus the function  $f$  of Eq. (A-1) should express not simply a physical law but a measurement process, and in particular, it should contain all quantities that can contribute a significant uncertainty to the measurement result.

A.2 An estimate of the measurand or *output quantity*  $Y$ , denoted by  $y$ , is obtained from Eq. (A-1) using *input estimates*  $x_1, x_2, \dots, x_N$  for the values of the  $N$  *input quantities*  $X_1, X_2, \dots, X_N$ . Thus the *output estimate*  $y$ , which is the result of the measurement, is given by

$$y = f(x_1, x_2, \dots, x_N). \quad (\text{A-2})$$

A.3 The combined standard uncertainty of the measurement result  $y$ , designated by  $u_c(y)$  and taken to represent the estimated standard deviation of the result, is the positive square root of the estimated variance  $u_c^2(y)$  obtained from

$$u_c^2(y) = \sum_{i=1}^N \left[ \frac{\partial f}{\partial x_i} \right]^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j). \quad (\text{A-3})$$

Equation (A-3) is based on a first-order Taylor series approximation of  $Y = f(X_1, X_2, \dots, X_N)$  and is conveniently referred to as the *law of propagation of uncertainty*. The partial derivatives  $\partial f / \partial x_i$  (often referred to as *sensitivity coefficients*) are equal to  $\partial f / \partial X_i$  evaluated at  $X_i = x_i$ ;  $u(x_i)$  is the standard uncertainty associated with the input estimate  $x_i$ ; and  $u(x_i, x_j)$  is the estimated covariance associated with  $x_i$  and  $x_j$ .

A.4 As an example of a Type A evaluation, consider an input quantity  $X_i$  whose value is estimated from  $n$  independent observations  $X_{i,k}$  of  $X_i$  obtained under the same conditions of measurement. In this case the input estimate  $x_i$  is usually the sample mean

$$x_i = \bar{X}_i = \frac{1}{n} \sum_{k=1}^n X_{i,k}, \quad (\text{A-4})$$

and the standard uncertainty  $u(x_i)$  to be associated with  $x_i$  is the estimated standard deviation of the mean

$$u(x_i) = s(\bar{X}_i) = \left[ \frac{1}{n(n-1)} \sum_{k=1}^n (X_{i,k} - \bar{X}_i)^2 \right]^{1/2}. \quad (\text{A-5})$$

A.5 As an example of a Type B evaluation, consider an input quantity  $X_i$  whose value is estimated from an assumed rectangular probability distribution of lower limit  $a_-$  and upper limit  $a_+$ . In this case the input estimate is usually the expectation of the distribution

$$x_i = (a_+ + a_-) / 2, \quad (\text{A-6})$$

and the standard uncertainty  $u(x_i)$  to be associated with  $x_i$  is the positive square root of the variance of the distribution

$$u(x_i) = a / \sqrt{3}, \quad (\text{A-7})$$

where  $a = (a_+ - a_-) / 2$  (see subsection 4.6).



NOTE - When  $x_i$  is obtained from an assumed distribution, the associated variance is appropriately written as  $u^2(X_i)$  and the associated standard uncertainty as  $u(X_i)$ , but for simplicity,  $u^2(x_i)$  and  $u(x_i)$  are used. Similar considerations apply to the symbols  $u_c^2(y)$  and  $u_c(y)$ .

## Appendix B

### Coverage Factors

**B.1** This appendix summarizes a conventional procedure, given by the *Guide* [2] and Dietrich [10], intended for use in calculating a coverage factor  $k$  when the conditions of the Central Limit Theorem are met (see subsection 5.4) and (1) a value other than  $k = 2$  is required for a specific application dictated by an established and documented requirement; and (2) that value of  $k$  must provide an interval having a level of confidence close to a specified value. More specifically, it is intended to yield a coverage factor  $k_p$  that produces an expanded uncertainty  $U_p = k_p u_c(y)$  that defines an interval  $y - U_p \leq Y \leq y + U_p$ , which is commonly written as  $Y = y \pm U_p$ , having an approximate level of confidence  $p$ .

The four-step procedure is included in these guidelines because it is expected to find broad acceptance internationally, due in part to its computational convenience, in much the same way that  $k = 2$  has become the conventional coverage factor. However, although the procedure is based on a proven approximation, it should not be interpreted as being rigorous because the approximation is extrapolated to situations where its applicability has yet to be fully investigated.

**B.2** To estimate the value of such a coverage factor requires taking into account the uncertainty of  $u_c(y)$ , that is, how well  $u_c(y)$  estimates the standard deviation associated with the measurement result. For an estimate of the standard deviation of a normal distribution, the degrees of freedom of the estimate, which depends on the size of the sample on which the estimate is based, is a measure of its uncertainty. For a combined standard uncertainty  $u_c(y)$ , the “effective degrees of freedom”  $\nu_{\text{eff}}$  of  $u_c(y)$ , which is approximated by appropriately combining the degrees of freedom of its components, is a measure of its uncertainty. Hence  $\nu_{\text{eff}}$  is a key factor in determining  $k_p$ . For example, if  $\nu_{\text{eff}}$  is less than about 11, simply assuming that the uncertainty of  $u_c(y)$  is negligible and taking  $k = 2$  may be inadequate if an expanded uncertainty  $U = k u_c(y)$  that defines an interval

having a level of confidence close to 95 percent is required for a specific application. More specifically, according to Table B.1 (to be discussed below), if  $\nu_{\text{eff}} = 8$ ,  $k_{95} = 2.3$  rather than 2.0. In this case, and in other similar cases where  $\nu_{\text{eff}}$  of  $u_c(y)$  is comparatively small and an interval having a level of confidence close to a specified level is required, it is unlikely that the uncertainty of  $u_c(y)$  would be considered negligible. Instead, the small value of  $\nu_{\text{eff}}$ , and thus the uncertainty of  $u_c(y)$ , would probably be taken into account when determining  $k_p$ .

**B.3** The four-step procedure for calculating  $k_p$  is as follows:

- 1) Obtain  $y$  and  $u_c(y)$  as indicated in Appendix A.
- 2) Estimate the effective degrees of freedom  $\nu_{\text{eff}}$  of  $u_c(y)$  from the Welch-Satterthwaite formula

$$\nu_{\text{eff}} = \frac{u_c^4(y)}{\sum_{i=1}^N \frac{c_i^4 u^4(x_i)}{\nu_i}}, \quad (\text{B-1})$$

where  $c_i \equiv \partial f / \partial x_i$ , all of the  $u(x_i)$  are mutually statistically independent,  $\nu_i$  is the degrees of freedom of  $u(x_i)$ , and

$$\nu_{\text{eff}} \leq \sum_{i=1}^N \nu_i. \quad (\text{B-2})$$

The degrees of freedom of a standard uncertainty  $u(x_i)$  obtained from a Type A evaluation is determined by appropriate statistical methods [7]. In the common case discussed in subsection A.4 where  $x_i = \bar{X}_i$  and  $u(x_i) = s(\bar{X}_i)$ , the degrees of freedom of  $u(x_i)$  is  $\nu_i = n - 1$ . If  $m$  parameters are estimated by fitting a curve to  $n$  data points by the method of least squares, the degrees of freedom of the standard uncertainty of each parameter is  $n - m$ .

The degrees of freedom to associate with a standard uncertainty  $u(x_i)$  obtained from a Type B evaluation is more problematic. However, it is common practice to carry out such evaluations in a manner that ensures that an underestimation is avoided. For example, when lower and upper limits  $a_-$  and  $a_+$  are set as in the case discussed in subsection A.5, they are usually chosen in such a way that the probability of the quantity in question lying outside these limits is in fact extremely small. Under the assumption that

this practice is followed, the degrees of freedom of  $u(x_i)$  may be taken to be  $\nu_i \rightarrow \infty$ .

NOTE – See the *Guide* [2] for a possible way to estimate  $\nu_i$  when this assumption is not justified.

3) Obtain the  $t$ -factor  $t_p(\nu_{\text{eff}})$  for the required level of

confidence  $p$  from a table of values of  $t_p(\nu)$  from the  $t$ -distribution, such as Table B.1 of this Appendix. If  $\nu_{\text{eff}}$  is not an integer, which will usually be the case, either interpolate or truncate  $\nu_{\text{eff}}$  to the next lower integer.

4) Take  $k_p = t_p(\nu_{\text{eff}})$  and calculate  $U_p = k_p u_c(y)$ .

**Table B.1** — Value of  $t_p(\nu)$  from the  $t$ -distribution for degrees of freedom  $\nu$  that defines an interval  $-t_p(\nu)$  to  $+t_p(\nu)$  that encompasses the fraction  $p$  of the distribution

Degrees of freedom $\nu$	Fraction $p$ in percent					
	68.27 <sup>(a)</sup>	90	95	95.45 <sup>(a)</sup>	99	99.73 <sup>(a)</sup>
1	1.84	6.31	12.71	13.97	63.66	235.8
2	1.32	2.92	4.30	4.53	9.92	19.21
3	1.20	2.35	3.18	3.31	5.84	9.22
4	1.14	2.13	2.78	2.87	4.60	6.62
5	1.11	2.02	2.57	2.65	4.03	5.51
6	1.09	1.94	2.45	2.52	3.71	4.90
7	1.08	1.89	2.36	2.43	3.50	4.53
8	1.07	1.86	2.31	2.37	3.36	4.28
9	1.06	1.83	2.26	2.32	3.25	4.09
10	1.05	1.81	2.23	2.28	3.17	3.96
11	1.05	1.80	2.20	2.25	3.11	3.85
12	1.04	1.78	2.18	2.23	3.05	3.76
13	1.04	1.77	2.16	2.21	3.01	3.69
14	1.04	1.76	2.14	2.20	2.98	3.64
15	1.03	1.75	2.13	2.18	2.95	3.59
16	1.03	1.75	2.12	2.17	2.92	3.54
17	1.03	1.74	2.11	2.16	2.90	3.51
18	1.03	1.73	2.10	2.15	2.88	3.48
19	1.03	1.73	2.09	2.14	2.86	3.45
20	1.03	1.72	2.09	2.13	2.85	3.42
25	1.02	1.71	2.06	2.11	2.79	3.33
30	1.02	1.70	2.04	2.09	2.75	3.27
35	1.01	1.70	2.03	2.07	2.72	3.23
40	1.01	1.68	2.02	2.06	2.70	3.20
45	1.01	1.68	2.01	2.06	2.69	3.18
50	1.01	1.68	2.01	2.05	2.68	3.16
100	1.005	1.660	1.984	2.025	2.626	3.077
$\infty$	1.000	1.645	1.960	2.000	2.576	3.000

<sup>(a)</sup>For a quantity  $z$  described by a normal distribution with expectation  $\mu_z$  and standard deviation  $\sigma$ , the interval  $\mu_z \pm k\sigma$  encompasses  $p = 68.27, 95.45,$  and  $99.73$  percent of the distribution for  $k = 1, 2,$  and  $3,$  respectively.





## APPENDIX C

NIST Technical Communications Program

## APPENDIX E

### STATEMENTS OF UNCERTAINTY ASSOCIATED WITH MEASUREMENT RESULTS

A measurement result is complete only when accompanied by a quantitative statement of its uncertainty. This policy requires that NIST measurement results be accompanied by such statements and that a uniform approach to expressing measurement uncertainty be followed.

#### 1. Background

Since the early 1980s, an international consensus has been developing on a uniform approach to the expression of uncertainty in measurement. Many of NIST's sister national standards laboratories as well as a number of important metrological organizations, including the Western European Calibration Cooperation (WECC) and EUROMET, have adopted the approach recommended by the International Committee for Weights and Measures (CIPM) in 1981 [1] and reaffirmed by the CIPM in 1986 [2].

Equally important, the CIPM approach has come into use in a significant number of areas at NIST and is also becoming accepted in U.S. industry. For example, the National Conference of Standards Laboratories (NCSL) is using it to develop a Recommended Practice on measurement uncertainty for NCSL member laboratories.

The CIPM approach is based on Recommendation INC-1 (1980) of the Working Group on the Statement of Uncertainties [3]. This group was convened in 1980 by the International Bureau of Weights and Measures (BIPM) in response to a request by the CIPM. More recently, at the request of the CIPM, a joint BIPM/IEC/ISO/OIML working group developed a comprehensive reference document on the general application of the CIPM approach titled *Guide to the Expression of Uncertainty in Measurement* [4] (IEC: International Electrotechnical Commission; ISO:

International Organization for Standardization; OIML: International Organization of Legal Metrology). The development of the *Guide* is providing further impetus to the worldwide adoption of the CIPM approach.

#### 2. Policy

All NIST measurement results are to be accompanied by quantitative statements of uncertainty. To ensure that such statements are consistent with each other and with present international practice, this NIST policy adopts in substance the approach to expressing measurement uncertainty recommended by the International Committee for Weights and Measures (CIPM). The CIPM approach as adapted for use by NIST is:

- 1) *Standard Uncertainty*: Represent each component of uncertainty that contributes to the uncertainty of the measurement result by an estimated standard deviation  $u_i$ , termed **standard uncertainty**, equal to the positive square root of the estimated variance  $u_i^2$ .
- 2) *Combined Standard Uncertainty*: Determine the **combined standard uncertainty**  $u_c$  of the measurement result, taken to represent the estimated standard deviation of the result, by combining the individual standard uncertainties  $u_i$  (and covariances as appropriate) using the usual "root-sum-of-squares" method, or equivalent established and documented methods.

Commonly,  $u_c$  is used for reporting results of determinations of fundamental constants, fundamental metrological research, and international comparisons of realizations of SI units.

- 3) *Expanded Uncertainty*: Determine an **expanded uncertainty**  $U$  by multiplying  $u_c$  by a **coverage factor**  $k$ :  $U = ku_c$ . The purpose of  $U$  is to provide an interval  $y - U$  to  $y + U$  about the result  $y$  within which the value of  $Y$ , the specific quantity subject to measurement and estimated by  $y$ , can be asserted to lie with a high level of confidence. Thus one can confidently assert that  $y - U \leq Y \leq y + U$ , which is commonly written as  $Y = y \pm U$ .

Use expanded uncertainty  $U$  to report the results of all NIST measurements other than those for which  $u_c$  has traditionally been employed. To be consistent with current international practice, the value of  $k$  to be used at NIST for calculating  $U$  is, by convention,  $k = 2$ . Values of  $k$  other than 2 are only to be used for specific applications dictated by established and documented requirements.

- 4) *Reporting Uncertainty*: Report  $U$  together with the coverage factor  $k$  used to obtain it, or report  $u_c$ .

When reporting a measurement result and its uncertainty, include the following information in the report itself or by referring to a published document:

- A list of all components of standard uncertainty, together with their degrees of freedom where appropriate, and the resulting value of  $u_c$ . The components should be identified according to the method used to estimate their numerical values:
  - A. those which are evaluated by statistical methods,
  - B. those which are evaluated by other means.
- A detailed description of how each component of standard uncertainty was evaluated.
- A description of how  $k$  was chosen when  $k$  is not taken equal to 2.

It is often desirable to provide a probability interpretation, such as a level of confidence, for the interval defined by  $U$  or  $u_c$ . When this is done, the basis for such a statement must be given.

Additional guidance on the use of the CIPM approach at NIST may be found in *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results* [5]. A more detailed discussion of the CIPM approach is given in the *Guide to the Expression of Uncertainty in Measurement* [4]. Classic expositions of the statistical evaluation of measurement processes are given in references [6-8].

### 3. Responsibilities

- a. Operating Unit Directors are responsible for compliance with this policy.
- b. The Statistical Engineering Division, Computing and Applied Mathematics Laboratory, is responsible for providing technical advice on statistical methods for evaluating and expressing the uncertainty of NIST measurement results.
- c. NIST Editorial Review Boards are responsible for ensuring that statements of measurement uncertainty are included in NIST publications and other technical outputs under their jurisdiction which report measurement results and that such statements are in conformity with this policy.
- d. The Calibrations Advisory Group is responsible for ensuring that calibration and test reports and other technical outputs under its jurisdiction are in compliance with this policy.
- e. The Standard Reference Materials and Standard Reference Data programs are responsible for ensuring that technical outputs under their jurisdiction are in compliance with this policy.
- f. Authors, as part of the process of preparing manuscripts and other technical outputs, are responsible for formulating measurement uncertainty statements consistent with this policy. These statements must be present in drafts submitted for NIST review and approval.



## 4. Exceptions

It is understood that any valid statistical method that is technically justified under the existing circumstances may be used to determine the equivalent of  $u_i$ ,  $u_c$ , or  $U$ . Further, it is recognized that international, national, or contractual agreements to which NIST is a party may occasionally require deviation from this policy. In both cases, the report of uncertainty must document what was done and why.

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