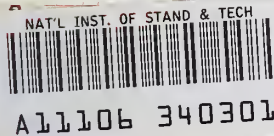


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U.S. DEPARTMENT OF COMMERCE / National Bureau of Standards

## The Thermodynamic Properties of Helium II from 0 K to the Lambda Transitions

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# The Thermodynamic Properties of Helium II from 0 K to the Lambda Transitions

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# THE THERMODYNAMIC PROPERTIES OF HELIUM II FROM 0 K TO THE LAMBDA TRANSITIONS

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The equation of state of He-II is modeled by an equation of state explicit in pressure as a function of density and temperature. The equation of state is divided into three regions of temperature 0 to .8 K; .8 to 1.2 K and 1.2 to the lambda temperature for which similar functional forms are used with different adjustable parameters. The combined functions are valid over the entire PT range of the superfluid and may be used for all classical thermodynamic properties. Comparisons between calculated and experimental data are presented. A computer program for calculation of thermodynamic properties (PVT, isochoric heat capacity, isobaric heat capacity, internal energy, enthalpy, entropy, and velocity of sound) is included.

Key words: Computer program; equation of state; deviation plots; helium II, mathematical model; superfluid; thermodynamic properties.

## 1. Introduction

A reliable mathematical model of the equation of state of a fluid has become an important tool in nearly all applications of the classical thermodynamic properties. Until now, no such model has been available for superfluid helium II. Because of this situation the NASA-Ames Research Center issued a contract to the National Bureau of Standards in Boulder, Colorado to develop such a model. As a result, a model has been developed which is valid for all of the thermodynamic properties over the entire range of pressure and temperature of the superfluid helium II. It is not valid however for the prediction of the anomalous behavior

of the superfluid transition (the  $\lambda$ -line). The purpose of this report is to present the results of the study conducted for NASA.

## 2. Survey of the Literature

A search of the world's scientific literature began with a computerized search of about 120,000 articles at the Cryogenic Data Center of the National Bureau of Standards in Boulder, Colorado. This search produced about 250 references. Each of these articles was considered for possible applicable information and leads to articles not revealed in the original computer search. The literature search produced an extensive set of experimental data which covered the entire pressure and temperature range of He II, and in some cases regions of overlap between the data sets.

Two extensive sources of data were found in the literature; the data by Brooks and Donnelly [3] and the data by Maynard [7]. The Brooks and Donnelly [3] reference contains extensive tabulations of thermodynamic properties over the entire PT range of the superfluid phase and is truly a monumental effort on the part of the authors. The Maynard reference [7] also contains tabulations of the thermodynamic properties of the superfluid but only for temperatures of 1.2 K and above. The tabulated properties in these two references are of sufficient quality and quantity to satisfy almost any need for tabular properties. However, neither of the references contain a satisfactory mathematical model of the equation of state which is the primary purpose of this work. For reasons discussed later the data of Brooks and Donnelly [3] were selected as the primary source of data for the least squares estimation procedures. In addition, the 1958 helium temperature scale [2] and the saturated liquid densities by Ker and Taylor [5], were used as input data.



### 3. Vapor Pressure

An expression for the 1958 helium temperature scale from McCarty [8] was used to predict the vapor pressure of helium between .8 and 2.172 K. The expression is

$$\ln P = \sum_{i=1}^{14} B_i T^{(2-i)} \quad (1)$$

Table 1. Coefficients for Eq. 1 (Vapor Pressure)

$B_1$	=	-	49.510540356
$B_2$	=		651.9236417
$B_3$	=	-	3707.5430856
$B_4$	=		12880.673491
$B_5$	=	-	30048.545554
$B_6$	=		49532.267436
$B_7$	=	-	59337.558548
$B_8$	=		52311.296025
$B_9$	=	-	33950.233134
$B_{10}$	=		16028.674003
$B_{11}$	=	-	5354.1038967
$B_{12}$	=		1199.0301906
$B_{13}$	=	-	161.46362959
$B_{14}$	=		9.8811553386

where  $P$  is in  $\mu\text{m Hg} = 133.322 \text{ Pa}$  and  $T$  is in kelvin. The range of validity for eq (1) is 0.5 to 2.172 K. The coefficients to eq (1) are given in table 1. The vapor pressures in table 7 below 0.8 K are calculated via the thermodynamic

conditions for the coexistence of two phases in equilibrium assuming the ideal gas law for gaseous helium and using the Gibbs free energy (G) of Brooks and Donnelly [3] for the liquid phase. The agreement between the vapor pressures calculated from eq (1) and those calculated as described above between 2.172 and 0.5 K is good, with typical differences of 0.1 percent or less. Table 2 gives a detailed comparison between the 1958 helium scale and pressures calculated via the phase rule using the Brooks and Donnelly [3] data for the saturated liquid and the PVT surface of McCarty [8] for the saturated vapor. The agreement (as shown in table 2) between the 1958 scale and this work is a little surprising since the real gas contribution at 2.0 K on the vapor side amounts to a 3.8 percent of the pressure (i.e., the difference between using an ideal gas equation of state and the one from McCarty). At 1.5 K this real gas contribution has fallen to 1.1 percent and at 0.8 K it has disappeared entirely. Below 1 K the value of the Gibbs energy of the saturated liquid has become so small that it has little effect on the solution and the vapor pressure is determined almost entirely by the choice of  $L_0$ , the latent heat of vaporization at 0 K. A value of 59.62 joules/mole was used in the 1958 helium vapor pressure scale and a value of 59.60202 joules/mole was used here. In using the Brooks and Donnelly [3] tables for this calculation, both the tabulated G and the tabulated H-TS were used for comparative purposes. The results are similar but the procedure of using the tabulated G produced results slightly closer to the 58 scale. The G obtained by forming H-TS agrees quite well with the tabulated G down to 0.5 K but begins to differ from the tabulated G below 0.5 K and the two values actually have opposite signs at 0.2 K and below. Since the values of G are so small at these temperatures the resulting vapor pressure is the same for either the - or the + value of G.

Table 2. Comparison of 1958 He Vapor-Pressure-Temperature Scale and Values From This Work.

T	P, bar 1958 Scale (eq 1)	P, bar This Work	%
2.10	$0.41906 \times 10^{-1}$	$0.41780 \times 10^{-1}$	0.29
2.00	$0.31688 \times 10^{-1}$	$0.31692 \times 10^{-1}$	0.00
1.9	$0.23303 \times 10^{-1}$	$0.23281 \times 10^{-1}$	0.09
1.8	$0.16618 \times 10^{-1}$	$0.16616 \times 10^{-1}$	0.01
1.7	$0.11451 \times 10^{-1}$	$0.11455 \times 10^{-1}$	-0.04
1.6	$0.75857 \times 10^{-2}$	$0.75889 \times 10^{-2}$	-0.04
1.5	$0.47985 \times 10^{-2}$	$0.48012 \times 10^{-2}$	-0.05
1.4	$0.28736 \times 10^{-2}$	$0.28756 \times 10^{-2}$	-0.07
1.3	$0.16111 \times 10^{-2}$	$0.16123 \times 10^{-2}$	-0.08
1.2	$0.83320 \times 10^{-3}$	$0.83357 \times 10^{-3}$	-0.07
1.1	$0.38952 \times 10^{-3}$	$0.38967 \times 10^{-3}$	-0.04
1.0	$0.15999 \times 10^{-3}$	$0.16001 \times 10^{-3}$	-0.01
.9	$0.55431 \times 10^{-4}$	$0.55436 \times 10^{-4}$	-0.01
.8	$0.15258 \times 10^{-4}$	$0.15258 \times 10^{-4}$	0.00
.7	$0.30376 \times 10^{-5}$	$0.30385 \times 10^{-5}$	-0.01
.6	$0.37487 \times 10^{-6}$	$0.37496 \times 10^{-6}$	-0.02
.5	$0.21770 \times 10^{-7}$	$0.21792 \times 10^{-7}$	-0.10
.4		$0.34626 \times 10^{-9}$	
.3		$0.42918 \times 10^{-12}$	
.2		$0.10086 \times 10^{-17}$	
.1		$0.48395 \times 10^{-34}$	

#### 4. Density of the Saturated Liquid and Gaseous Phases

The density of the saturated liquid phase was taken from Ker and Taylor [5] and is given by

$$V_{\text{sat liq}} = \ell_1 + \ell_2 |T - T_\lambda| + \ell_3 |T - T_\lambda| \ln |T - T_\lambda| \quad (2)$$

Table 3. Coefficients for Eq. 2 (Saturated Liquid)

$\ell_1$	=	3.31007
$\ell_2$	=	0.00742434913
$\ell_3$	= -	0.0059164737553

where  $V$  is in liters/mole,  $T$  is in kelvin and the  $\ell_1$ ,  $\ell_2$ ,  $\ell_3$ , and  $T_\lambda$  are given in table 3.

The density of the saturated gaseous state was calculated using the equation of state from McCarty [8] between 0.8 and 2.172 K. Below 0.8 K the density of the saturated gas was calculated using the vapor pressure (as described in the previous section) and the ideal gas equation of state.

#### 5. The Mathematical Model of the Equation of State

It is desirable to model the equation of state of a fluid with a single mathematical function. A single function insures thermodynamic consistency and eliminates potential mathematical continuity problems going from one function to another. Unfortunately a single function for the helium IV equation of state was not found; instead it was necessary to divide the surface into three regions: one from 0.0 to 0.8 K (Region I), one from 0.8 to 1.2 K (Region II), and from 1.2 to the lambda line (Region III). Although mathematical continuity is not achieved at the boundaries, the fit is good enough at the boundaries of each

region to minimize the thermodynamic inconsistencies. For the region of 0 to 0.8 K the function is

$$P = [P_0(\Delta\rho) + F_S(\Delta\rho, T)] 1.01325 \quad (3)$$

where

$$P_0(\Delta\rho) = \sum_{i=1}^3 a_i \Delta\rho^i \quad (4)$$

and

$$\begin{aligned} F_S(\Delta\rho, T) = & - F_{S,1} D^2 T^4 / 4 - F_{S,2} D^2 T^5 / 5 - F_{S,3} D^2 T^6 / 6 \\ & - 2F_{S,4}(\Delta\rho) D^2 T^3 / 3 - 2F_{S,5}(\Delta\rho) D^2 T^4 / 4 \\ & - 2F_{S,6}(\Delta\rho) D^2 T^5 / 5 - 2F_{S,7}(\Delta\rho) D^2 T^6 / 6 - 2F_{S,8}(\Delta\rho) D^2 T^7 / 7 \\ & - 3F_{S,9}(\Delta\rho)^2 D^2 T^3 / 3 - 3F_{S,10}(\Delta\rho)^2 D^2 T^4 / 4 - 3F_{S,11}(\Delta\rho)^2 D^2 T^5 / 5 \\ & - 3F_{S,12}(\Delta\rho)^2 D^2 T^6 / 6 - 3F_{S,13}(\Delta\rho)^2 D^2 T^7 / 7 - 4F_{S,14}(\Delta\rho)^3 D^2 T^3 / 3 \quad (5) \\ & - 4F_{S,15}(\Delta\rho)^3 D^2 T^5 / 5 - 4F_{S,16}(\Delta\rho)^3 D^2 T^7 / 7 - 4F_{S,17}(\Delta\rho)^3 D^2 T^9 / 9 \\ & - 4F_{S,18}(\Delta\rho)^3 D^2 T^{11} / 11 - 5F_{S,19}(\Delta\rho)^4 D^2 T^3 / 3 \\ & - 5F_{S,20}(\Delta\rho)^4 D^2 T^5 / 5 - 5F_{S,21}(\Delta\rho)^4 D^2 T^7 / 7 \\ & - 5F_{S,22}(\Delta\rho)^4 D^2 T^9 / 9 - 5F_{S,23}(\Delta\rho)^4 D^2 T^{11} / 11 \end{aligned}$$

where P is the pressure in bar, D is the density in moles/liter, T is temperature in kelvin and  $\Delta\rho = D - 36.27877$ . The coefficients for Region I are given in table 4.

Table 4. Coefficients to Equations for Region I (0 to 0.8 K).

Coefficients for eq 4 (Region I)

(note  $\Delta\rho$  for Region I is defined as

D - 36.27877)

$$a_1 = 2.281877372$$

$$a_2 = 0.16820886$$

$$a_3 = 0.005277884968$$

Coefficients for eq 5 (Region I, 0 to 0.8 K)

$$F_{s,1} = - .776003592103 \times 10^{-4}$$

$$F_{s,2} = .516985343553 \times 10^{-4}$$

$$F_{s,3} = - .185460414352 \times 10^{-5}$$

$$F_{s,4} = .993150555179 \times 10^{-6}$$

$$F_{s,5} = - .372729528003 \times 10^{-5}$$

$$F_{s,6} = .905240314118 \times 10^{-4}$$

$$F_{s,7} = - .263138088468 \times 10^{-3}$$

$$F_{s,8} = .210133644446 \times 10^{-3}$$

$$F_{s,9} = - .251675888508 \times 10^{-6}$$

$$F_{s,10} = .246805662352 \times 10^{-5}$$

$$F_{s,11} = - .235766906295 \times 10^{-4}$$

$$F_{s,12} = .636877273619 \times 10^{-4}$$

$$F_{s,13} = - .464000281660 \times 10^{-4}$$

$$F_{s,14} = .133504455025 \times 10^{-7}$$

$$F_{s,15} = .441252121325 \times 10^{-6}$$

$$F_{s,16} = - .390205136440 \times 10^{-5}$$

$$F_{s,17} = .569946840678 \times 10^{-5}$$

$$F_{s,18} = - .219762939629 \times 10^{-5}$$



Table 4. Continued

$$\begin{aligned}
 F_{s,19} &= - .581270462264 \times 10^{-9} \\
 F_{s,20} &= - .191711245461 \times 10^{-7} \\
 F_{s,21} &= .144897551106 \times 10^{-6} \\
 F_{s,22} &= - .793219612515 \times 10^{-7} \\
 F_{s,23} &= - .390940913081 \times 10^{-7}
 \end{aligned}$$

The model for Regions II and III is given by

$$P = [VP(T) + P_o(\Delta D) + P_r(\Delta D, T)] 1.01325 \quad (6)$$

where P is pressure in bar, D is density in moles/liter, T is temperature in kelvin and VP(T) is given by eq (1). The  $P_o(\Delta D)$  is of the same functional form as eq (4) but with different adjustable parameters and a new definition of the independent variable which is  $\Delta D = D - D_{\text{sat liq}}$ . D is as before the density in moles/liter and  $D_{\text{sat liq}}$  is  $1/V_{\text{sat liq}}$  which is given by eq (2). The new  $a_i$  are given in table 5. The  $P_r(\Delta D, T)$  is defined as

$$P_r(\Delta D, T) = f_p(\Delta D, T) + f_s(\Delta \rho, T) \quad (7)$$

where

$$\begin{aligned}
 f_p(\Delta D, T) &= F_{p,1} \Delta D^3 T^3 + F_{p,2} \Delta D^3 T^{2.5} + F_{p,3} \Delta D^3 T^2 \\
 &\quad + F_{p,4} \Delta D^2 T^3 + F_{p,5} \Delta D^2 T^2 \\
 &\quad + F_{p,6} \Delta D T^3 + F_{p,7} \Delta D T^{2.5} + F_{p,8} \Delta D T^2
 \end{aligned} \quad (8)$$

The  $\Delta D$  and T are as above and the  $F_{p,i}$  are given in table 5.

The  $f_s(\Delta\rho, T)$  in eq (7) is of the same functional form as eq (5) with different  $F_{S,i}$ . All of the parameters for Regions II and III are given in table 5.

Table 5. Coefficients to Equations for Regions II and III

Coefficients for eq 4 (Regions II and III, 0.8 to  $T_\lambda$ )

$$\begin{aligned} a_1 &= 2.241456 \\ a_2 &= 0.1757482 \\ a_3 &= 0.00470035 \end{aligned}$$

Coefficients to eq 5 (Region III, 1.2 to  $T_\lambda$ )

$$\begin{aligned} F_{S,1} &= - .299775895293 \times 10^{-3} \\ F_{S,2} &= .261528116001 \times 10^{-3} \\ F_{S,3} &= - .451073420829 \times 10^{-4} \\ F_{S,4} &= - .179926805218 \times 10^{-3} \\ F_{S,5} &= .268760818966 \times 10^{-3} \\ F_{S,6} &= .153832317516 \times 10^{-4} \\ F_{S,7} &= - .162726148595 \times 10^{-3} \\ F_{S,8} &= .477756675722 \times 10^{-4} \\ F_{S,9} &= - .356060361531 \times 10^{-4} \\ F_{S,10} &= .407625370109 \times 10^{-3} \\ F_{S,11} &= - .713769173335 \times 10^{-3} \\ F_{S,12} &= .449456804718 \times 10^{-3} \\ F_{S,13} &= - .913635541095 \times 10^{-4} \\ F_{S,14} &= - .975555037829 \times 10^{-5} \\ F_{S,15} &= .121659779679 \times 10^{-4} \\ F_{S,16} &= - .528306039117 \times 10^{-5} \end{aligned}$$

Table 5. Continued

$$\begin{aligned}
 F_{s,17} &= .311573112016 \times 10^{-6} \\
 F_{s,18} &= .613299771434 \times 10^{-7} \\
 F_{s,19} &= .506000325098 \times 10^{-6} \\
 F_{s,20} &= - .612590386700 \times 10^{-6} \\
 F_{s,21} &= .230922759488 \times 10^{-6} \\
 F_{s,22} &= .327499222785 \times 10^{-8} \\
 F_{s,23} &= - .515534867647 \times 10^{-8}
 \end{aligned}$$

Coefficients for eq 5 (Region II, 0.8 to 1.2 K)

$$\begin{aligned}
 F_{s,1} &= .108660418499 \times 10^{-2} \\
 F_{s,2} &= - .217871751436 \times 10^{-2} \\
 F_{s,3} &= .102911648479 \times 10^{-2} \\
 F_{s,4} &= .189253572751 \times 10^{-2} \\
 F_{s,5} &= - .674364748289 \times 10^{-2} \\
 F_{s,6} &= .798926642309 \times 10^{-2} \\
 F_{s,7} &= - .344107467055 \times 10^{-2} \\
 F_{s,8} &= .299781633163 \times 10^{-3} \\
 F_{s,9} &= - .214084674667 \times 10^{-3} \\
 F_{s,10} &= .335439600940 \times 10^{-3} \\
 F_{s,11} &= .559092792724 \times 10^{-3} \\
 F_{s,12} &= - .119903558078 \times 10^{-2}
 \end{aligned}$$

Table 5. Continued

$$\begin{aligned}
 F_{s,13} &= .526331681180 \times 10^{-3} \\
 F_{s,14} &= .118775501632 \times 10^{-4} \\
 F_{s,15} &= - .459408808154 \times 10^{-4} \\
 F_{s,16} &= .519701003921 \times 10^{-4} \\
 F_{s,17} &= - .196070771338 \times 10^{-4} \\
 F_{s,18} &= .731453369826 \times 10^{-6} \\
 F_{s,19} &= - .526985760908 \times 10^{-6} \\
 F_{s,20} &= .169561251135 \times 10^{-5} \\
 F_{s,21} &= - .131795348291 \times 10^{-5} \\
 F_{s,22} &= - .714287537326 \times 10^{-7} \\
 F_{s,23} &= .258759130915 \times 10^{-6}
 \end{aligned}$$

Coefficients for eq 9 (Regions II and III, 0.8 to  $T_\lambda$ )

$$\begin{aligned}
 F_{p,1} &= - .320783527549 \\
 F_{p,2} &= .580145141306 \\
 F_{p,3} &= - .294344361744 \\
 F_{p,4} &= .290449403103 \\
 F_{p,5} &= .801446582474 \\
 F_{p,6} &= - .175703015761 \\
 F_{p,7} &= .400129303603 \\
 F_{p,8} &= - .255176262894
 \end{aligned}$$

## 6. The Derived Thermodynamic Properties

All of the He II property values tabulated here have been calculated using the equation of state, as described in the previous sections, applied to the following relationships

$$\Delta S = - \int_{\rho_{\text{sat}}}^{\rho} \frac{1}{\rho^2} \left( \frac{\partial p}{\partial T} \right)_{\rho} d\rho \quad (9)$$

$$\Delta H = \int_{\rho_{\text{sat}}}^{\rho} \left[ \frac{p}{\rho^2} - \frac{T}{\rho^2} \left( \frac{\partial p}{\partial T} \right)_{\rho} \right] d\rho + p/\rho \quad (10)$$

$$\Delta C_V = - T \int_{\rho_{\text{sat}}}^{\rho} \frac{1}{\rho^2} \left( \frac{\partial^2 p}{\partial T^2} \right)_{\rho} d\rho \quad (11)$$

$$C_p = C_v + \left( \frac{\partial p}{\partial T} \right)^2 / \left( \frac{\partial p}{\partial \rho} \right)_T \frac{1}{\rho^2} \quad (12)$$

$$W = \left[ \left( \frac{C_p}{C_v} \right) \left( \frac{\partial p}{\partial \rho} \right)_T \right]^{1/2} \quad (13)$$

The  $S$ ,  $H$  and  $C_v$  of the saturated liquid state were taken from Brooks and Donnelly [3]. The tabulated reference values of Brooks and Donnelly [3] and those of Maynard [7] must be adjusted to be consistent with the published tables of McCarty [8]. The adjustment of 59.869851 joules/mole must be added to the enthalpy. The saturated vapor values tabulated here were calculated using the equation of state by McCarty. Computer program listings for all of the He-II properties are given in Appendix A.

## 7. Computer Programs

The listings of the computer programs for the properties of He-II in Appendix A are the programs used to calculate the various thermodynamic properties in tables 7 and 8. In general the Fortran functions have been named using the first letter or two to describe the property and the last three letters to describe the fluid. For example FUNCTION CVHE2(D,T) is the routine for specific

heat capacity at constant volume and the fluid is He-II. In addition to the subprograms used in the example in Appendix A, there are a few other routines which are not used in the calculation of the properties but have been included here for the reader's convenience. These routines are PMELT2(T), FINDTD(D), FINDTP(P), DENLAM(T) and PRSLAM(T). The PMELT2(T) routine calculates the melting pressure of He-II according to the work of Grilly [4]. The other four routines mentioned above give the PVT of the lambda line. FINDTP(P) calculates the temperature given a pressure; PRSLAM(T) calculates the pressure given a temperature; DENLAM(T) calculates the density given a temperature; and FINDTD(D) calculates the temperature given a density. The PVT relations of the lambda line are from the work of Kierstead [6]. The remainder of the routines are used in the calculation of the He-II properties.

#### 8. Estimated Accuracy of the Thermodynamic Properties

The accuracy of the thermodynamic properties in tables 7 and 8 depends almost entirely upon the accuracy of the data used in developing the mathematical models. Figures 1 through 9 illustrate the deviations between properties calculated using the methods described in this work and data from the two primary sources of Brooks and Donnelly [3] and Maynard [7]. In viewing these deviation plots, it should be kept in mind that the Brooks and Donnelly [3] data were used in the fit, over the entire temperature range but comparisons to the Maynard data are included to illustrate the differences (or agreement) between the two major sources in their range of overlapping pressures and temperatures.

No attempt was made to force the equation of state to reproduce the asymptotic behavior of He-II as the lambda line is approached. However, as may be seen in table 6, with the exception of the specific heat capacities, the performance of the equation of state along the lambda line is comparable with the



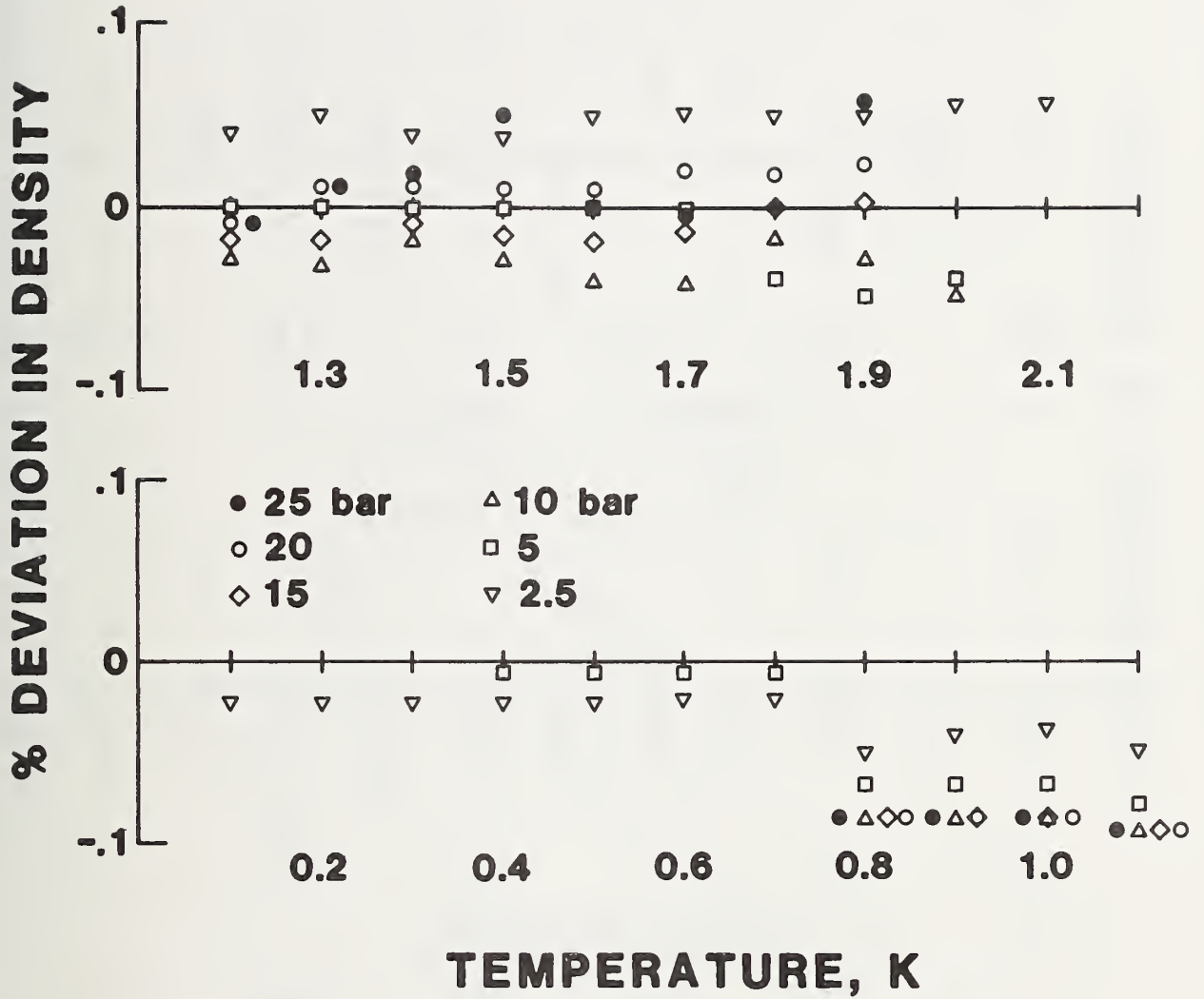


Figure 1. Differences between calculated densities and those of Brooks and Donnelly [3].

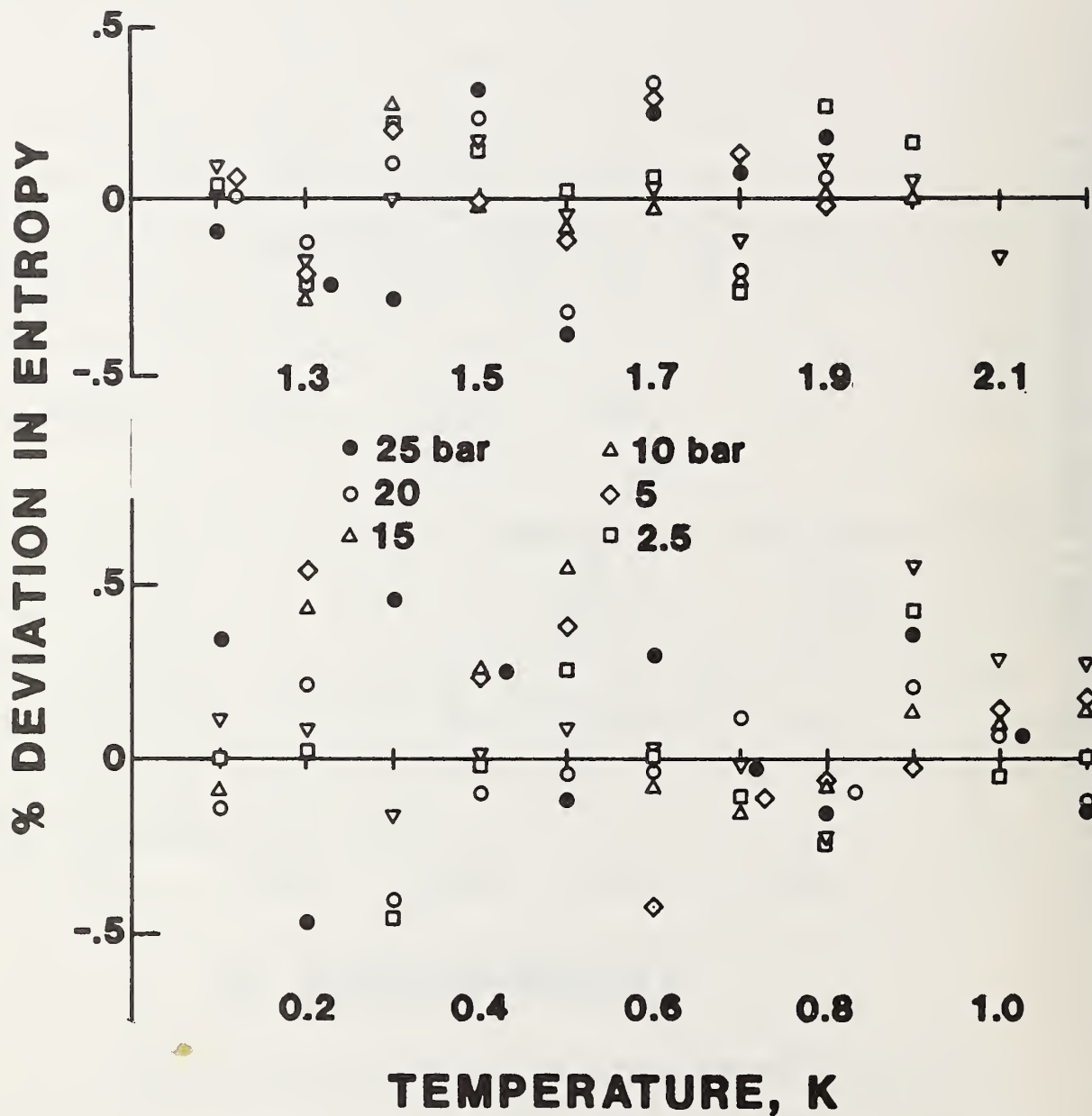


Figure 2. Differences between calculated entropy and those of Brooks and Donnelly [3].

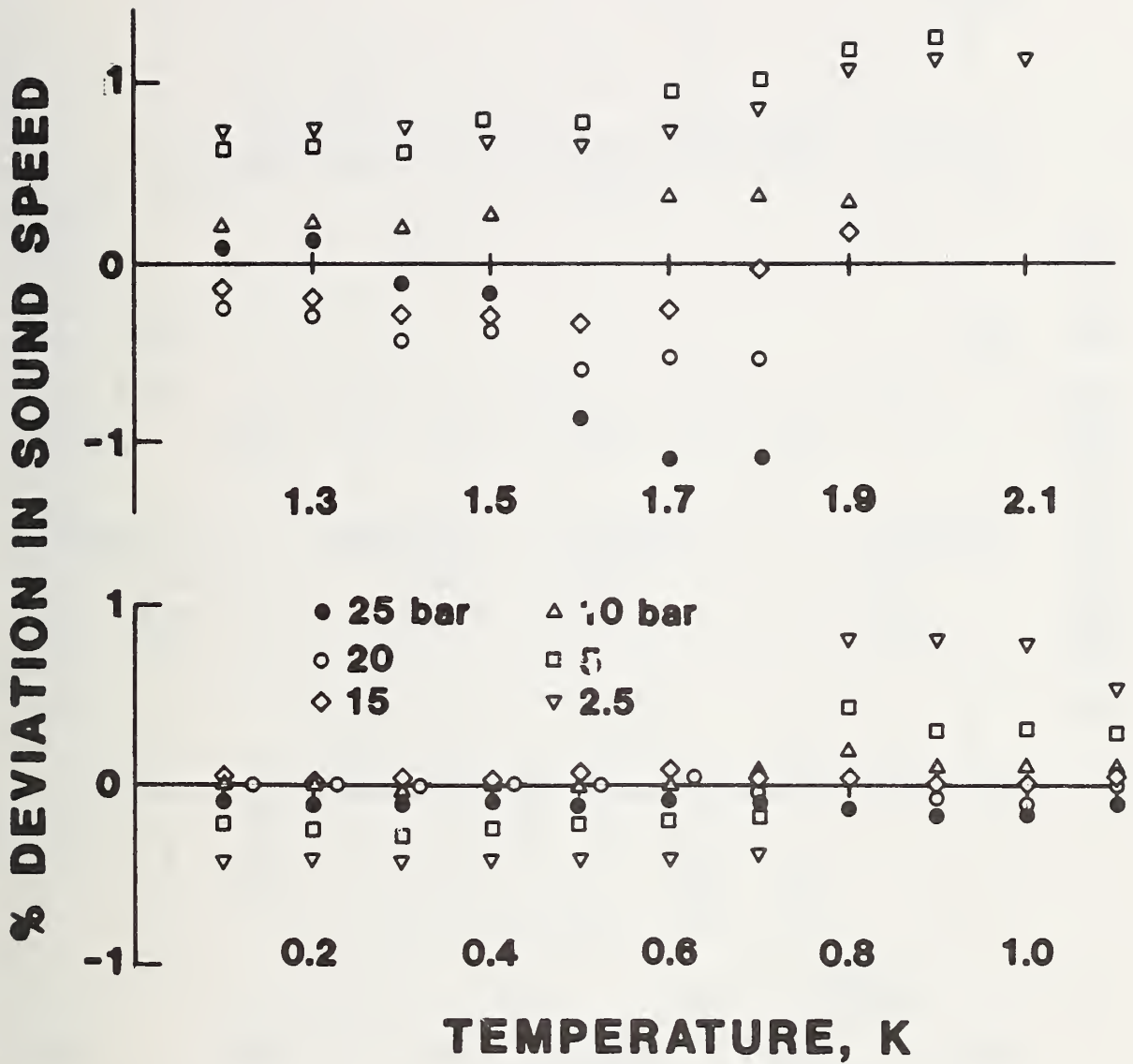


Figure 3. Differences between calculated sound speed and those of Brooks and Donnelly [3].

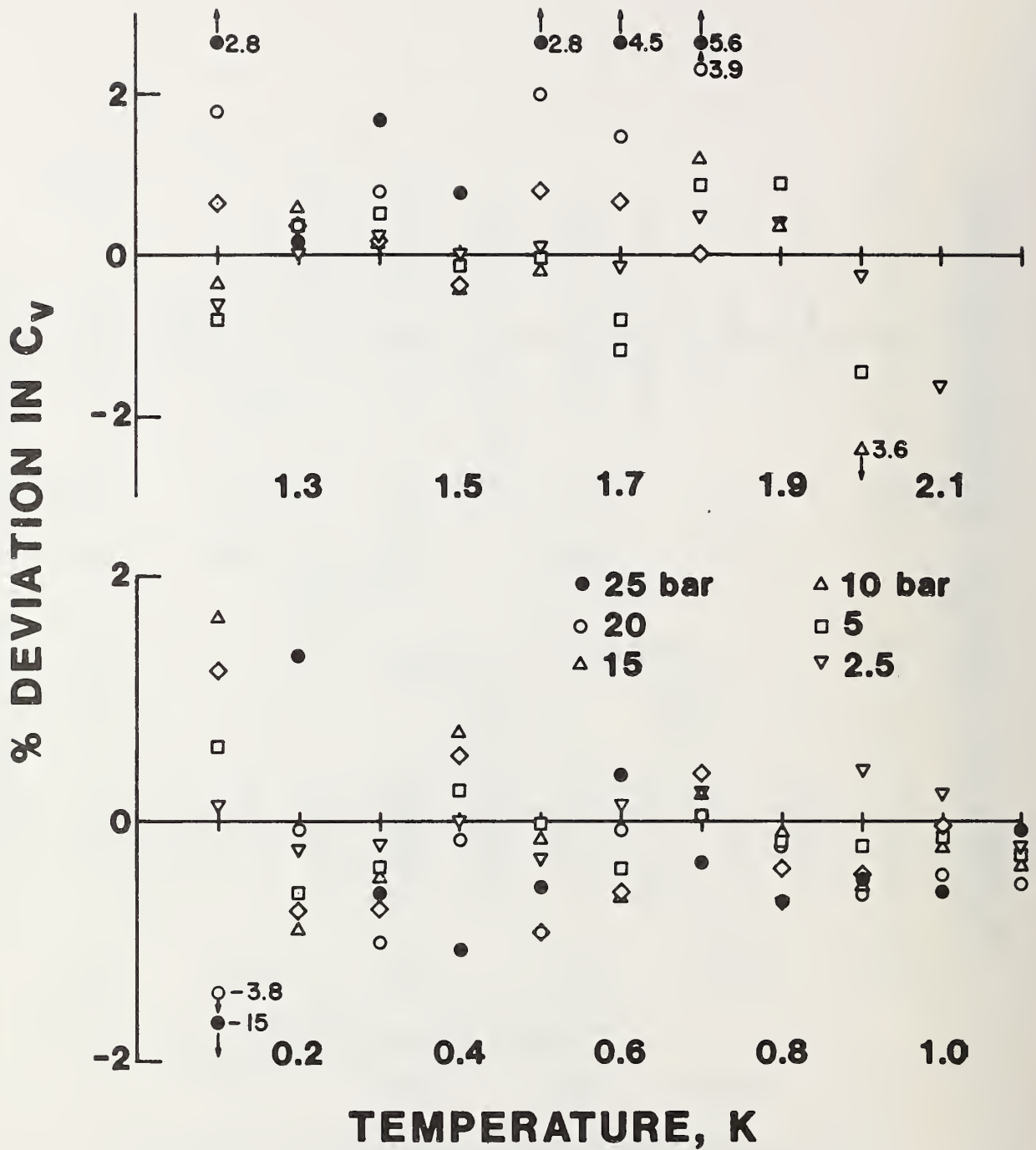


Figure 4. Differences between calculated specific heat capacity at constant value and those of Brooks and Donnelly [3].

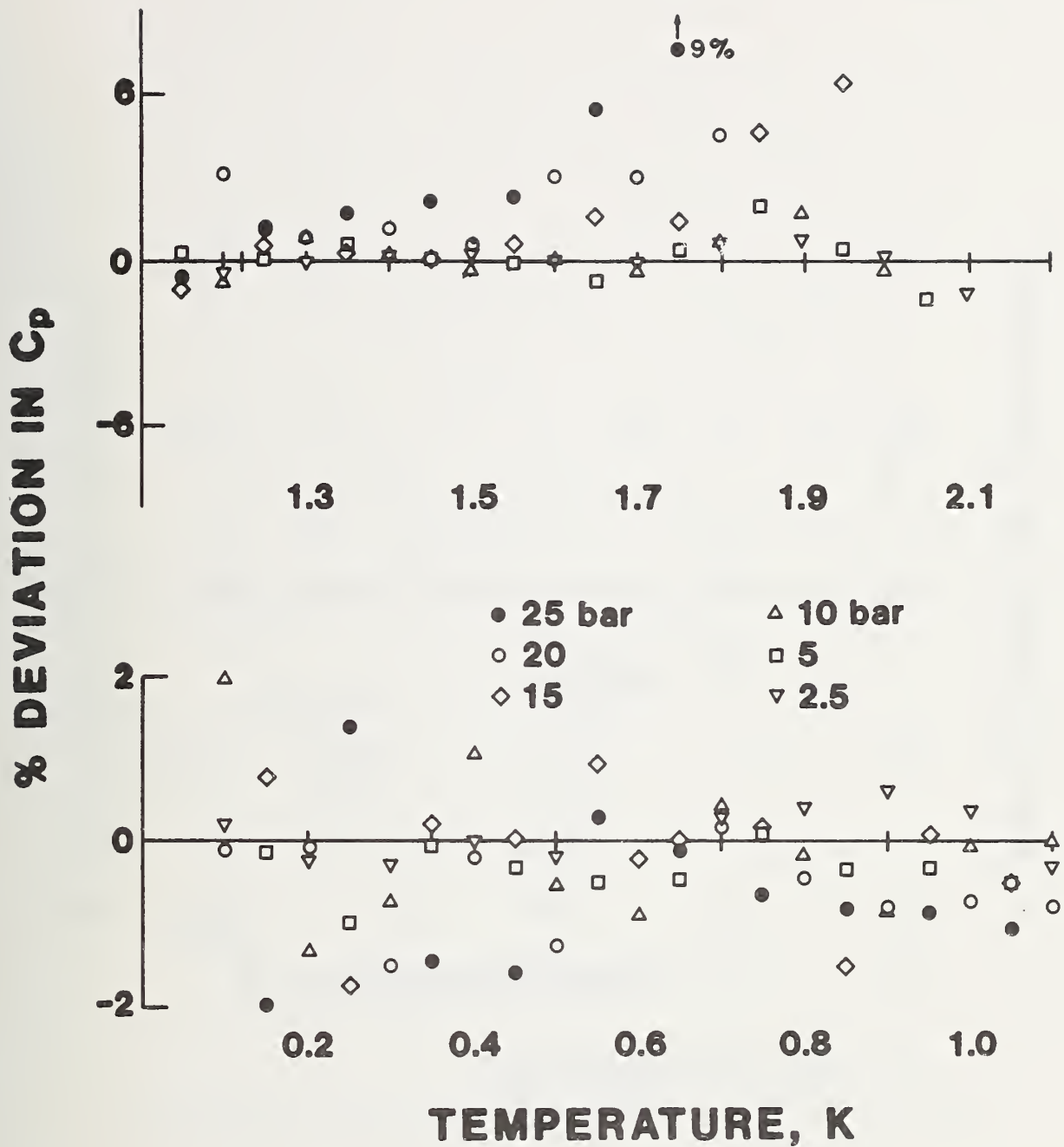


Figure 5. Differences between calculated specific heat capacity at constant pressure and those of Brooks and Donnelly [3].

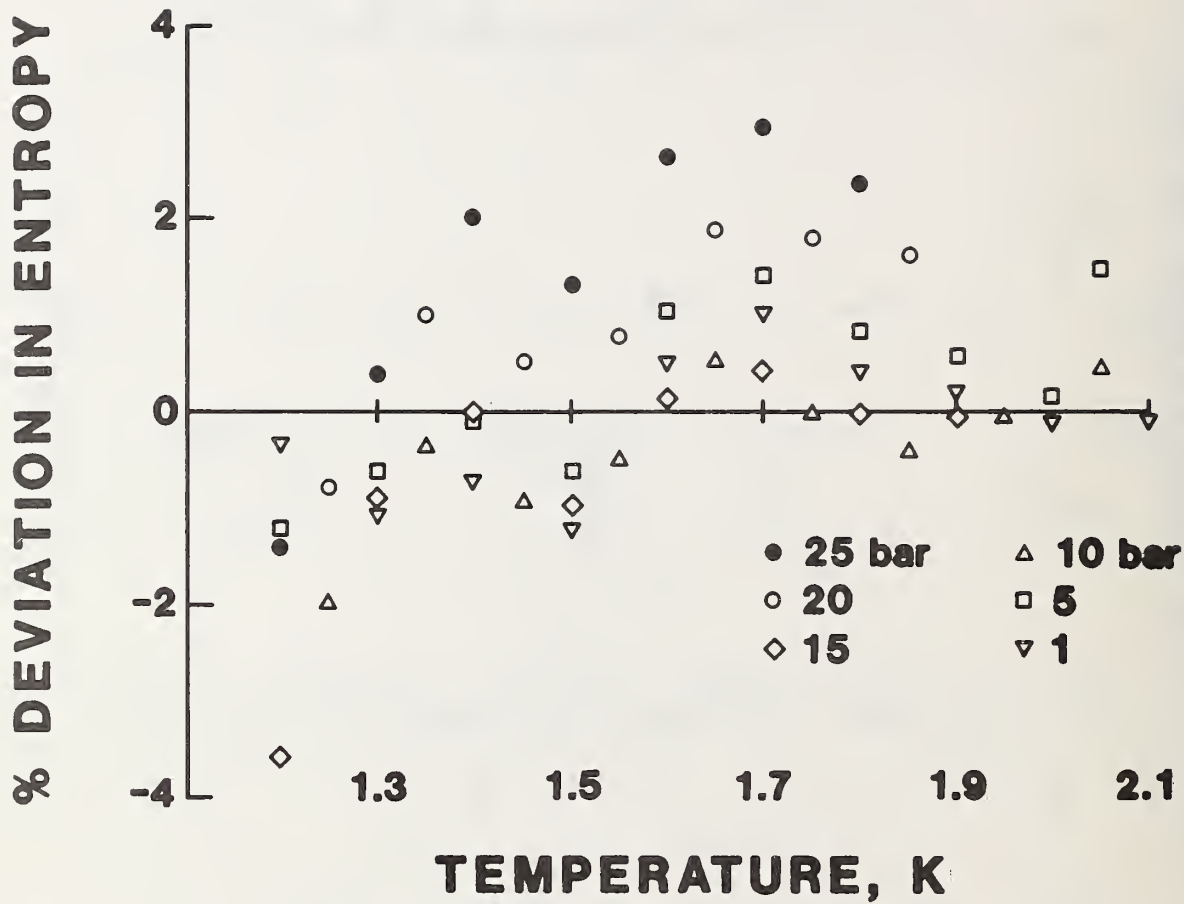


Figure 6. Differences between calculated entropy and those of Maynard [7].



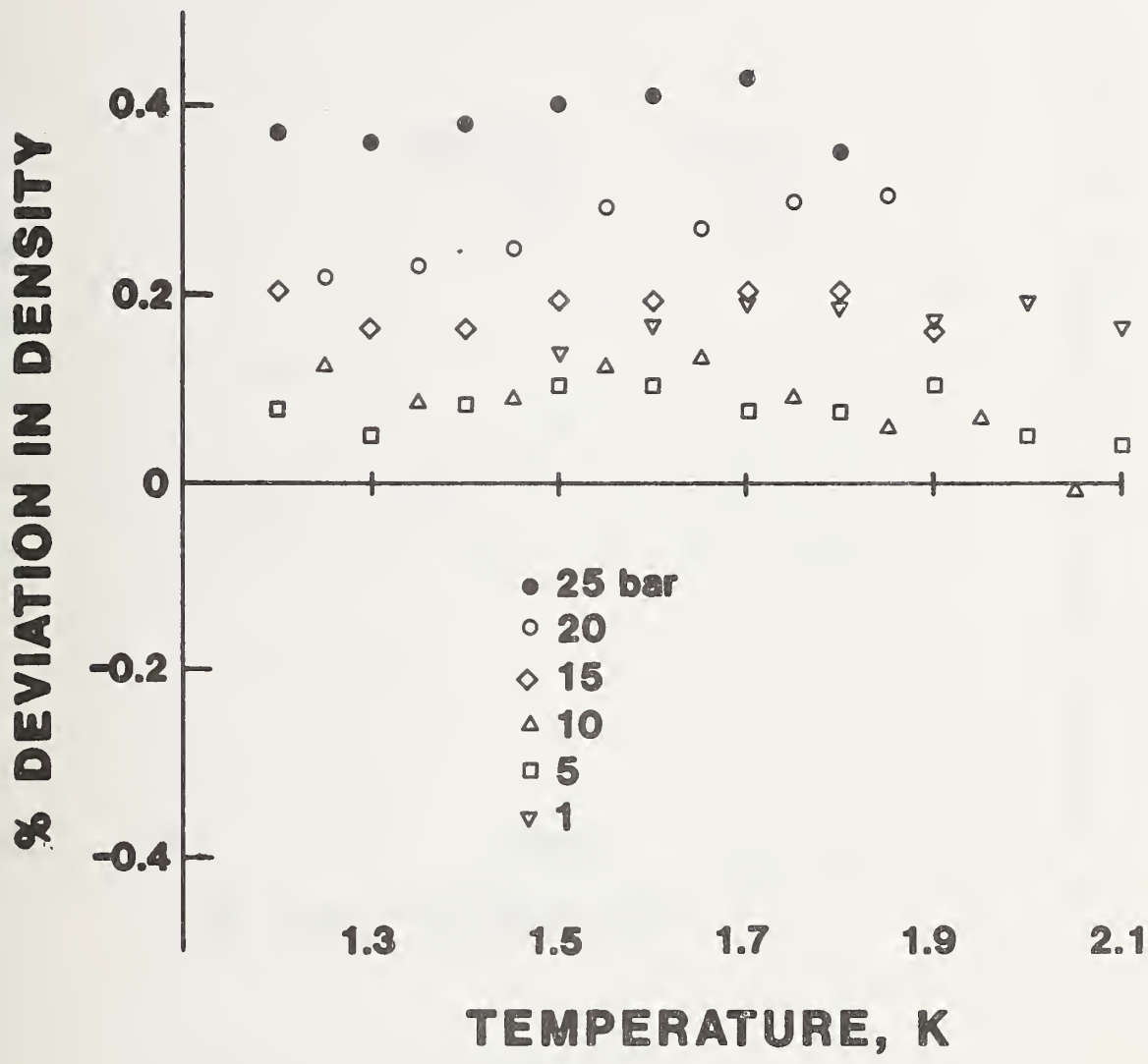


Figure 7. Differences between calculated density and those of Maynard [7].

**% DEVIATION IN SOUND SPEED**

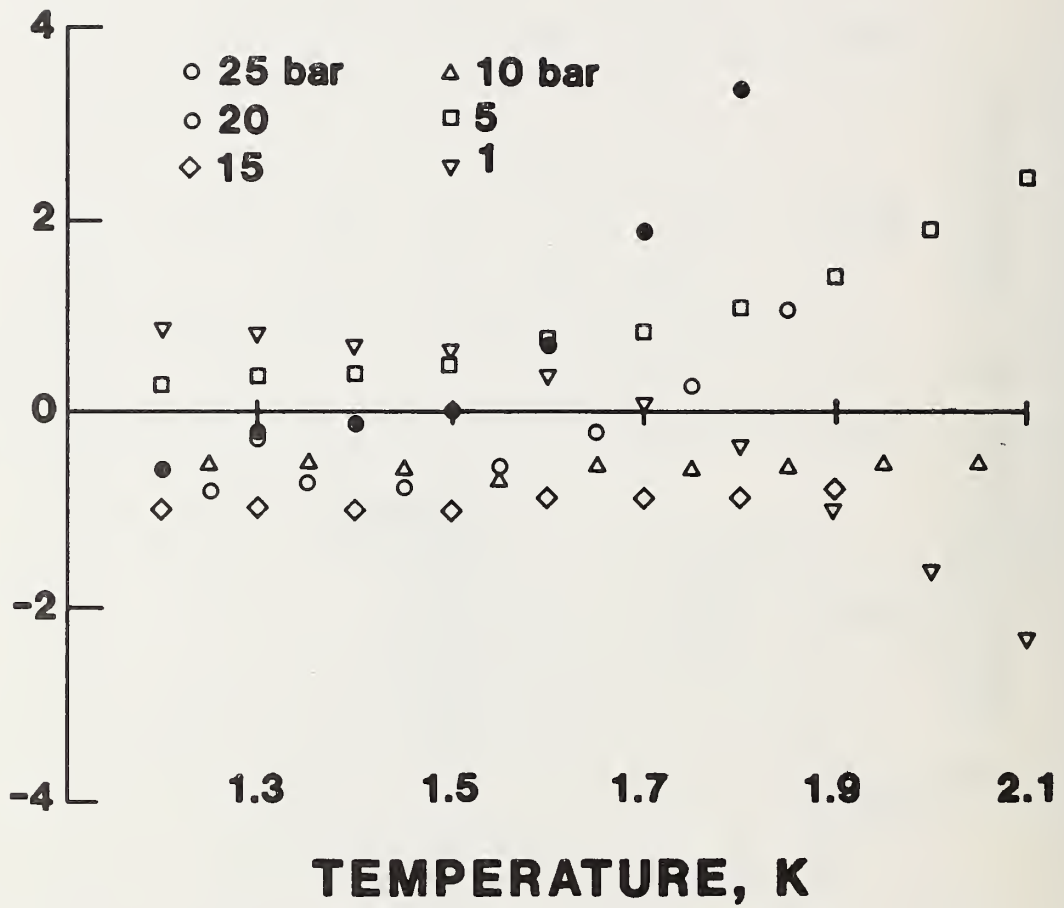


Figure 8. Differences between calculated sound speed and those of Maynard [7].

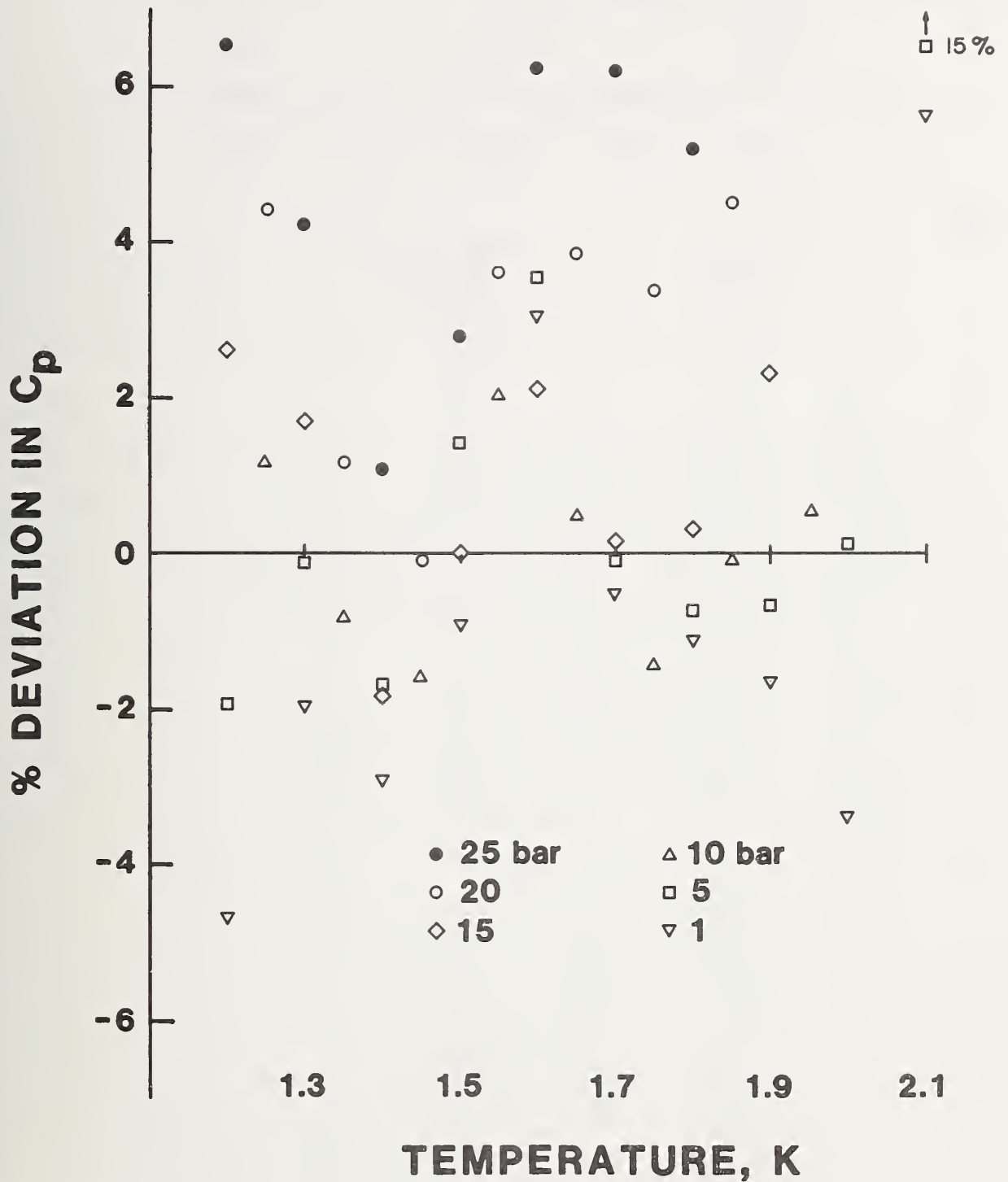


Figure 9. Differences between calculated specific heat capacity at constant pressure and those of Maynard [7].

rest of the surface. The PVT in table 6 are from Kierstead [6] and the entropies and sound velocity are from Ahlers [1].

Table 6. Selected Properties of He Along the Lambda Line With Comparison to the Equation of State in This Work.

$P_{\text{bar}}$	T K	$\rho_1$ <u>moles</u> <u>liter</u>	%	$C_2$ <u>meters</u> <u>second</u>	%diff	$S_2$ <u>joules</u> <u>mole</u>	%
1.	2.163	37.001	- .2	225.3	-3.4	6.1560	1.9
2.	2.154	37.451	- .2	233.4	- .3	6.0719	3.0
3.	2.143	37.900	.2	241.0	2.0	5.9879	4.0
4.	2.133	38.300	.15	248.1	2.5	5.9118	4.7
5.	2.122	38.675	.06	254.8	2.2	5.8398	3.8
6.	2.111	39.050	.07	261.0	1.5	5.7717	3.2
7.	2.099	39.399	0.04	266.9	0.7	5.7037	2.7
8.	2.087	39.724	0.06	272.5	0.1	5.6397	1.9
9.	2.075	40.049	.07	277.7	-0.4	5.5796	1.8
10.	2.063	40.349	.1	282.6	-0.9	5.5196	1.5
11.	2.051	40.649	.1	287.3	-1.2	5.4635	1.2
12.	2.038	40.923	0.15	291.8	-1.3	5.4075	1.3
13.	2.025	41.198	0.16	296.0	-1.2	5.3515	1.2
14.	2.012	41.473	0.2	300.1	-1.0	5.2994	1.3
15.	1.998	41.748	0.3	303.9	-0.7	5.2474	1.6
16.	1.985	41.998	0.3	307.6	-0.2	5.1954	1.6
17.	1.971	42.248	0.3	311.1	0.2	5.1473	1.8
18.	1.957	42.497	+ .3	314.5	0.8	5.0953	1.9
19.	1.942	42.722	- .3	317.7	1.4	5.0473	2.3

Table 6. Continued

$P_{\text{bar}}$	T	$\rho_1$	%	$C_2$	%diff	$S_2$	%
	K	$\frac{\text{moles}}{\text{liter}}$		$\frac{\text{meters}}{\text{second}}$		$\frac{\text{joules}}{\text{mole}}$	
20.	1.928	42.947	0.3	320.7	1.9	4.9992	2.5
21.	1.913	43.172	0.4	323.6	2.5	4.9512	2.2
22.	1.897	43.397	.04	326.3	3.0	4.9032	2.5
23.	1.882	43.622	0.4	328.8	3.5	4.8552	2.3
24.	1.866	43.846	0.4	331.1	3.8	4.8071	2.4
25.	1.850	44.046	0.4	333.3	4.0	4.7591	2.6

When using the equation of state, the best accuracy will result when the input variables are pressure and temperature rather than density and temperature. This is true of all equations of state in the liquid region and is a consequence of the nature of the liquid surface rather than the inadequacies of a mathematical model.

Figures 1 through 9 illustrate a wide variation in the ability of the model to reproduce some of the properties. Below 1.2 K the deviations in general between calculated and input properties are noticeably less than those above 1.2 K. The thermodynamic consistency of the data seems to be better at the lower temperatures, and Brooks and Donnelly [3] estimate the accuracy of the data is better at the lower temperatures. At the beginning of the project, Maynard [7] was chosen as the primary source of data above 1.2 K. This decision was based on preliminary analysis of the data themselves and on the claims of accuracy made by the author. During the course of the investigation it became apparent that the Brooks and Donnelly [3] data are the more internally consistent of the two sets and these data were therefore chosen to be used in the fit over the entire

temperature range. The problem with the Maynard [7] data seems to be with the density. Fitting the Maynard data by itself using a simultaneous data fitting technique (which is necessary for this thermodynamic surface) where the PVT and the entropy are used in the least squares estimation of the adjustable parameters, produces a reasonably good fit to those two properties. Comparisons to the other thermodynamic properties such as  $C_p$  and velocity of sound with corresponding calculated values indicate large systematic deviations. Further these deviation patterns are the same when comparing with the Brooks and Donnelly data except for PVT, where there is a definite difference of about 0.3 percent between Maynard and Brooks and Donnelly. Performing the same estimation procedure as before (i.e., using PVT and entropy as input) but using the Brooks and Donnelly data instead of the Maynard data produces a better overall fit to both sets of data except again for the PVT of Maynard. This strongly suggests that the Maynard PVT are inconsistent with the true thermodynamic surfaces. Quite a lot of experimentation using various different combinations of the data as input to the least squares estimation procedure indicated several other points. First it is impossible to produce an equation of state for Helium-II which is adequate to calculate all the thermodynamic properties using only PVT data as input to the estimation procedure. This is true because the  $(\partial P/\partial T)_\rho$  and  $(\partial^2 P/\partial T^2)_\rho$  are so small that several orders of magnitude of precision in the PVT would be required to determine them from the PVT alone. Second using more than two types of data, for example, PVT, entropy and  $C_v$  simultaneously, improved the  $C_v$  representation (but only marginally) over the case where only PVT and entropy were used, however, the slight improvement of  $C_v$  degraded the entropy representation noticeably. Below 0.8 kelvin the entropy may be fit independently of the PVT using eq (5) and then simply added to eq (4) without any noticeable effect on the representation of pressure.



An indication of the accuracy of the properties calculated from the equation of state may be found by consulting the deviation plots, figs. 1 through 9.

## 9. Property Tables

The thermodynamic properties for He-II given here in tables 7 and 8, with one exception, have been calculated using the computer programs given in the appendix. The properties of the saturated vapor phase in tables 7 and 8 have been calculated using the equation of state from McCarty [8]. To achieve continuity between this work and the enthalpy of the author's 1973 work, a value of 59.869851 joules/mole must be added to the enthalpy of the 1973 work. No adjustment to the entropy is required.

The tables and equations given here are based on the 1958 helium vapor pressure scale and the temperature adjustment made in the author's previous helium work (1973) has not been made here.

The molecular weight of helium is taken to be 4.0026 and the value of R used is .0831434 liter•bar/mole•kelvin.

Table 7. Properties of Coexisting Liquid and Vapor.

PRESSURE BAR	TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE·K	CP	SOUND SPEED METERS/SECOND
.4707E-01	2.15	.2777E+00	.1021E+03	.4846E+02	.1278E+02	.2239E+02	83.7
.4707E-01	2.15	.3647E+02	.1023E+02	.5656E+01	.3317E+02	.3329E+02	238.0
.4133E-01	2.10	.2486E+00	.1013E+03	.4910E+02	.1276E+02	.2227E+02	82.9
.4133E-01	2.10	.3643E+02	.8903E+01	.4959E+01	.2657E+02	.2664E+02	238.5
.3605E-01	2.05	.2212E+00	.1005E+03	.4977E+02	.1274E+02	.2214E+02	82.1
.3605E-01	2.05	.3640E+02	.7664E+01	.4369E+01	.2361E+02	.2365E+02	238.7
.3123E-01	2.00	.1957E+00	.9962E+02	.5050E+02	.1272E+02	.2202E+02	81.3
.3123E-01	2.00	.3638E+02	.6518E+01	.3804E+01	.2076E+02	.2080E+02	238.5
.2687E-01	1.95	.1720E+00	.9875E+02	.5126E+02	.1270E+02	.2191E+02	80.4
.2687E-01	1.95	.3635E+02	.5525E+01	.3305E+01	.1805E+02	.1808E+02	238.1
.2295E-01	1.90	.1502E+00	.9787E+02	.5207E+02	.1268E+02	.2180E+02	79.6
.2295E-01	1.90	.3633E+02	.4706E+01	.2887E+01	.1545E+02	.1547E+02	237.6
.1945E-01	1.85	.1303E+00	.9698E+02	.5293E+02	.1266E+02	.2169E+02	78.7
.1945E-01	1.85	.3631E+02	.3985E+01	.2507E+01	.1333E+02	.1335E+02	237.1
.1635E-01	1.80	.1122E+00	.9608E+02	.5384E+02	.1264E+02	.2159E+02	77.8
.1635E-01	1.80	.3630E+02	.3354E+01	.2166E+01	.1171E+02	.1172E+02	236.5
.1363E-01	1.75	.9591E-01	.9517E+02	.5480E+02	.1262E+02	.2150E+02	76.8
.1363E-01	1.75	.3628E+02	.2803E+01	.1860E+01	.1018E+02	.1019E+02	236.0
.1126E-01	1.70	.8131E-01	.9424E+02	.5582E+02	.1260E+02	.2141E+02	75.8
.1126E-01	1.70	.3627E+02	.2318E+01	.1582E+01	.8745E+01	.8753E+01	235.6
.9205E-02	1.65	.6833E-01	.9331E+02	.5690E+02	.1258E+02	.2133E+02	74.8
.9205E-02	1.65	.3626E+02	.1910E+01	.1342E+01	.7395E+01	.7401E+01	235.2
.7449E-02	1.60	.5688E-01	.9236E+02	.5805E+02	.1257E+02	.2126E+02	73.8
.7449E-02	1.60	.3625E+02	.1572E+01	.1137E+01	.6200E+01	.6204E+01	234.9
.5958E-02	1.55	.4686E-01	.9140E+02	.5927E+02	.1255E+02	.2119E+02	72.7
.5958E-02	1.55	.3624E+02	.1290E+01	.9605E+00	.5266E+01	.5268E+01	234.7
.4707E-02	1.50	.3818E-01	.9044E+02	.6057E+02	.1254E+02	.2112E+02	71.6
.4707E-02	1.50	.3623E+02	.1046E+01	.8028E+00	.4532E+01	.4534E+01	234.6
.3667E-02	1.45	.3071E-01	.8946E+02	.6196E+02	.1252E+02	.2107E+02	70.5
.3667E-02	1.45	.3623E+02	.8313E+00	.6591E+00	.3879E+01	.3880E+01	234.5
.2815E-02	1.40	.2438E-01	.8848E+02	.6345E+02	.1251E+02	.2102E+02	69.3
.2815E-02	1.40	.3622E+02	.6483E+00	.5320E+00	.3236E+01	.3236E+01	234.6
.2125E-02	1.35	.1906E-01	.8748E+02	.6505E+02	.1250E+02	.2097E+02	68.1
.2125E-02	1.35	.3622E+02	.5017E+00	.4270E+00	.2629E+01	.2629E+01	234.7
.1576E-02	1.30	.1466E-01	.8648E+02	.6676E+02	.1249E+02	.2093E+02	66.9
.1576E-02	1.30	.3622E+02	.3860E+00	.3411E+00	.2105E+01	.2105E+01	234.8
.1145E-02	1.25	.1106E-01	.8548E+02	.6862E+02	.1248E+02	.2090E+02	65.6
.1145E-02	1.25	.3622E+02	.2916E+00	.2677E+00	.1685E+01	.1685E+01	235.0
.8134E-03	1.20	.8180E-02	.8447E+02	.7062E+02	.1248E+02	.2087E+02	64.3
.8134E-03	1.20	.3622E+02	.2160E+00	.2069E+00	.1340E+01	.1341E+01	235.2
.5635E-03	1.15	.5908E-02	.8345E+02	.7280E+02	.1247E+02	.2085E+02	63.0
.5635E-03	1.15	.3624E+02	.1570E+00	.1572E+00	.1043E+01	.1043E+01	232.0
.3794E-03	1.10	.4156E-02	.8243E+02	.7517E+02	.1247E+02	.2083E+02	61.7
.3794E-03	1.10	.3624E+02	.1113E+00	.1170E+00	.7931E+00	.7932E+00	232.3
.2474E-03	1.05	.2837E-02	.8140E+02	.7777E+02	.1247E+02	.2081E+02	60.3
.2474E-03	1.05	.3624E+02	.7737E-01	.8588E-01	.5864E+00	.5864E+00	232.6
.1554E-03	1.00	.1871E-02	.8037E+02	.8062E+02	.1247E+02	.2080E+02	58.8
.1554E-03	1.00	.3624E+02	.5289E-01	.6228E-01	.4204E+00	.4205E+00	232.8
.9358E-04	.95	.1185E-02	.7934E+02	.8378E+02	.1247E+02	.2079E+02	57.3
.9358E-04	.95	.3624E+02	.3578E-01	.4493E-01	.2802E+00	.2802E+00	233.0
.5365E-04	.90	.7173E-03	.7830E+02	.8728E+02	.1247E+02	.2079E+02	55.8
.5365E-04	.90	.3624E+02	.2401E-01	.3234E-01	.1905E+00	.1905E+00	233.2
.2905E-04	.85	.4111E-03	.7727E+02	.9120E+02	.1247E+02	.2079E+02	54.2
.2905E-04	.85	.3624E+02	.1611E-01	.2341E-01	.1271E+00	.1271E+00	233.2
.1526E-04	.80	.2294E-03	.7623E+02	.9529E+02	.2079E+02	.2910E+02	48.2
.1526E-04	.80	.3624E+02	.1085E-01	.1709E-01	.8428E-01	.8431E-01	233.3
.7145E-05	.75	.1146E-03	.7519E+02	.1003E+03	.2079E+02	.2910E+02	46.7
.7145E-05	.75	.3628E+02	.7223E-02	.1260E-01	.5521E-01	.5523E-01	240.3
.3038E-05	.70	.5220E-04	.7415E+02	.1059E+03	.2079E+02	.2910E+02	45.1
.3038E-05	.70	.3628E+02	.4973E-02	.9495E-02	.3659E-01	.3660E-01	240.3
.1148E-05	.65	.2125E-04	.7311E+02	.1125E+03	.2079E+02	.2910E+02	43.5

Table 7. (Continued)

.1148E-05	.65	.3628E+02	.3467E-02	.7249E-02	.2501E-01	.2502E-01	240.3
.3750E-06	.60	.7517E-05	.7207E+02	.1201E+03	.2079E+02	.2910E+02	41.8
.3750E-06	.60	.3628E+02	.2428E-02	.5569E-02	.1764E-01	.1765E-01	240.3
.1018E-06	.55	.2227E-05	.7103E+02	.1292E+03	.2079E+02	.2910E+02	40.0
.1018E-06	.55	.3628E+02	.1688E-02	.4260E-02	.1283E-01	.1284E-01	240.3
.2180E-07	.50	.5243E-06	.6999E+02	.1400E+03	.2079E+02	.2910E+02	38.1
.2180E-07	.50	.3628E+02	.1151E-02	.3210E-02	.9454E-02	.9456E-02	240.3
.3405E-08	.45	.9101E-07	.6896E+02	.1532E+03	.2079E+02	.2910E+02	36.2
.3405E-08	.45	.3628E+02	.7607E-03	.2357E-02	.6898E-02	.6899E-02	240.3
.3463E-09	.40	.1041E-07	.6792E+02	.1698E+03	.2079E+02	.2910E+02	34.1
.3463E-09	.40	.3628E+02	.4840E-03	.1671E-02	.4892E-02	.4893E-02	240.3
.1917E-10	.35	.6588E-09	.6688E+02	.1911E+03	.2079E+02	.2910E+02	31.9
.1917E-10	.35	.3628E+02	.2955E-03	.1130E-02	.3319E-02	.3319E-02	240.3
.4293E-12	.30	.1721E-10	.6584E+02	.2195E+03	.2079E+02	.2910E+02	29.5
.4293E-12	.30	.3628E+02	.1748E-03	.7190E-03	.2118E-02	.2118E-02	240.3
.2287E-14	.25	.1100E-12	.6480E+02	.2592E+03	.2079E+02	.2910E+02	27.0
.2287E-14	.25	.3628E+02	.1039E-03	.4202E-03	.1243E-02	.1243E-02	240.3
.1009E-17	.20	.6066E-16	.6376E+02	.3188E+03	.2079E+02	.2910E+02	24.1
.1009E-17	.20	.3628E+02	.6696E-04	.2170E-03	.6446E-03	.6447E-03	240.3
.3181E-23	.15	.2551E-21	.6272E+02	.4181E+03	.2079E+02	.2910E+02	28.9
.3181E-23	.15	.3628E+02	.5082E-04	.9239E-04	.2751E-03	.2751E-03	240.3
.4839E-34	.10	.5821E-32	.6168E+02	.6168E+03	.2079E+02	.2910E+02	17.1
.4839E-34	.10	.3628E+02	.4464E-04	.2757E-04	.8244E-04	.8244E-04	240.3



Table 8. Properties of Superfluid Helium.

## 1. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE·K	CP	SOUND SPEED METERS/SECOND
2.16	.36916E+02	.13784E+02	.60442E+01	.47138E+02	.47350E+02	233.09
2.00	.36804E+02	.92334E+01	.38383E+01	.20947E+02	.20987E+02	237.85
1.95	.36781E+02	.82447E+01	.33344E+01	.18197E+02	.18227E+02	238.65
1.90	.36762E+02	.74296E+01	.29137E+01	.15573E+02	.15596E+02	239.27
1.85	.36744E+02	.67125E+01	.25300E+01	.13443E+02	.13460E+02	239.76
1.80	.36729E+02	.60850E+01	.21863E+01	.11807E+02	.11821E+02	240.14
1.75	.36716E+02	.55369E+01	.18776E+01	.10270E+02	.10280E+02	240.45
1.70	.36705E+02	.50546E+01	.15973E+01	.88219E+01	.88297E+01	240.71
1.65	.36695E+02	.46488E+01	.13554E+01	.74637E+01	.74696E+01	240.94
1.60	.36686E+02	.43126E+01	.11484E+01	.62609E+01	.62652E+01	241.14
1.55	.36679E+02	.40315E+01	.97026E+00	.53194E+01	.53225E+01	241.33
1.50	.36673E+02	.37884E+01	.81092E+00	.45794E+01	.45816E+01	241.51
1.45	.36668E+02	.35743E+01	.66568E+00	.39210E+01	.39225E+01	241.70
1.40	.36663E+02	.33916E+01	.53716E+00	.32723E+01	.32733E+01	241.88
1.35	.36660E+02	.32452E+01	.43093E+00	.26610E+01	.26616E+01	242.08
1.30	.36658E+02	.31295E+01	.34387E+00	.21325E+01	.21328E+01	242.27
1.25	.36656E+02	.30350E+01	.26952E+00	.17083E+01	.17084E+01	242.47
1.20	.36654E+02	.29593E+01	.20776E+00	.13608E+01	.13608E+01	242.67
1.15	.36684E+02	.28983E+01	.15756E+00	.10610E+01	.10610E+01	241.44
1.10	.36684E+02	.28524E+01	.11676E+00	.80340E+00	.80340E+00	241.67
1.05	.36684E+02	.28185E+01	.85279E-01	.59168E+00	.59168E+00	241.90
1.00	.36684E+02	.27941E+01	.61507E-01	.42254E+00	.42254E+00	242.10
.95	.36684E+02	.27771E+01	.44109E-01	.28050E+00	.28050E+00	242.25
.90	.36684E+02	.27654E+01	.31526E-01	.19003E+00	.19004E+00	242.36
.85	.36685E+02	.27576E+01	.22621E-01	.12659E+00	.12660E+00	242.43
.80	.36685E+02	.27524E+01	.16325E-01	.84241E-01	.84254E-01	242.49
.75	.36696E+02	.27473E+01	.11933E-01	.54126E-01	.54138E-01	247.74
.70	.36697E+02	.27451E+01	.89057E-02	.35443E-01	.35453E-01	247.77
.65	.36697E+02	.27437E+01	.67429E-02	.23909E-01	.23916E-01	247.78
.60	.36697E+02	.27427E+01	.51472E-02	.16650E-01	.16655E-01	247.79
.55	.36697E+02	.27420E+01	.39187E-02	.11987E-01	.11990E-01	247.80
.50	.36698E+02	.27415E+01	.29415E-02	.87629E-02	.87650E-02	247.80
.45	.36698E+02	.27411E+01	.21536E-02	.63590E-02	.63602E-02	247.80
.40	.36698E+02	.27409E+01	.15224E-02	.44903E-02	.44910E-02	247.80
.35	.36698E+02	.27407E+01	.10272E-02	.30354E-02	.30357E-02	247.79
.30	.36698E+02	.27406E+01	.65160E-03	.19305E-02	.19307E-02	247.79
.25	.36698E+02	.27405E+01	.37975E-03	.11285E-02	.11285E-02	247.79
.20	.36698E+02	.27405E+01	.19550E-03	.58332E-03	.58333E-03	247.79
.15	.36698E+02	.27405E+01	.83009E-04	.24795E-03	.24795E-03	247.79
.10	.36698E+02	.27405E+01	.24711E-04	.73982E-04	.73982E-04	247.79

Table 8. (Continued)

## 2. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY Joules/mole	ENTROPY	CV Joules/mole·K	CP	SOUND SPEED METERS/SECOND
2.15	.37369E+02	.16013E+02	.58774E+01	.38667E+02	.38902E+02	232.63
2.00	.37244E+02	.12013E+02	.38774E+01	.21156E+02	.21215E+02	239.83
1.95	.37216E+02	.11013E+02	.33681E+01	.18363E+02	.18406E+02	241.47
1.90	.37193E+02	.10189E+02	.29429E+01	.15711E+02	.15743E+02	242.87
1.85	.37172E+02	.94651E+01	.25553E+01	.13559E+02	.13584E+02	244.06
1.80	.37155E+02	.88316E+01	.22083E+01	.11908E+02	.11927E+02	245.06
1.75	.37139E+02	.82784E+01	.18968E+01	.10358E+02	.10372E+02	245.91
1.70	.37126E+02	.77918E+01	.16140E+01	.88984E+01	.89091E+01	246.63
1.65	.37115E+02	.73821E+01	.13698E+01	.75308E+01	.75388E+01	247.23
1.60	.37105E+02	.70426E+01	.11608E+01	.63198E+01	.63257E+01	247.75
1.55	.37096E+02	.67587E+01	.98082E+00	.53711E+01	.53754E+01	248.18
1.50	.37089E+02	.65130E+01	.81983E+00	.46248E+01	.46279E+01	248.55
1.45	.37083E+02	.62968E+01	.67311E+00	.39608E+01	.39630E+01	248.88
1.40	.37078E+02	.61122E+01	.54325E+00	.33071E+01	.33087E+01	249.16
1.35	.37074E+02	.59642E+01	.43581E+00	.26914E+01	.26924E+01	249.41
1.30	.37071E+02	.58470E+01	.34767E+00	.21590E+01	.21596E+01	249.64
1.25	.37068E+02	.57512E+01	.27233E+00	.17313E+01	.17316E+01	249.86
1.20	.37066E+02	.56745E+01	.20969E+00	.13807E+01	.13809E+01	250.06
1.15	.37098E+02	.56106E+01	.15914E+00	.10747E+01	.10748E+01	250.04
1.10	.37097E+02	.55641E+01	.11778E+00	.81470E+00	.81477E+00	250.23
1.05	.37096E+02	.55297E+01	.85827E-01	.60070E+00	.60073E+00	250.40
1.00	.37096E+02	.55049E+01	.61667E-01	.42940E+00	.42941E+00	250.55
.95	.37095E+02	.54876E+01	.43970E-01	.28536E+00	.28536E+00	250.67
.90	.37095E+02	.54758E+01	.31174E-01	.19312E+00	.19312E+00	250.76
.85	.37096E+02	.54678E+01	.22137E-01	.12819E+00	.12820E+00	250.82
.80	.37096E+02	.54626E+01	.15780E-01	.84734E-01	.84741E-01	250.87
.75	.37092E+02	.54573E+01	.11396E-01	.53810E-01	.53818E-01	254.82
.70	.37092E+02	.54551E+01	.84063E-02	.34735E-01	.34743E-01	254.85
.65	.37093E+02	.54537E+01	.63028E-02	.23055E-01	.23061E-01	254.87
.60	.37093E+02	.54528E+01	.47759E-02	.15803E-01	.15807E-01	254.88
.55	.37093E+02	.54521E+01	.36174E-02	.11232E-01	.11235E-01	254.88
.50	.37093E+02	.54516E+01	.27060E-02	.81390E-02	.81407E-02	254.89
.45	.37093E+02	.54513E+01	.19764E-02	.58736E-02	.58746E-02	254.88
.40	.37093E+02	.54511E+01	.13945E-02	.41334E-02	.41339E-02	254.88
.35	.37093E+02	.54509E+01	.93933E-03	.27877E-02	.27879E-02	254.88
.30	.37094E+02	.54508E+01	.59476E-03	.17695E-02	.17696E-02	254.88
.25	.37094E+02	.54508E+01	.34588E-03	.10321E-02	.10321E-02	254.88
.20	.37094E+02	.54507E+01	.17760E-03	.53195E-03	.53195E-03	254.88
.15	.37094E+02	.54507E+01	.75221E-04	.22524E-03	.22524E-03	254.88
.10	.37094E+02	.54507E+01	.22363E-04	.66931E-04	.66931E-04	254.87

Table 8. (Continued)

## 4. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE·K	CP	SOUND SPEED METERS/SECOND
2.13	.38244E+02	.20861E+02	.56718E+01	.31043E+02	.31451E+02	241.72
2.00	.38085E+02	.17533E+02	.39826E+01	.21684E+02	.21826E+02	249.79
1.95	.38043E+02	.16502E+02	.34577E+01	.18786E+02	.18890E+02	252.08
1.90	.38007E+02	.15654E+02	.30199E+01	.16064E+02	.16142E+02	254.06
1.85	.37977E+02	.14910E+02	.26217E+01	.13860E+02	.13919E+02	255.78
1.80	.37950E+02	.14260E+02	.22656E+01	.12168E+02	.12211E+02	257.25
1.75	.37928E+02	.13693E+02	.19462E+01	.10583E+02	.10615E+02	258.52
1.70	.37908E+02	.13194E+02	.16565E+01	.90943E+01	.91183E+01	259.61
1.65	.37892E+02	.12774E+02	.14063E+01	.77010E+01	.77187E+01	260.54
1.60	.37878E+02	.12426E+02	.11920E+01	.64674E+01	.64802E+01	261.32
1.55	.37866E+02	.12135E+02	.10074E+01	.54986E+01	.55079E+01	261.96
1.50	.37855E+02	.11883E+02	.84228E+00	.47345E+01	.47412E+01	262.49
1.45	.37847E+02	.11662E+02	.69196E+00	.40548E+01	.40595E+01	262.93
1.40	.37840E+02	.11473E+02	.55895E+00	.33874E+01	.33907E+01	263.30
1.35	.37834E+02	.11321E+02	.44874E+00	.27596E+01	.27618E+01	263.60
1.30	.37829E+02	.11200E+02	.35817E+00	.22167E+01	.22183E+01	263.86
1.25	.37825E+02	.11102E+02	.28071E+00	.17801E+01	.17811E+01	264.08
1.20	.37822E+02	.11023E+02	.21621E+00	.14219E+01	.14226E+01	264.28
1.15	.37850E+02	.10953E+02	.16471E+00	.11025E+01	.11030E+01	265.31
1.10	.37848E+02	.10905E+02	.12212E+00	.84151E+00	.84181E+00	265.44
1.05	.37846E+02	.10869E+02	.88954E-01	.62520E+00	.62536E+00	265.54
1.00	.37845E+02	.10844E+02	.63674E-01	.45046E+00	.45054E+00	265.63
.95	.37844E+02	.10825E+02	.45002E-01	.30216E+00	.30219E+00	265.70
.90	.37844E+02	.10813E+02	.31424E-01	.20520E+00	.20520E+00	265.75
.85	.37844E+02	.10804E+02	.21836E-01	.13549E+00	.13549E+00	265.80
.80	.37844E+02	.10799E+02	.15175E-01	.87621E-01	.87622E-01	265.83
.75	.37823E+02	.10795E+02	.10650E-01	.54844E-01	.54847E-01	268.03
.70	.37823E+02	.10793E+02	.76436E-02	.34351E-01	.34356E-01	268.06
.65	.37823E+02	.10792E+02	.55978E-02	.21989E-01	.21993E-01	268.08
.60	.37824E+02	.10791E+02	.41674E-02	.14521E-01	.14524E-01	268.09
.55	.37824E+02	.10790E+02	.31197E-02	.10002E-01	.10004E-01	268.10
.50	.37824E+02	.10790E+02	.23173E-02	.70922E-02	.70934E-02	268.10
.45	.37824E+02	.10790E+02	.16859E-02	.50551E-02	.50558E-02	268.10
.40	.37824E+02	.10789E+02	.11869E-02	.35363E-02	.35366E-02	268.10
.35	.37824E+02	.10789E+02	.79811E-03	.23792E-02	.23794E-02	268.10
.30	.37824E+02	.10789E+02	.50433E-03	.15084E-02	.15084E-02	268.10
.25	.37824E+02	.10789E+02	.29242E-03	.87810E-03	.87813E-03	268.09
.20	.37824E+02	.10789E+02	.14950E-03	.45076E-03	.45076E-03	268.09
.15	.37824E+02	.10789E+02	.63056E-04	.18951E-03	.18952E-03	268.09
.10	.37824E+02	.10789E+02	.18749E-04	.55869E-04	.55869E-04	268.09



Table 8. (Continued)

## 3. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE·K	CP	SOUND SPEED METERS/SECOND
2.14	.37816E+02	.18384E+02	.57551E+01	.33558E+02	.33863E+02	235.85
2.00	.37673E+02	.14777E+02	.39249E+01	.21398E+02	.21491E+02	244.02
1.95	.37638E+02	.13764E+02	.34086E+01	.18556E+02	.18625E+02	246.13
1.90	.37609E+02	.12929E+02	.29778E+01	.15872E+02	.15923E+02	247.96
1.85	.37584E+02	.12196E+02	.25855E+01	.13696E+02	.13735E+02	249.53
1.80	.37562E+02	.11555E+02	.22344E+01	.12026E+02	.12055E+02	250.87
1.75	.37543E+02	.10996E+02	.19193E+01	.10460E+02	.10482E+02	252.02
1.70	.37527E+02	.10504E+02	.16335E+01	.89874E+01	.90035E+01	253.00
1.65	.37513E+02	.10090E+02	.13866E+01	.76082E+01	.76202E+01	253.82
1.60	.37501E+02	.97463E+01	.11752E+01	.63871E+01	.63959E+01	254.52
1.55	.37491E+02	.94590E+01	.99305E+00	.54295E+01	.54359E+01	255.10
1.50	.37483E+02	.92105E+01	.83020E+00	.46754E+01	.46800E+01	255.57
1.45	.37476E+02	.89918E+01	.68182E+00	.40044E+01	.40077E+01	255.97
1.40	.37469E+02	.88052E+01	.55049E+00	.33448E+01	.33470E+01	256.31
1.35	.37464E+02	.86553E+01	.44175E+00	.27237E+01	.27252E+01	256.60
1.30	.37460E+02	.85367E+01	.35246E+00	.21866E+01	.21877E+01	256.85
1.25	.37457E+02	.84396E+01	.27610E+00	.17549E+01	.17556E+01	257.07
1.20	.37454E+02	.83616E+01	.21255E+00	.14010E+01	.14014E+01	257.27
1.15	.37485E+02	.82949E+01	.16160E+00	.10879E+01	.10882E+01	257.96
1.10	.37484E+02	.82477E+01	.11965E+00	.82735E+00	.82753E+00	258.11
1.05	.37482E+02	.82127E+01	.87133E-01	.61211E+00	.61221E+00	258.25
1.00	.37481E+02	.81873E+01	.62455E-01	.43907E+00	.43911E+00	258.36
.95	.37481E+02	.81696E+01	.44313E-01	.29294E+00	.29295E+00	258.45
.90	.37481E+02	.81574E+01	.31168E-01	.19845E+00	.19845E+00	258.52
.85	.37481E+02	.81493E+01	.21892E-01	.13131E+00	.13131E+00	258.57
.80	.37481E+02	.81439E+01	.15409E-01	.85866E-01	.85869E-01	258.61
.75	.37467E+02	.81393E+01	.10974E-01	.54094E-01	.54099E-01	261.57
.70	.37467E+02	.81371E+01	.79888E-02	.34393E-01	.34399E-01	261.60
.65	.37467E+02	.81358E+01	.59231E-02	.22425E-01	.22429E-01	261.62
.60	.37467E+02	.81348E+01	.44509E-02	.15097E-01	.15101E-01	261.63
.55	.37468E+02	.81342E+01	.33525E-02	.10572E-01	.10575E-01	261.64
.50	.37468E+02	.81338E+01	.24992E-02	.75831E-02	.75846E-02	261.64
.45	.37468E+02	.81336E+01	.18217E-02	.54402E-02	.54410E-02	261.64
.40	.37468E+02	.81333E+01	.12837E-02	.38169E-02	.38173E-02	261.64
.35	.37468E+02	.81332E+01	.86373E-03	.25704E-02	.25706E-02	261.64
.30	.37468E+02	.81331E+01	.54622E-03	.16300E-02	.16301E-02	261.64
.25	.37468E+02	.81331E+01	.31713E-03	.94952E-03	.94955E-03	261.63
.20	.37468E+02	.81330E+01	.16247E-03	.48830E-03	.48830E-03	261.63
.15	.37468E+02	.81330E+01	.68659E-04	.20601E-03	.20601E-03	261.63
.10	.37468E+02	.81330E+01	.20406E-04	.60969E-04	.60969E-04	261.63

Table 8. (Continued)

## 6. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE·K	CP	SOUND SPEED METERS/SECOND
2.11	.39023E+02	.25953E+02	.55803E+01	.30491E+02	.31156E+02	256.93
2.00	.38852E+02	.23021E+02	.41277E+01	.22415E+02	.22691E+02	263.59
1.95	.38795E+02	.21947E+02	.35806E+01	.19368E+02	.19568E+02	265.80
1.90	.38747E+02	.21065E+02	.31249E+01	.16547E+02	.16695E+02	267.68
1.85	.38706E+02	.20292E+02	.27119E+01	.14272E+02	.14382E+02	269.30
1.80	.38671E+02	.19619E+02	.23431E+01	.12523E+02	.12605E+02	270.69
1.75	.38641E+02	.19032E+02	.20126E+01	.10892E+02	.10953E+02	271.90
1.70	.38616E+02	.18517E+02	.17134E+01	.93634E+01	.94081E+01	272.96
1.65	.38594E+02	.18084E+02	.14548E+01	.79346E+01	.79672E+01	273.88
1.60	.38575E+02	.17724E+02	.12332E+01	.66690E+01	.66926E+01	274.66
1.55	.38560E+02	.17422E+02	.10422E+01	.56714E+01	.56882E+01	275.31
1.50	.38546E+02	.17162E+02	.87155E+00	.48813E+01	.48932E+01	275.85
1.45	.38535E+02	.16933E+02	.71644E+00	.41783E+01	.41866E+01	276.29
1.40	.38526E+02	.16738E+02	.57931E+00	.34903E+01	.34961E+01	276.66
1.35	.38519E+02	.16582E+02	.46560E+00	.28444E+01	.28484E+01	276.96
1.30	.38512E+02	.16457E+02	.37206E+00	.22861E+01	.22888E+01	277.21
1.25	.38507E+02	.16356E+02	.29210E+00	.18363E+01	.18381E+01	277.42
1.20	.38503E+02	.16274E+02	.22550E+00	.14673E+01	.14685E+01	277.60
1.15	.38526E+02	.16199E+02	.17237E+00	.11386E+01	.11394E+01	278.66
1.10	.38523E+02	.16149E+02	.12822E+00	.87465E+00	.87517E+00	278.76
1.05	.38521E+02	.16112E+02	.93583E-01	.65498E+00	.65528E+00	278.83
1.00	.38519E+02	.16084E+02	.66941E-01	.47623E+00	.47639E+00	278.88
.95	.38518E+02	.16065E+02	.47062E-01	.32323E+00	.32330E+00	278.92
.90	.38517E+02	.16051E+02	.32484E-01	.22106E+00	.22108E+00	278.95
.85	.38517E+02	.16042E+02	.22147E-01	.14588E+00	.14589E+00	278.98
.80	.38517E+02	.16036E+02	.15021E-01	.92703E-01	.92703E-01	279.00
.75	.38488E+02	.16036E+02	.10239E-01	.57341E-01	.57342E-01	280.18
.70	.38488E+02	.16034E+02	.71325E-02	.34953E-01	.34955E-01	280.20
.65	.38488E+02	.16033E+02	.50842E-02	.21590E-01	.21593E-01	280.22
.60	.38488E+02	.16032E+02	.37060E-02	.13698E-01	.13700E-01	280.23
.55	.38488E+02	.16031E+02	.27352E-02	.90977E-02	.90990E-02	280.24
.50	.38489E+02	.16031E+02	.20154E-02	.62831E-02	.62840E-02	280.24
.45	.38489E+02	.16031E+02	.14607E-02	.44117E-02	.44121E-02	280.24
.40	.38489E+02	.16030E+02	.10269E-02	.30667E-02	.30669E-02	280.24
.35	.38489E+02	.16030E+02	.69015E-03	.20607E-02	.20609E-02	280.24
.30	.38489E+02	.16030E+02	.43574E-03	.13072E-02	.13072E-02	280.24
.25	.38489E+02	.16030E+02	.25216E-03	.76087E-03	.76088E-03	280.24
.20	.38489E+02	.16030E+02	.12846E-03	.38950E-03	.38950E-03	280.24
.15	.38489E+02	.16030E+02	.54003E-04	.16272E-03	.16272E-03	280.24
.10	.38489E+02	.16030E+02	.16096E-04	.47624E-04	.47624E-04	280.24

Table 8. (Continued)

## 10. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE·K	CP	SOUND SPEED METERS/SECOND
2.06	.40313E+02	.35772E+02	.54414E+01	.31007E+02	.32182E+02	285.22
2.00	.40180E+02	.33914E+02	.45147E+01	.24677E+02	.25332E+02	289.49
1.95	.40099E+02	.32711E+02	.39028E+01	.21072E+02	.21518E+02	291.74
1.90	.40032E+02	.31734E+02	.33976E+01	.17896E+02	.18212E+02	293.47
1.85	.39976E+02	.30887E+02	.29448E+01	.15381E+02	.15611E+02	294.80
1.80	.39928E+02	.30153E+02	.25430E+01	.13460E+02	.13630E+02	295.87
1.75	.39887E+02	.29517E+02	.21843E+01	.11698E+02	.11824E+02	296.79
1.70	.39852E+02	.28959E+02	.18605E+01	.10062E+02	.10155E+02	297.60
1.65	.39821E+02	.28489E+02	.15803E+01	.85417E+01	.86103E+01	298.32
1.60	.39796E+02	.28098E+02	.13398E+01	.71950E+01	.72449E+01	298.94
1.55	.39774E+02	.27771E+02	.11321E+01	.61238E+01	.61596E+01	299.47
1.50	.39756E+02	.27488E+02	.94696E+00	.52664E+01	.52917E+01	299.92
1.45	.39741E+02	.27241E+02	.77930E+00	.45019E+01	.45195E+01	300.30
1.40	.39728E+02	.27031E+02	.63143E+00	.37583E+01	.37704E+01	300.63
1.35	.39717E+02	.26862E+02	.50863E+00	.30629E+01	.30712E+01	300.91
1.30	.39709E+02	.26727E+02	.40751E+00	.24612E+01	.24668E+01	301.14
1.25	.39702E+02	.26618E+02	.32130E+00	.19745E+01	.19783E+01	301.32
1.20	.39696E+02	.26530E+02	.24963E+00	.15748E+01	.15774E+01	301.47
1.15	.39713E+02	.26443E+02	.19165E+00	.12396E+01	.12413E+01	301.53
1.10	.39710E+02	.26388E+02	.14334E+00	.95978E+00	.96077E+00	301.60
1.05	.39707E+02	.26347E+02	.10506E+00	.72620E+00	.72676E+00	301.65
1.00	.39705E+02	.26316E+02	.75246E-01	.53481E+00	.53510E+00	301.68
.95	.39703E+02	.26294E+02	.52667E-01	.36996E+00	.37010E+00	301.69
.90	.39702E+02	.26279E+02	.35872E-01	.25644E+00	.25650E+00	301.70
.85	.39701E+02	.26268E+02	.23827E-01	.17036E+00	.17038E+00	301.71
.80	.39701E+02	.26261E+02	.15537E-01	.10686E+00	.10687E+00	301.72
.75	.39667E+02	.26269E+02	.10067E-01	.64699E-01	.64699E-01	301.97
.70	.39667E+02	.26267E+02	.66195E-02	.37923E-01	.37923E-01	301.98
.65	.39667E+02	.26265E+02	.44538E-02	.22084E-01	.22084E-01	301.99
.60	.39667E+02	.26264E+02	.30912E-02	.13000E-01	.13001E-01	302.01
.55	.39667E+02	.26264E+02	.22035E-02	.79908E-02	.79914E-02	302.01
.50	.39667E+02	.26264E+02	.15908E-02	.51870E-02	.51874E-02	302.02
.45	.39667E+02	.26263E+02	.11421E-02	.35083E-02	.35085E-02	302.02
.40	.39667E+02	.26263E+02	.80050E-03	.24003E-02	.24004E-02	302.02
.35	.39667E+02	.26263E+02	.53772E-03	.16083E-02	.16083E-02	302.02
.30	.39667E+02	.26263E+02	.33920E-03	.10219E-02	.10219E-02	302.02
.25	.39667E+02	.26263E+02	.19574E-03	.59501E-03	.59502E-03	302.02
.20	.39667E+02	.26263E+02	.99195E-04	.30328E-03	.30328E-03	302.02
.15	.39667E+02	.26263E+02	.41529E-04	.12544E-03	.12544E-03	302.01
.10	.39667E+02	.26263E+02	.12451E-04	.36373E-04	.36373E-04	302.01



Table 8. (Continued)

## 12. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE * K	CP	SOUND SPEED METERS/SECOND
2.04	.40865E+02	.40506E+02	.53391E+01	.30103E+02	.31499E+02	295.59
2.00	.40775E+02	.39333E+02	.47542E+01	.26324E+02	.27274E+02	298.68
1.95	.40679E+02	.38040E+02	.40959E+01	.22242E+02	.22856E+02	301.60
1.90	.40603E+02	.36998E+02	.35578E+01	.18770E+02	.19191E+02	303.71
1.85	.40539E+02	.36105E+02	.30802E+01	.16064E+02	.16364E+02	305.26
1.80	.40486E+02	.35335E+02	.26585E+01	.14016E+02	.14235E+02	306.42
1.75	.40440E+02	.34669E+02	.22834E+01	.12164E+02	.12326E+02	307.39
1.70	.40401E+02	.34088E+02	.19456E+01	.10461E+02	.10581E+02	308.21
1.65	.40368E+02	.33597E+02	.16532E+01	.88876E+01	.89759E+01	308.93
1.60	.40340E+02	.33189E+02	.14018E+01	.74954E+01	.75599E+01	309.56
1.55	.40315E+02	.32846E+02	.11847E+01	.63837E+01	.64303E+01	310.08
1.50	.40295E+02	.32551E+02	.99118E+00	.54894E+01	.55224E+01	310.52
1.45	.40278E+02	.32293E+02	.81621E+00	.46908E+01	.47140E+01	310.90
1.40	.40264E+02	.32074E+02	.66204E+00	.39158E+01	.39318E+01	311.23
1.35	.40252E+02	.31897E+02	.53388E+00	.31920E+01	.32029E+01	311.52
1.30	.40242E+02	.31757E+02	.42827E+00	.25650E+01	.25724E+01	311.75
1.25	.40234E+02	.31643E+02	.33837E+00	.20563E+01	.20612E+01	311.94
1.20	.40228E+02	.31551E+02	.26371E+00	.16380E+01	.16414E+01	312.09
1.15	.40246E+02	.31458E+02	.20286E+00	.13019E+01	.13040E+01	311.59
1.10	.40241E+02	.31401E+02	.15202E+00	.10110E+01	.10123E+01	311.67
1.05	.40238E+02	.31357E+02	.11158E+00	.76749E+00	.76821E+00	311.72
1.00	.40236E+02	.31325E+02	.79964E-01	.56738E+00	.56776E+00	311.75
.95	.40234E+02	.31301E+02	.55912E-01	.39499E+00	.39517E+00	311.76
.90	.40233E+02	.31284E+02	.37943E-01	.27500E+00	.27508E+00	311.77
.85	.40232E+02	.31273E+02	.25001E-01	.18334E+00	.18337E+00	311.78
.80	.40232E+02	.31266E+02	.16075E-01	.11502E+00	.11502E+00	311.79
.75	.40197E+02	.31277E+02	.10202E-01	.69128E-01	.69129E-01	311.86
.70	.40197E+02	.31275E+02	.65370E-02	.40004E-01	.40004E-01	311.87
.65	.40197E+02	.31273E+02	.42744E-02	.22772E-01	.22772E-01	311.88
.60	.40197E+02	.31272E+02	.28894E-02	.12974E-01	.12975E-01	311.89
.55	.40197E+02	.31272E+02	.20189E-02	.76778E-02	.76782E-02	311.90
.50	.40197E+02	.31272E+02	.14400E-02	.48196E-02	.48199E-02	311.90
.45	.40197E+02	.31271E+02	.10278E-02	.31905E-02	.31907E-02	311.90
.40	.40198E+02	.31271E+02	.71893E-03	.21624E-02	.21625E-02	311.90
.35	.40198E+02	.31271E+02	.48263E-03	.14457E-02	.14457E-02	311.90
.30	.40198E+02	.31271E+02	.30426E-03	.91863E-03	.91865E-03	311.90
.25	.40198E+02	.31271E+02	.17537E-03	.53456E-03	.53457E-03	311.90
.20	.40198E+02	.31271E+02	.88722E-04	.27188E-03	.27188E-03	311.90
.15	.40198E+02	.31271E+02	.37127E-04	.11214E-03	.11214E-03	311.90
.10	.40198E+02	.31271E+02	.11148E-04	.32511E-04	.32511E-04	311.90

Table 8. (Continued)

## 14. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE·K	CP	SOUND SPEED METERS/SECOND
2.01	.41378E+02	.45148E+02	.52267E+01	.29756E+02	.31369E+02	303.18
2.00	.41346E+02	.44765E+02	.50345E+01	.28446E+02	.29861E+02	304.46
1.95	.41232E+02	.43349E+02	.43147E+01	.23691E+02	.24551E+02	308.74
1.90	.41143E+02	.42229E+02	.37353E+01	.19811E+02	.20375E+02	311.71
1.85	.41071E+02	.41280E+02	.32279E+01	.16849E+02	.17239E+02	313.81
1.80	.41011E+02	.40468E+02	.27835E+01	.14634E+02	.14913E+02	315.34
1.75	.40961E+02	.39770E+02	.23902E+01	.12670E+02	.12874E+02	316.55
1.70	.40918E+02	.39162E+02	.20371E+01	.10888E+02	.11038E+02	317.55
1.65	.40881E+02	.38650E+02	.17316E+01	.92541E+01	.93647E+01	318.40
1.60	.40850E+02	.38223E+02	.14688E+01	.78131E+01	.78941E+01	319.11
1.55	.40823E+02	.37864E+02	.12415E+01	.66592E+01	.67179E+01	319.70
1.50	.40801E+02	.37555E+02	.10391E+01	.57268E+01	.57688E+01	320.19
1.45	.40782E+02	.37286E+02	.85631E+00	.48932E+01	.49228E+01	320.60
1.40	.40766E+02	.37057E+02	.69534E+00	.40859E+01	.41063E+01	320.96
1.35	.40753E+02	.36872E+02	.56136E+00	.33323E+01	.33463E+01	321.28
1.30	.40742E+02	.36725E+02	.45085E+00	.26786E+01	.26880E+01	321.54
1.25	.40733E+02	.36606E+02	.35689E+00	.21463E+01	.21527E+01	321.74
1.20	.40726E+02	.36510E+02	.27891E+00	.17080E+01	.17123E+01	321.91
1.15	.40746E+02	.36410E+02	.21505E+00	.13700E+01	.13727E+01	320.98
1.10	.40741E+02	.36350E+02	.16143E+00	.10672E+01	.10689E+01	321.06
1.05	.40738E+02	.36304E+02	.11862E+00	.81266E+00	.81358E+00	321.12
1.00	.40735E+02	.36269E+02	.85046E-01	.60252E+00	.60300E+00	321.16
.95	.40733E+02	.36244E+02	.59418E-01	.42146E+00	.42169E+00	321.18
.90	.40732E+02	.36227E+02	.40219E-01	.29426E+00	.29436E+00	321.19
.85	.40731E+02	.36214E+02	.26352E-01	.19672E+00	.19676E+00	321.20
.80	.40731E+02	.36207E+02	.16766E-01	.12363E+00	.12364E+00	321.21
.75	.40696E+02	.36222E+02	.10452E-01	.74012E-01	.74013E-01	321.19
.70	.40696E+02	.36219E+02	.65420E-02	.42438E-01	.42438E-01	321.20
.65	.40696E+02	.36218E+02	.41606E-02	.23703E-01	.23703E-01	321.21
.60	.40696E+02	.36217E+02	.27370E-02	.13113E-01	.13113E-01	321.22
.55	.40696E+02	.36216E+02	.18718E-02	.74792E-02	.74795E-02	321.23
.50	.40696E+02	.36216E+02	.13173E-02	.45354E-02	.45356E-02	321.23
.45	.40696E+02	.36216E+02	.93420E-03	.29341E-02	.29342E-02	321.24
.40	.40696E+02	.36216E+02	.65185E-03	.19684E-02	.19685E-02	321.24
.35	.40696E+02	.36216E+02	.43719E-03	.13126E-02	.13126E-02	321.23
.30	.40696E+02	.36216E+02	.27537E-03	.83345E-03	.83346E-03	321.23
.25	.40696E+02	.36216E+02	.15855E-03	.48426E-03	.48426E-03	321.23
.20	.40696E+02	.36216E+02	.80156E-04	.24575E-03	.24575E-03	321.23
.15	.40696E+02	.36216E+02	.33575E-04	.10131E-03	.10131E-03	321.23
.10	.40696E+02	.36216E+02	.10076E-04	.29496E-04	.29496E-04	321.23

Table 8. (Continued)

## 16. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE·K	CP	SOUND SPEED METERS/SECOND
1.98	.41864E+02	.49718E+02	.51062E+01	.29285E+02	.31137E+02	308.48
1.00	.41207E+02	.41156E+02	.90552E-01	.64049E+00	.64110E+00	330.02
.95	.41205E+02	.41130E+02	.63229E-01	.44974E+00	.45004E+00	330.05
.90	.41203E+02	.41111E+02	.42724E-01	.31454E+00	.31467E+00	330.07
.85	.41202E+02	.41098E+02	.27886E-01	.21069E+00	.21074E+00	330.09
.80	.41202E+02	.41089E+02	.17608E-01	.13277E+00	.13278E+00	330.10
.75	.41167E+02	.41109E+02	.10815E-01	.79423E-01	.79427E-01	330.05
.70	.41167E+02	.41105E+02	.66270E-02	.45261E-01	.45262E-01	330.06
.65	.41167E+02	.41104E+02	.41032E-02	.24887E-01	.24887E-01	330.07
.60	.41167E+02	.41103E+02	.26250E-02	.13408E-01	.13408E-01	330.08
.55	.41167E+02	.41102E+02	.17543E-02	.73795E-02	.73797E-02	330.08
.50	.41167E+02	.41102E+02	.12165E-02	.43181E-02	.43183E-02	330.09
.45	.41167E+02	.41102E+02	.85646E-03	.27254E-02	.27255E-02	330.09
.40	.41167E+02	.41102E+02	.59592E-03	.18082E-02	.18083E-02	330.09
.35	.41167E+02	.41102E+02	.39917E-03	.12020E-02	.12020E-02	330.09
.30	.41167E+02	.41102E+02	.25114E-03	.76212E-03	.76213E-03	330.09
.25	.41167E+02	.41102E+02	.14449E-03	.44171E-03	.44171E-03	330.09
.20	.41167E+02	.41102E+02	.73081E-04	.22367E-03	.22367E-03	330.09
.15	.41167E+02	.41102E+02	.30697E-04	.92414E-04	.92414E-04	330.09
.10	.41167E+02	.41102E+02	.91828E-05	.27170E-04	.27170E-04	330.09



Table 8. (Continued)

## 18. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE·K	CP	SOUND SPEED METERS/SECOND
1.96	.42330E+02	.54239E+02	.49873E+01	.28661E+02	.30808E+02	312.02
1.00	.41654E+02	.45990E+02	.96570E-01	.68247E+00	.68226E+00	338.43
.95	.41652E+02	.45961E+02	.67417E-01	.48020E+00	.48058E+00	338.48
.90	.41651E+02	.45941E+02	.45510E-01	.33624E+00	.33641E+00	338.51
.85	.41650E+02	.45927E+02	.29635E-01	.22558E+00	.22565E+00	338.53
.80	.41649E+02	.45918E+02	.18616E-01	.14262E+00	.14265E+00	338.55
.75	.41614E+02	.45941E+02	.11297E-01	.85490E-01	.85496E-01	338.48
.70	.41614E+02	.45938E+02	.67927E-02	.48552E-01	.48553E-01	338.49
.65	.41614E+02	.45936E+02	.40989E-02	.26361E-01	.26361E-01	338.50
.60	.41614E+02	.45935E+02	.25484E-02	.13869E-01	.13869E-01	338.51
.55	.41614E+02	.45934E+02	.16612E-02	.73748E-02	.73749E-02	338.52
.50	.41614E+02	.45934E+02	.11330E-02	.41587E-02	.41588E-02	338.52
.45	.41614E+02	.45934E+02	.79109E-03	.25550E-02	.25550E-02	338.52
.40	.41614E+02	.45934E+02	.54860E-03	.16743E-02	.16744E-02	338.52
.35	.41614E+02	.45934E+02	.36688E-03	.11089E-02	.11089E-02	338.52
.30	.41614E+02	.45934E+02	.23053E-03	.70140E-03	.70141E-03	338.52
.25	.41614E+02	.45934E+02	.13258E-03	.40507E-03	.40507E-03	338.52
.20	.41614E+02	.45934E+02	.67188E-04	.20470E-03	.20470E-03	338.52
.15	.41614E+02	.45934E+02	.28361E-04	.85065E-04	.85065E-04	338.52
.10	.41614E+02	.45934E+02	.84294E-05	.25414E-04	.25414E-04	338.52

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## APPENDIX A. Computer Program Listings

```

FUNCTION HE2(D,TT)
C MAIN PROPERTIES ROUTINE, FOR THE THERMODYNAMIC PROPERTIES
C OF HELIUM II.
C INPUT IS DENSITY IN MOLES/LITER, AND TEMPERATURE IN KELVIN
C OUTPUT IS PRESSURE, DPDT, DPDD, ENTROPY, ENTHALPY
C OR CV. OUTPUT UNITS ARE ATMOSPHERES, MOLES/LITER-KELVIN
C AND JOULES.
C
DIMENSION G(8),P(69),PR(23)
COMMON/DATA 2/PR,R,F
COMMON/DATA 3/G,RR
DATA(NR=0),(IH=0)
1 IF(NR.GT.0)GO TO 2
CALL DATA HE2
N0=47
2 T=TT
IF(T.LT..1)T=.1
IF(T.GT.1.199)GO TO 3
N0=24
IF(T.GT..799)GO TO 4
N0=1
GO TO 4
3 N0=47
4 IF(NR.NE.N0)GO TO 5
GO TO 7
5 NR=N0
NF=NR+22
J=0
DO 6 I=NR,NF
J=J+1
6 PR(J)=P(I)
7 CONTINUE
GO TO (11,12,13,14,15,16),M
ENTRY PHE2
M=1
GO TO 1
ENTRY DPDHE2
M=2
GO TO 1
ENTRY OPTHE2
M=3
GO TO 1
ENTRY SHE2
M=4
GO TO 1
ENTRY HHE2
IH=1
M=5
GO TO 1
ENTRY CVHE2
M=6
GO TO 1
11 CALL PRESS2(P1,D,T)
IF(NR.LT.24)GO TO 111
CALL PRESS3(P3,D,T)
HE2=P1+P3
IF(IH.EQ.1)GO TO 17
RETURN
111 HE2=P00(D)+P1
IF(IH.EQ.1)GO TO 17
RETURN
12 CALL DPDD2(P1,D,T)
IF(NR.LT.24)GO TO 112
CALL DPDD3(P3,D,T)

```

```

HE2=P1+P3
RETURN
112 HE2=P1+DP00(D)
RETURN
13 CALL DPDT2(P1,D,T)
IF(NR.LT.24)GO TO 113
CALL DPDT3(P3,D,T)
HE2=P1+P3
RETURN
113 HE2=P1
IF(TT.LT..1)HE2=HE2*TT*10.
RETURN
14 DS=SATL2(T)
CALL DSON2(P1S,DS,T)
CALL DSON2(P1,D,T)
IF(NR.LT.24)GO TO 114
CALL DSON3(P3S,DS,T)
CALL DSON3(P3,D,T)
HE2=(P1-P1S+P3-P3S+DVPHE2(T)*(1./D-1./DS))*101.325+S0(T)
RETURN
114 HE2=S0(T)+(P1-P1S)*101.325
IF(TT.LT..1)HE2=HE2*TT*10.
RETURN
15 DS=SATL2(T)
GO TO 11
17 CONTINUE
CALL DSON2(P1S,DS,T)
CALL PID2(P2S,DS,T)
CALL DSON2(P1,D,T)
CALL PID2(P2,D,T)
IF(NR.LT.24)GO TO 115
CALL DSON3(P3S,DS,T)
CALL PIC3(P4S,DS,T)
CALL DSON3(P3,D,T)
CALL PID3(P4,D,T)
HE2=((P1-P1S+P3-P3S+DVPHE2(T)*(1./D-1./DS))*T+HE2/D
1+P2-P2S+P4-P4S)*101.325+H0(T)
IH=0
RETURN
115 HE2=H0(T)+((P1-P1S)*T+(P2-P2S))*101.325+(P00H(D)+HE2/D)*101.325
HE2=HE2-P00H(D)*101.325
IH=0
RETURN
16 DS=SATL2(T)
CALL DSOT2(P1S,DS,T)
CALL DSOT2(P1,D,T)
IF(NR.LT.24)GO TO 116
CALL DSOT3(P2S,DS,T)
CALL DSOT3(P2,D,T)
HE2=(P1-P1S+P2-P2S)*101.325+C0(T)+CVVP(D,T)
RETURN
116 HE2=(P1-P1S)*101.325+C0(T)
IF(TT.LT..1)HE2=HE2*TT*10.
99 CONTINUE
RETURN
END
SUBROUTINE DATA HE2
C SETS UP THE CONSTANTS FOR THE EQUATION OF STATE
DIMENSION G(8),P(69),PR(23)
COMMON/DATA 2/PR,R,P
COMMON/DATA 3/G,RR
DATA(A0=2.241456)
DATA(A1=.1757482)
DATA(A2=.00470035)

```

R=.08205616

RR=R

P( 1) = -.776003592103E-04  
P( 2) = .516985343553E-04  
P( 3) = -.185460414352E-05  
P( 4) = .993150555179E-06  
P( 5) = -.372729528003E-05  
P( 6) = .905240314118E-04  
P( 7) = -.263138088468E-03  
P( 8) = .210133644446E-03  
P( 9) = -.251675888508E-06  
P(10) = .246805662352E-05  
P(11) = -.235766906295E-04  
P(12) = .636877273619E-04  
P(13) = -.464000281660E-04  
P(14) = .133504455025E-07  
P(15) = .441252121325E-06  
P(16) = -.390205136440E-05  
P(17) = .569946840678E-05  
P(18) = -.219762939629E-05  
P(19) = -.581270462264E-09  
P(20) = -.191711245461E-07  
P(21) = .144897551106E-06  
P(22) = -.793219612515E-07  
P(23) = -.390940913081E-07  
P(24) = .108660418499E-02  
P(25) = -.217871751436E-02  
P(26) = .102911648479E-02  
P(27) = .189253572751E-02  
P(28) = -.674364748289E-02  
P(29) = .798926642309E-02  
P(30) = -.344107467055E-02  
P(31) = .299781633163E-03  
P(32) = -.214084674667E-03  
P(33) = .335439600940E-03  
P(34) = .559092792724E-03  
P(35) = -.119903558078E-02  
P(36) = .526331681180E-03  
P(37) = .118775501632E-04  
P(38) = -.459408808154E-04  
P(39) = .519701003921E-04  
P(40) = -.196070771338E-04  
P(41) = .731453369826E-06  
P(42) = -.526985760908E-06  
P(43) = .169561251135E-05  
P(44) = -.131795348291E-05  
P(45) = -.714287537326E-07  
P(46) = .258759130915E-06  
P(47) = -.299775895293E-03  
P(48) = .261528116001E-03  
P(49) = -.451073420829E-04  
P(50) = -.179926805218E-03  
P(51) = .268760818966E-03  
P(52) = .153832317516E-04  
P(53) = -.162726148595E-03  
P(54) = .477756675722E-04  
P(55) = -.356060361531E-04  
P(56) = .407625370109E-03  
P(57) = -.713769173335E-03  
P(58) = .449456804718E-03  
P(59) = -.913635541095E-04  
P(60) = -.975555037829E-05  
P(61) = .121659779679E-04  
P(62) = -.528306039117E-05



```

P(63)= .311573112016E-06
P(64)= .613299771434E-07
P(65)= .506000325098E-06
P(66)= -.612590386700E-06
P(67)= .230922759488E-06
P(68)= .327499222785E-08
P(69)= -.515534867647E-08
G( 1)= -.320783527549E+01
G( 2)= .580145141306E+01
G( 3)= -.294344361744E+01
G( 4)= .290449403103E-01
G( 5)= .801446582474E-01
G( 6)= -.175703015761E+00
G( 7)= .400129303603E+00
G( 8)= -.255176262894E+00

```

```

RETURN
END

```

```

SUBROUTINE HE2E(P,Q,T)

```

```

C CALCULATE ALL PROPERTIES OF THE F SUB S PART OF THE EQUATION
C OF STATE, UNITS ARE THE SAME AS IN FUNCTION HE2

```

```

DIMENSION G(8),A(8)

```

```

COMMON/DATA3/G,R

```

```

DATA(A0=2.241456)

```

```

DATA(A1=.1757482)

```

```

DATA(A2=.00470035)

```

```

1 N=1

```

```

IE=1

```

```

Q2=Q*Q

```

```

Q3=Q2*Q

```

```

D0=SATL2(T)

```

```

D02=D0*D0

```

```

D03=D02*D0

```

```

DD0=DSATL2(T)

```

```

DD02=DD0*DD0

```

```

D20=D2SATL(T)

```

```

D=Q

```

```

T2=T*T

```

```

T3=T2*T

```

```

T4=T3*T

```

```

T5=T4*T

```

```

T6=T5*T

```

```

D=D-SATL2(T)

```

```

D2=D**2

```

```

D3=D2*D

```

```

D4=D3*D

```

```

GO TO (10,20,30,40,50,60),II

```

```

ENTRY PRESS3

```

```

II=1

```

```

GO TO 1

```

```

10 A(1)=A2*D3*T3

```

```

A(2)=A2*D3*T**2.5

```

```

A(3)=A2*D3*T2

```

```

A(4)=A1*D2*T3

```

```

A(5)=A1*D2*T2

```

```

A(6)=A0*D*T3

```

```

A(7)=A0*D*T**2.5

```

```

A(8)=A0*D*T2

```

```

2 SUM=0

```

```

DO 3 I=1,8

```

```

3 SUM=SUM+A(I)*G(I)

```

```

P=SUM+P0(Q,T)+VPNHE2(T)

```

```

RETURN

```

```

ENTRY DSDN3

```

```

II=2

```



```

GO TO 1
20 A(1)=-A2*(Q2*T2*3/2.-3*Q*(D00*T3+D0*T2*3)+3*(2*D0*D00*T3
1 +3*D02*T2)*ALOG(Q)+(3*C02*D00*T3+D03*T2*3)/Q)
A(2)=-A2*(Q2*T**1.5*2.5/2.-3*Q*(D00*T**2.5+D0*T**1.5*2.5)
1 +3*ALOG(Q)*(2*D00*D00*T**2.5+2.5*D02*T**1.5)
2 +(3.*D02*D00*T**2.5+2.5*D03*T**1.5)/Q)
A(3)=-A2*(Q2*T-3*Q*(D00*T2+D0*T2)+3*ALOG(Q)*(2*D00*D00*T2
1 +2*D02*T)+(3*D02*D00*T2+2*D03*T)/Q)
A(4)=-A1*(3*T2*Q-ALOG(Q)*(6*T2*D0+2*T3*D00)
1 -(3*T2*D02+T3*D00*D0*2)/Q)
A(5)=-A1*(T*Q*2-ALOG(Q)*(4*T*D0+2*T2*D00)
1 -(2*T*D02+T2*D00*D0*2)/Q)
A(6)=-A0*(3*ALOG(Q)*T2+(D00*T3+T2*D0*3)/Q)
A(7)=-A0*(2.5*ALOG(Q)*T**1.5+(D00*T**2.5+2.5*D00*T**1.5)/Q)
A(8)=-A0*(2*ALOG(Q)*T+(D00*T2+T*D0*2)/Q)
SUM=-A0*D00/Q+A1*2*D00*ALOG(Q)+2*A1*D0*D00/Q
1 -6*A2*D00*D00*ALOG(Q)-3*A2*D02*D00/Q+3*A2*D00*Q
DO 22 I=1,8
22 SUM=SUM+A(I)*G(I)
P=SUM
RETURN
ENTRY DSOT3
II=3
GO TO 1
30 A(1)=-A2*(Q2*T*3-3*Q*(C20*T3+D00*T2*6+D0*T*6)+3*(2*D002*T3
1 +2*D00*D02*T3+6*D00*D00*T2+6*D00*D0*T2+6*D02*T)*ALOG(Q)
2 +(3*D00*D002*T3*2+3*D02*D00*T3+9*D02*D00*T2+9*D02*D00*T2
3 +6*D03*T)/Q)*T
A(2)=-A2*(Q2*T**.5*3.75/2.-3*Q*(D20*T**2.5+D00*T**1.5*2.5
1 +D00*T**1.5*2.5+D0*T**.5*3.75)+3*ALOG(Q)*(2*D002*T**2.5
2 +2*D00*D20*T**2.5+5*D00*D00*T**1.5+5*D00*D0*T**1.5+3.75*
3 D02*T**1.5)+(6*D002*D00*T**2.5+3*D02*D20*T**2.5+7.5*D002*
4 D00*T**1.5+7.5*D00*D02*T**1.5+3.75*D03*T**1.5)/Q)*T
A(3)=-A2*(Q2-3*Q*(D20*T2+D00*T*4+D0*2)+3*ALOG(Q)*(2*D002*T2
1 +D0*D20*T2*2+8*D00*D00*T+2*D02)+(3*D02*D20*T2+6*D002*D0*T2
2 +12*D02*D00*T+2*C03)/Q)*T
A(4)=-A1*(6*T*Q-ALOG(Q)*(6*T2*D00+12*T*D0+6*T2*D00+2*T3*D20
1 )-(6*D00*D00*T2+6*T*D02+6*T2*D00*D0+2*T3*D20*D0+2*T3*D002)/Q)*T
A(5)=-A1*(Q*2-ALOG(Q)*(8*T*D00+4*D0+2*T2*D20)-(2*D02
1 +8*T*D00*D00+2*T2*D20*D0+2*T2*D002)/Q)*T
A(6)=-A0*(6*ALOG(Q)*T+(C20*T3+3*D00*T2+6*T*D0+3*T2*D00)/Q)*T
A(7)=-A0*(3.75*ALOG(Q)*T**1.5+(D20*T**2.5+D00*T**1.5*2.5
1 +2.5*D00*T**1.5+3.75*D0*T**1.5)/Q)*T
A(8)=-A0*(2*ALOG(Q)+(D20*T2+2*C00*T+2*D0+2*T*D00)/Q)*T
SUM=-A0*D20/Q+A1*D20*ALOG(Q)*2+2*A1*(D002+D0*C20)/Q
1 -6*A2*ALOG(Q)*(D002+D0*D20)-3*A2*(D20*D02+2*D002*D0)/Q
2 +3*A2*D20*Q
SUM=SUM*T
DO 32 I=1,8
32 SUM=SUM+A(I)*G(I)
P=SUM
RETURN
ENTRY DPOC3
II=4
GO TO 1
40 A(1)=A2*D2*T3*3.
A(2)=A2*D2*3.*T**2.5
A(3)=A2*D2*T2*3.
A(4)=A1*D*T3*2.
A(5)=A1*D*T2*2.
A(6)=A0*T3
A(7)=A0*T**2.5
A(8)=A0*T2
41 SUM=0

```

```

DO 42 I=1,8
42 SUM=SUM+A(I)*G(I)
P=SUM+DP0(Q,T)
RETURN
ENTRY DPDT3
II=5
GO TO 1
50 A(1)=A2*(3*D2*(-D00)*T3+3*T2*D3)
A(2)=A2*(3*D2*(-D00)*T**2.5+2.5*T**1.5*C3)
A(3)=A2*(3*D2*(-D00)*T2+2*T*D3)
A(4)=A1*(2*D*(-D00)*T3 + 3*T2*D2)
A(5)=A1*(2*D*(-D00)*T2 +2*T*D2)
A(6)=A0*((-D00)*T3+3*T2*D )
A(7)=A0*((-D00)*T**2.5+2.5*T**1.5*D)
A(8)=A0*((-D00)*T2+2*T*C)
51 SUM=0
DO 52 I=1,8
52 SUM=SUM+A(I)*G(I)
P=SUM+DVPHE2(T)+DPODT(Q,T)
RETURN
ENTRY PID3
C INTEGRATE P/Q**2 WITH RESPECT TO Q
II=6
GOTO 1
60 QINTGR= .5*Q**2-3*Q*D0+3*ALOG(Q)*D0**2+D0**3/Q
A(1)=A2*T**3.0*QINTGR
A(2)=A2*T**2.5*QINTGR
A(3)=A2*T**2.0*QINTGR
QINTGR=Q-2*ALOG(Q)*D0-D0**2/Q
A(4)=A1*T**3.0*QINTGR
A(5)=A1*T**2.0*QINTGR
QINTGR=ALOG(Q)+D0/Q
A(6)=A0*T**3.0*QINTGR
A(7)=A0*T**2.5*QINTGR
A(8)=A0*T**2.0*QINTGR
SUM=0
DO 62 I=1,8
62 SUM=SUM+A(I)*G(I)
P=SUM-VPNHE2(T)/Q+A0*ALOG(Q)+A1*Q-A1**2*D0*ALOG(Q)
+A0*D0/Q-D0**2*A1/Q+D0**3*A2/Q
1+A2*Q**2/2-A2**3*D0*Q+A2**3*D0**2*ALOG(Q)
RETURN
END
FUNCTION P0(DI,T)
C CALCULATE THE PRESSURE FOR THE P0 PART OF THE EQUATION OF
C STATE,UNITS ARE THE SAME AS IN FUNCTION HE2
DATA(A0=2.241456)
DATA(A1=.1757482)
DATA(A2=.00470035)
X=DI-SATL2(T)
P0=A0*X+A1*X**2+A2*X**3
RETURN
END
FUNCTION DPODT(DI,T)
C CALCULATE DPODT, UNITS ARE THE SAME AS IN FUNCTION HE2
DATA(A0=2.241456)
DATA(A1=.1757482)
DATA(A2=.00470035)
X=DI-SATL2(T)
DXDT=-DSATL2(T)
DPODT=A0*CXDT+A1**2*X*DXCT+A2**3*X**2*DXDT
RETURN
END
FUNCTION DP0(DI,T)

```

```

C   CALCULATE DPOCD, UNITS ARE THE SAME AS IN THE FUNCTION HE2
DATA(A0=2.241456)
DATA(A1=.1757482)
DATA(A2=.00470035)
X=DI-SATL2(T)
DP0=A0+A1*2.*X+A2*3.*X**2
RETURN
END
FUNCTION SATL2(T)
C   CALCULATE THE SATUARATED LIQUID DENSITY IN MOLES/LITER FOR
C   A TEMPERATURE IN KELVIN
X=2.172-T
D=-3.31007-.0074243913*X+.0059164737533*X*ALOG(X)
D=EXP(D)*1000.
SATL2=D
IF(T.LT..8)D=36.27877
RETURN
END
FUNCTION DSATL2(T)
C   1ST DERIVATIVE OF SATL2
X=2.172-T
DSATL2=+.00742434913-.0059164737533*(1.+ALOG(X))
DSATL2=DSATL2*SATL2(T)
RETURN
END
FUNCTION D2SATL(T)
C   2ND DERIVATIVE OF SATL2
X=2.172-T
D=SATL2(T)
D2=.0059164737553*D/X+(.00742434913-.0059164737553*(ALOG(X)+1))
1*DSATL2(T)
D2SATL=D2
RETURN
END
FUNCTION CVVP(D,T)
C   CALCULATES THE CONTRIBUTION OF THE VAPOR PRESSURE TERM TO CV
D0=SATL2(T)
CVVP=DVPHE2(T+.005)-DVPHE2(T-.005)
CVVP=CVVP*T*101.325*(1./D-1./D0)/.01
RETURN
END
SUBROUTINE HE2EG(P,Q,T)
C   CALCULATES THE F SUB S TERM IN THE EQUATION OF STATE
C   UNITS ARE THE SAME AS IN THE FUNCTION HE2
DIMENSION G(23),A(23)
COMMON/DATA 2/G,R
1 N=1
O=Q-36.27877
T2=T*T
T3=T2*T
T4=T3*T
T5=T4*T
T6=T5*T
T7=T6*T
T8=T6*T2
T9=T8*T
T10=T8*T2
T11=T10*T
O2=O*O
O3=O2*O
O4=O3*O
O5=O4*O
O6=O5*O
GO TO (10,20,30,40,50,60),II

```

```

ENTRY PRESS2
II=1
GO TO 1
10 A(1)=-D2*T4/4.
A(2)=-D2*T5/5.
A(3)=-D2*T6/6.
A(4)=-D3*T3*2./3.
A(5)=-D3*T4*2./4.
A(6)=-2.*D3*T5/5.
A(7)=-2.*D3*T6/6.
A(8)=-2.*D3*T7/7.
A(9)=-3.*D4*T3/3.
A(10)=-3.*D4*T4/4.
A(11)=-3.*D4*T5/5.
A(12)=-3.*D4*T6/6.
A(13)=-3.*D4*T7/7.
A(14)=-4.*D5*T3/3.
A(15)=-4.*D5*T5/5.
A(16)=-4.*D5*T7/7.
A(17)=-4.*D5*T9/9.
A(18)=-4.*D5*T11/11.
A(19)=-5.*D6*T3/3.
A(20)=-5.*D6*T5/5.
A(21)=-5.*D6*T7/7.
A(22)=-5.*D6*T9/9.
A(23)=-5.*D6*T11/11.
2 SUM=0
DO 3 I=1,23
3 SUM=SUM+A(I)*G(I)*Q**2/D2
P=SUM
RETURN
ENTRY DSDN2
II=2
GO TO 1
20 A(1)=T3*D
A(2)=D*T4
A(3)=D*T5
A(4)=D2*T2
A(5)=D2*T3
A(6)=D2*T4
A(7)=D2*T5
A(8)=D2*T6
A(9)=D3*T2
A(10)=D3*T3
A(11)=D3*T4
A(12)=D3*T5
A(13)=D3*T6
A(14)=D4*T2
A(15)=D4*T4
A(16)=D4*T6
A(17)=D4*T8
A(18)=D4*T10
A(19)=D5*T2
A(20)=D5*T4
A(21)=D5*T6
A(22)=D5*T8
A(23)=D5*T10
21 SUM=0
DO 22 I=1,23
22 SUM=SUM+A(I)*G(I)
P=SUM
RETURN
ENTRY DSDT2
II=3

```

```

GO TO 1
30 A(1)=3*T2*D
   A(2)=4*T3*D
   A(3)=5*T4*D
   A(4)=2*D2*T
   A(5)=3*T2*D2
   A(6)=4*T3*D2
   A(7)=5*T4*D2
   A(8)=6*T5*D2
   A(9)=2*T*D3
   A(10)=3*T2*D3
   A(11)=4*T3*D3
   A(12)=5*T4*D3
   A(13)=6*T5*D3
   A(14)=2*T*D4
   A(15)=4*T3*D4
   A(16)=6*T5*D4
   A(17)=8*T7*D4
   A(18)=10*T9*D4
   A(19)=2*T*D5
   A(20)=4*T3*D5
   A(21)=6*T5*D5
   A(22)=8*T7*D5
   A(23)=10*T9*D5
31 SUM=0
   DO 32 I=1,23
32 SUM=SUM+A(I)*G(I)
   P=SUM*T
   RETURN
   ENTRY DPOC2
   II=4

```

```

GO TO 1
40 A(1)=-2.*Q*T4/4.
   A(2)=-2.*Q*T5/5.
   A(3)=-2.*Q*T6/6.
   A(4)=- (Q**2+2*Q*D)*2*T3/3.
   A(5)=- (Q**2+2*Q*D)*2*T4/4.
   A(6)=- (Q**2+2*Q*D)*2*T5/5.
   A(7)=- (Q**2+2*Q*D)*2*T6/6.
   A(8)=- (Q**2+2*Q*D)*2*T7/7.
   A(9)=- (Q**2*2*D+2*Q*D2)*3*T3/3.
   A(10)=- (Q**2*2*D+2*Q*D2)*3*T4/4.
   A(11)=- (Q**2*2*D+2*Q*D2)*3*T5/5.
   A(12)=- (Q**2*2*D+2*Q*D2)*3*T6/6.
   A(13)=- (Q**2*2*D+2*Q*D2)*3*T7/7.
   A(14)=- (Q**2*D2*3+2*Q*D3)*4*T3/3.
   A(15)=- (Q**2*D2*3+2*Q*D3)*4*T5/5.
   A(16)=- (Q**2*D2*3+2*Q*D3)*4*T7/7.
   A(17)=- (Q**2*D2*3+2*Q*D3)*4*T9/9.
   A(18)=- (Q**2*D2*3+2*Q*D3)*4*T11/11.
   A(19)=- (Q**2*D3*4+2*Q*D4)*5*T3/3.
   A(20)=- (Q**2*D3*4+2*Q*D4)*5*T5/5.
   A(21)=- (Q**2*D3*4+2*Q*D4)*5*T7/7.
   A(22)=- (Q**2*D3*4+2*Q*D4)*5*T9/9.
   A(23)=- (Q**2*D3*4+2*Q*D4)*5*T11/11.

```

```

41 SUM=0
   DO 42 I=1,23
42 SUM=SUM+A(I)*G(I)
   P=SUM
   RETURN
   ENTRY DPDT2
   II=5
   GO TO 1
50 A(1)=-D2*T3

```



```

A(2)=-D2*T4
A(3)=-D2*T5
A(4)=-2*D3*T2
A(5)=-2*D3*T3
A(6)=-2*D3*T4
A(7)=-2*D3*T5
A(8)=-2*D3*T6
A(9)=-3*D4*T2
A(10)=-3*D4*T3
A(11)=-3*D4*T4
A(12)=-3*D4*T5
A(13)=-3*D4*T6
A(14)=-4*D5*T2
A(15)=-4*D5*T4
A(16)=-4*D5*T6
A(17)=-4*D5*T8
A(18)=-4*D5*T10
A(19)=-5*D6*T2
A(20)=-5*D6*T4
A(21)=-5*D6*T6
A(22)=-5*D6*T8
A(23)=-5*D6*T10
51 SUM=0
   DD 52 I=1,23
52 SUM=SUM+A(I)*G(I)*Q**2/D2
   P=SUM
   RETURN
   ENTRY PID2
   II=6
   GO TO 1
60 A(1)=-D*T4/4
   A(2)=-D*T5/5
   A(3)=-D*T6/6
   A(4)=-D2*T3/3
   A(5)=-D2*T4/4
   A(6)=-D2*T5/5
   A(7)=-D2*T6/6
   A(8)=-D2*T7/7
   A(9)=-D3*T3/3
   A(10)=-D3*T4/4
   A(11)=-D3*T5/5
   A(12)=-D3*T6/6
   A(13)=-D3*T7/7
   A(14)=-D4*T3/3
   A(15)=-D4*T5/5
   A(16)=-D4*T7/7
   A(17)=-D4*T9/9
   A(18)=-D4*T11/11
   A(19)=-D5*T3/3
   A(20)=-D5*T5/5
   A(21)=-D5*T7/7
   A(22)=-D5*T9/9
   A(23)=-D5*T11/11
61 SUM=0
   DD 62 I=1,23
62 SUM=SUM+A(I)*G(I)
   P=SUM
   RETURN
   END
FUNCTION S0(TT)
C   CALCULATES THE 0 PRESSURE VALUE FOR S,H, AND CV
C   UNITS ARE THE SAME AS IN FUNCTION HE2
   DIMENSION S(44),C(44),H(44),T(44)
   DATA(T=0,.1,.15,.2,.25,.3,.35,.4,.45,.5,.55,.6,.65,.7,.75,.8,

```



```

1.85,.9,.95,1.,1.05,1.1,1.15,1.2,1.25,1.3,1.35,1.4,1.45,1.5,
21.55,1.6,1.65,1.7,1.75,1.8,1.85,1.9,1.95,2.,2.05,2.1,2.15,2.172)
DATA(C=0,.20597E-4,.68827E-4,.16143E-3,.31144E-3,.53134E-3,
1.83306E-3,.0012284,.0017327,.0023749,.0032236,.0044293,
2.8062754,.0091734,.01383,.021058,.031759,.047598,.070005,.10504,
3.1465,.19816,.26074,.33523,.42128,.52642,.65752,.80923,
4.97015,1.1334,1.3169,1.5506,1.8494,2.1868,2.5464,
52.9278,3.3332,3.8614,4.5113,5.19,5.9009,6.6428,
68.298466,14.)

```

```

DATA(S=0,.68889E-5,.23112E-4,.54338E-4,.10534E-3,.18041E-3,
1.28386E-3,.41983E-3,.59254E-3,.80709E-3,.10714E-2,.0014005,
2.0018226,.0023865,.0031649,.0042735,.0058517,.0080843,.01123,
3.815562,.021457,.029233,.039275,.051689,.066910,.085256,.10675,
4.13301,.16481,.20076,.24020,.28431,.33558,.39554,.46502,
5.54155,.62676,.72189,.82613,.95098,1.0920,1.2394,1.4137
679,1.559)

```

```

DATA(H=0,.4174E-5,.6261E-5,.1181E-4,.2337E-4,.4412E-4,.7787E-4,
1.1290E-3,.2025E-3,.3046E-3,.4436E-3,.6331E-3,.8974E-3,.1278E-2,
2.1844E-2,.2705E-2,.4008E-2,.5966E-2,.8878E-2,.01311,.01916,
3.02754,.03883,.05344,.0721,.09543,.124,.1602,.2054,.2584,.3186,
4.3882,.4715,.5721,.6917,.8276,.9832,1.161,1.363,1.608,1.891,
52.197,2.525,2.916)

```

```

SQ=ATKINT(TT,S,T,44,6,NES,.01)*4.0026
RETURN

```

```

ENTRY C0

```

```

SQ=ATKINT(TT,C,T,44,6,NES,.01)*4.0026
RETURN

```

```

ENTRY H0

```

```

SQ=ATKINT(TT,H,T,44,6,NES,.01)*4.0026
RETURN

```

```

END

```

```

FUNCTION FINDD2(PI,T)

```

```

C CALCULATES THE DENSITY IN MOLES/LITER FOR AN INPUT
C OF PRESSURE IN ATMOSPHERES AND TEMPERATURE IN KELVIN.

```

```

P=PI

```

```

IF(PI.LT..00001)P=.00001

```

```

D=SATL2(T)

```

```

D=D+(43.-D)*P/25.

```

```

ID=1

```

```

DO 10 I=1,25

```

```

PP=PHE2(D,T)

```

```

IF(ABS(PP-P).LE..00001*P)GO TO 11

```

```

DP=DPDHE2(D,T)

```

```

10 D=D-(PP-P)/DP

```

```

FINDD2=D

```

```

RETURN

```

```

11 FINDD2=0

```

```

RETURN

```

```

END

```

```

FUNCTION PMELT2(T)

```

```

C INPUT IN KELVIN, OUTPUT IN ATMOSPHERES (FROM GRILLY, 1972)

```

```

IF(T.LT.1.464)GO TO 1

```

```

PMELT2=31.168-17.122*T+9.292*T**2

```

```

RETURN

```

```

1 PMELT2=24.996+.0799014422*T-.6729427939*T**2+1.87853695*T**3

```

```

1-2.326509762*T**4+1.061136353*T**5

```

```

RETURN

```

```

END

```

```

FUNCTION FIND TD(CD)

```

```

C CALCULATE THE TEMPERATURE IN K ON THE LAMBDA LINE FOR
C AN INPUT OF DENSITY IN GM/CC (KIERSTEND 1967)

```

```

D=CD

```

```

T=2.172

```

```

DO 10 I=1,20

```

```

DC=DENLAM(T)
IF (ABS(DD-DC).LE..000001*DD)GO TO 11
10 T=T-(DENLAM(T)-DD)/(DELAMP(T))
GO TO 12
11 FIND TD=T
RETURN
12 FIND TD=0
RETURN
END
FUNCTION DENLAM(T)
C CALCULATE THE DENSITY IN GM/CC ON THE LAMBDA LINE FOR A
C TEMPERATURE IN K (KIERSTEND 1967)
D0=0.14841388
D1=-0.150735
D2=-0.3298225
D3=-0.53031333
D4=-0.383035
D5=-0.00226388
D6=36.7348
X=T-2.172
DENLAM=D0+D1*X+D2*X**2+D3*X**3+D4*X**4+D5*EXP(D6*X)
RETURN
END
FUNCTION DELAMP(T)
C CALCULATE THE DERIVATIVE OF DENLAM WITH RESPECT TO TEMPERATURE
D0=0.14841388
D1=-0.150735
D2=-0.3298225
D3=-0.53031333
D4=-0.383035
D5=-0.00226388
D6=36.7348
X=T-2.172
DELAMP=D1+2*D2*X+3*D3*X**2+4*D4*X**3+D5*C6*EXP(D6*X)
RETURN
END
FUNCTION FIND TP(PP)
C CALCULATE THE TEMPERATURE ON THE LAMBDA LINE FOR A GIVEN
C PRESSURE IN ATM (KIERSTEND 1967)
P=PP
T=2.172
DO 10 I=1,20
BC=PRSLAM(T)
IF (ABS(PP-BC).LE..000001*PP) GO TO 11
10 T=T-(PRSLAM(T)-PP)/(PRLAMP(T))
GO TO 12
11 FIND TP=T
RETURN
12 FIND TP=0
RETURN
END
FUNCTION PRSLAM(T)
C CALCULATE THE PRESSURE IN ATMOSPHERES FOR AN INPUT OF
C TEMPERATURE IN KELVIN (KIERSTEND 1967)
B0=0.42800749
B1=-95.0719
B2=-86.417
B3=-103.341
B4=-77.52175
B5=-0.37827065
B6=42.2507
X=T-2.172
P=PRSLAM=B0+B1*X+B2*X**2+B3*X**3+B4*X**4+B5*EXP(B6*X)
RETURN

```

```

END
FUNCTION PRLAMP (T)
C CALCULATE THE DERIVATIVE OF PRSLAM
B0=0.42800749
B1=-95.0719
B2=-86.417
B3=-103.341
B4=-77.52175
B5=-0.37827065
B6=42.2507
X=T-2.172
PFLAMP=B1+2*B2*X+3*B3*X**2+4*B4*X**3+B5*B6*EXP(B6*X)
RETURN
END
FUNCTION P00(DI)
C CALCULATE THE PRESSURE IN ATM AT T=0 FOR AN INPUT OF DENSITY
C IN MOLES/LITER
X=DI-36.27877
P00=2.281877372*X+.16820886*X**2+.005277884968*X**3
RETURN
END
FUNCTION P00H(DI)
C CALCULATE THE CONTRIBUTION OF P00 TO ENTHALPY
DATA(A=36.27877), (B=2.281877372), (C=.16820886), (F=.005277884968)
P00H=B*A*LOG(DI)+B*A/DI+C*DI-2*C*A*ALOG(DI)-C*A**2/DI+F*DI**2/2.
1-3*F*A*DI+3*F*A**2*ALOG(DI)+F*A**3/DI
RETURN
END
FUNCTION DP00(DI)
C CALCULATE THE DERIVATIVE OF P0 WITH RESPECT TO DENSITY
X=DI-36.27877
DP00=2.281877372+.16820886*2.*X+.005277884968*3.*X**2
RETURN
END
FUNCTION ATKINT(X,YMAT,XMAT,NELMTS,NMAX,NESSY,ACRCY)
DIMENSION YMAT(999), XMAT(999),A(21,20)
C FIRST TWO SUCCESSIVE VALUES OF THE XMATRIX THAT STRADDLE THE
C VALUE X WILL BE SOUGHT
JJ1=NELMTS-1
DO 20 I=1,JJ1
DIF1=X-XMAT(I)
DIF2=XMAT(I+1)-X
IF(DIF1)16,15,16
15 ATKINT=YMAT(I)
NESSY =NMAX
RETURN
16 IF(DIF2)18,17,18
17 ATKINT=YMAT(I+1)
NESSY =NMAX
RETURN
18 RATIO=DIF1/ DIF2
IF(RATIO)20,20,19
19 IMID=I
GO TO 32
20 CONTINUE
NESSY=1
ATKINT=0.0
RETURN
32 CONTINUE
C NOTE THAT RATIO IS POSITIVE IF THE TWO POINTS STRADDLE X
C REGARDLESS WHICH IS LARGER
JJJ=IMID
JUP=IMID
JDN=IMID

```

```

      IF (JJJ+NMAX-NELMTS+1) 98,98,102
98  DO 201 J=1,NMAX
      JJJ=IMID+J-1
      A(1,J)=XMAT(JJJ)
201  A(2,J)=YMAT(JJJ)
      GO TO 203
102  DO 41 J=1,NMAX
      JJ=J/2
      JOE=J-2*JJ
C    JOE IS 0 IF J IS EVEN AND 1 IF J IS ODD
      IF (J-1) 33,40,33
33  IF (JON-1) 34,36,34
34  IF (JUP-NELMTS) 35,37,35
35  IF (JOE) 37,36,37
36  JUP=JUP+1
      JJJ=JUP
      GO TO 40
37  JON=JON-1
      JJJ=JON
      GO TO 40
40  A(1,J)=XMAT(JJJ)
      A(2,J)=YMAT(JJJ)
41  CONTINUE
203  NNN=NMAX+1
      DO 6 J=3,NNN
      L=J-1
      DO 5 K=L,NMAX
C    J IS THE COLUMN NUMBER
C    K IS THE ROW NUMBER
      DA(J,K)=(A(J-1,K)-A(J-1,J-2))*(X-A(1,J-2))/(A(1,K)-A(1,J-2))
1    +A(J-1,J-2)
      IF (K-L) 3,2,3
2    IF (ABS((A(J,L)-A(J-1,L-1))/A(J,L))-ACRCY/100.0) 7,7,3
3    CONTINUE
5    CONTINUE
6    CONTINUE
      NESSY=0
      ATKINT=A(NNN,NMAX)
      RETURN
7    NESSY=J-1
      ATKINT=A(J,L)
      RETURN
      END
C    FUNCTION VPHHE2(TT)
C    GIVES A VAPOR PRESSURE FOR HELIUM IN ATMOSPHERES GIVEN A
C    TEMPERATURE IN KELVIN. THE FUNCTION REPRODUCES THE 1968 HELIUM
C    TEMPERATURE SCALE TO .0001 KELVIN
      DIMENSION C(12),D(14)
      DATA(C=-3.9394635287,141.27497598,-1640.7741565,11974.557102,
1-55283.309818,166219.56504,-325212.82840,398843.22750,
2-277718.06992,83395.204183)
      DATA(D=-49.510540356,651.9236417,-3707.5430856,12880.673491,
1-30048.545554,49532.267436,-59337.558548,52311.296025,
2-33950.233134,16028.674003,-5354.1038967,1199.0301906,
3-161.46362959,9.8811553386)
      T=TT
      P=0
      IF (T-2.1720) 10,10,1
1    T=T-DELT(T)
      DO 5 I=1,10
5    P=P+C(I)*T**(2-I)
      VP =EXP(P)/.76E+6
      VPHHE2=VP
      RETURN

```

```

10 CONTINUE
   T=T-DELT(T)
   IF(T.LT..8)GO TO 20
   DO 15 I=1,14
15  P=P+D(I)*T**(2-I)
   VP =EXP(P)/.76E+6
   VPNHE2=VP
   RETURN
20  VPNHE2=VPNLOW(TT)
   RETURN
   END
   FUNCTION DELT(TT)
   T=TT
   DELT=.001+.002*T
   RETURN
   END
   FUNCTION VPTHE2(PP)
C   SOLVES THE VAPOR PRESSURE EQUATION FOR TEMPERATURE GIVEN A PRESSURE
C   THE FLUID IS HELIUM AND THE UNITS ARE ATMOSPHERES AND KELVINS
   P=PP
   IF(P.LT..842105)GO TO 12
   T=5.0
   PCAL=VPNHE2(T)
   GO TO 13
12  PCAL=.049737
   IF(ABS(P-PCAL)-.0000001*PP) 11,11,1
   1  T=2.1720
13  DO 10 I=1,26
   DP=DVPHE2(T)
   DEL=(PCAL-P)/DP
   T=T-DEL
   PCAL=VPNHE2(T)
   IF(ABS(P-PCAL)-.0000001*P) 11,11,2
   2  IF(ABS(DEL)-.0000001*T) 11,11,10
10  CONTINUE
   PRINT 100,T
11  VPTHE2=T
   RETURN
100 FORMAT(* TEMPERATURE ITERATION FAILED AT T=*,E14.7)
   END
   FUNCTION CVPHE2(TT)
C   GIVES THE DERIVATIVE OF THE VAPOR PRESSURE FOR HELIUM GIVEN A
C   TEMPERATURE IN KELVIN9 PRESSURE IS IN ATMOSPHERES
   DIMENSION C(12),D(14)
   DATA(C=-3.9394635287,141.27497598,-1640.7741565,11974.557102,
1-55283.309818,166219.56504,-325212.82840,398843.22750,
2-277718.06992,83395.204183)
   DATA(D=-49.510540356,651.9236417,-3707.5430856,12880.673491,
1 -30048.545554,49532.267436,-59337.558548,52311.296025,
2-33950.233134,16028.674003,-5354.1038967,1199.0301906,
3 -161.46362959,9.8811553386)
   P=0
   IF(TT-2.1720) 10,10,1
   1  T=TT-DELT(TT)
   DO 5 I=1,10
   5  P=P+C(I)*T**(1-I)*(2-I)
   DVPHE2=P*VPNHE2(TT)
   RETURN
10  IF(TT.LT..8)GO TO 20
   T=TT-DELT(TT)
   DO 15 I=1,14
15  P=P+D(I)*T**(1-I)*(2-I)
   DVPHE2=P*VPNHE2(TT)
   RETURN

```



```

20 DVPHE2=DLOW(TT)
   RETURN
   END
   FUNCTION VPNLOW(T)
   R=.08205616
   H=H0(T)/4.0026
   S=S0(T)/4.0026
   GL=H-T*S
   D=.0001
   IF(T.LT..7)D=.0000001
   IF(T.LT..5)D=.1E-8
   IF(T.LT..35)D=1.E-12
   IF(T.LT..3)D=1.E-13
   IF(T.LT..25)D=1.E-16
   IF(T.LT..2)D=1.E-30
   IF(T.LT..15)D=.1E-34
   DO 10 I=1,20
   GV=36.805468+5.193043*(T-4.22-T*ALOG(T/4.22))-T*9.37855
1+25.31479*T*R*ALOG(R*T*D)
   IF(ABS(GL-GV).LE..001*(ABS(GL)))GO TO 11
   P=D*R*T
   DGV=25.31469*R*T/D
10 D=D-(GV-GL)/DGV
   PRINT 100
100 FORMAT(* NO CONVERGENCE*)
11 VPNLOW=D*R*T
   RETURN
   END
   FUNCTION DLOW(T)
   TUP=T+.05*T
   TDN=T-.05*T
   PUP=VPNLOW(TUP)
   PDN=VPNLOW(TDN)
   DLOW=(PUP-PDN)/(TUP-TDN)
   RETURN
   END
   FUNCTION ENTR2(D,T)
   DIMENSION A(20),G(20)
   COMMON/DATA2/G,A,F,N,IO
   IO=1
   CALL DSDN2(S1,D,T)
   DD=SATL2(T)
   CALL DSDN2(S0,DD,T)
   S=S0(T)+(DVPHE2(T)*(1./D-1./DD)+(S1-S0))*101.325
   ENTR2=S
   RETURN
   END
   FUNCTION CPHE2(D,T)
C   CALCULATE THE SPECIFIC HEAT CAPACITY AT CONSTANT PRESSURE
C   UNITS ARE THE SAME AS IN FUNCTION HE2
   CPHE2=CVHE2(D,T)+101.325*(DPHE2(D,T)**2/DPHE2(D,T)/D**2)
   RETURN
   END
   FUNCTION WHE2(D,T)
C   CALCULATE THE VELOCITY OF SOUND FOR AN INPUT OF DENSITY IN
C   MOLES/LITER
   WHE2=((CPHE2(D,T)/CVHE2(D,T))*DPHE2(D,T)*25311.)**.5
   RETURN
   END

```

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15. SUPPLEMENTARY NOTES <input type="checkbox"/> Document describes a computer program; SF-185, FIPS Software Summary, is attached.			

16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

The equation of state of He-II is modeled by an equation of state explicit in pressure as a function of density and temperature. The equation of state is divided into three regions of temperature 0 to .8 K; .8 to 1.2 K and 1.2 to the lambda temperature for which similar functional forms are used with different adjustable parameters. The combined functions are valid over the entire PT range of the superfluid and may be used for all classical thermodynamic properties. Comparisons between calculated and experimental data are presented. A computer program for calculation of thermodynamic properties (PVT, isochoric heat capacity, isobaric heat capacity, internal energy, enthalpy, entropy, and velocity of sound) is included.

17. KEY WORDS (six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)

Computer program; equation of state; deviation plots; helium II; mathematical model; superfluid; thermodynamic properties.

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