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PRECISE TIME SYNCHRONIZATION OF WIDELY SEPARATED CLOCKS

BY ALVIN H. MORGAN

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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SUMMARY

In many fields of science, such as in astronautics, missiles, astronomy, etc., which are being intensely investigated, the needs for new orders of magnitude of timing precisions have grown. For instance, the need has been greatly increased for more precise determination of the time of initiation and termination of certain events, which may occur several hundred miles apart. Or, in the location of bodies in space flight, the precise determination of their transit time at known sites is of major importance.

This paper describes known precise methods of setting a group of widely separated clocks to precisely the same time and keeping them in close agreement indefinitely; most of the proposed methods are now available. An estimate of the accuracies of each method are given. Also, there is some discussion of high frequency radio propagation theory pertinent to two of the methods and a few sets of measurements of the propagation delay time of high frequency signals from WWV to WWVH are given. Several graphs and tables are included to simplify some of the calculations.

Precise Time Synchronization Of Widely Separated Clocks

Alvin H. Morgan

I. INTRODUCTION

The need for higher precision of synchronization of highly accurate clocks, which are widely spaced from each other, has increased in many fields of science such as physics, astronomy, astronautics, etc. Only a few years ago the highest precision required was of the order of 10 milliseconds for most physical experiments. Today, the need for a precision of better than 100 microseconds is fairly common, and in a few instances a precision of 1 to 10 microseconds has been required.

At the present time quartz crystal clocks are the best working time standards for purposes such as those mentioned above. Atomic and molecular resonances are useful as frequency standards but not directly as time standards because instruments utilizing them are not able to run reliably for long periods of time. However, they are invaluable in maintaining the crystal clocks at a highly uniform rate and thus in effect making "atomic-time" available.

The problem we are concerned with here is not how to obtain uniform astronomical or atomic time, but how to precisely set widely spaced clocks to the same time and keep them in very close agreement, indefinitely. Of course, this requires very accurate and reliable clocks and a method of setting and continuously checking them relative to each other.

II. METHODS OF SYNCHRONIZING WIDELY SEPARATED CLOCKS

There are several known methods whereby it is possible to adjust the time of several clocks to a certain degree of agreement even when they are widely separated. However, most of these methods are not able to render the required accuracy when it is necessary to set the clocks to agree to better than one or two hundred microseconds.

One precise method is to transport a very accurate master clock around to each of the slave clocks, and set each one to it, in turn. The master clock would, of necessity, have to be controlled during the experiment by some very stable oscillations in order to achieve the required accuracy. A second method, which requires considerable apparatus, would be to transmit by radio a high resolution timing signal from the master clock to each slave clock and they in turn would immediately retransmit the timing signal back to the master clock. This would enable one to determine very accurately the signal propagation time and the great circle distance to each clock, without prior knowledge of the path lengths. A third method, which is less precise but which involves considerably less apparatus than either of the first two methods, is to transmit high resolution timing signals from the master to each slave clock. It requires knowledge of their exact longitude and latitude and that of the master clock, and the calculation of the great circle distance to each from the master clock. Using this information calculations are made of the delay time of the signal from the master to each slave clock, which is used to determine their corrections. This system may employ either ground wave or sky wave signals. Each is considered separately.

III. TRANSPORTING A MASTER TO EACH SLAVE CLOCK

a. General

This method of synchronizing widely separated clocks is limited now to clocks at rest on or near the surface of the earth. However, when space travel becomes a reality it may be possible to adapt it to use in outer space. The method at present requires considerable apparatus and is cumbersome, time consuming, and expensive. However, it appears to be the most accurate method of setting clocks, especially when really portable long running atomic clocks become available.

An appreciable period of time generally will elapse between the time the master clock is initially calibrated and the time it is used to correct the time of each of the slave clocks. Unless the master clock keeps time over long intervals to a precision better than that desired in the synchronization of the slave clocks, some corrections must be applied to it at the time of the setting of each clock. In order to do this, the initial rate of the master clock must be known to a high precision and it must be constant or predictable during the whole measuring interval, i. e., from the setting of the first to the last slave clock.

b. Clock Errors

The error observed in the master clock in μsec, at the end of a time interval P, in days, in terms of quantities observed at a previous time T_o, is approximated by:

$$\varepsilon_t = \left(\varepsilon_o + \Delta\varepsilon \right) + P\left(r_o + \alpha \frac{P}{2} \right) + \Delta P \qquad (1)$$

where: ε_o = error in clock at $t = T_o$, μsec, fast or slow

r_o = rate of clock at $t = T_o$, μsec/ day.

α = accel. of clock, μsec/day/day.

$\Delta\varepsilon$ = error made by observer in reading clock, μsec.

ΔP = error made in determining P, μsec.

Unless P is a very long interval of time, α may be considered to be zero in really high precision clocks, such as present-day quartz crystal clocks. In case an atomically controlled clock were used as the master clock, both the r_o and α terms would (so far as is known now) be equal to zero, regardless of the length of the time interval.

For quartz clocks, in present day high precision units, the r_o term is of the order of 10^{-10} per day (or less) which, in time increments, is around ± 10 μsec/day. Some special crystal units have been developed which have a relative rate, r_o, of 1 μsec/day, or less. When a high quality master clock is used, in many timing applications a period of from 10 to 30 hours may elapse between the setting of the first and the last slave clocks without the necessity of making any time corrections to the master clock.

c. Time Kept by a Clock

The time shown by a clock, at a time (T), may be expressed in terms of its rate r_o at T_o, acceleration α, change in acceleration β, elapsed time P, and the clock reading A_o at a time T_o, as:

$$T = A_o + P + r_o P + \frac{\alpha P^2}{2} + \frac{\beta P^3}{6} \qquad (2)$$

In high precision quartz crystal clocks, r_o is generally positive because of the aging characteristics of quartz crystals, although it may sometimes be negative; α may be either positive or negative; and, β, which is usually negative, may be neglected in all but very, very long time intervals, such as years.

IV. TWO-WAY TRANSMISSION OF TIME SIGNALS

a. HF Timing Signals

This method of synchronization has two attractive features inherent in it that may, in some instances, outweigh its disadvantages. First, because time signals are transmitted both ways over the given path, the one-way propagation time may be quite accurately determined without prior knowledge of the precise length of the path. Second, from this measured propagation time, it is then possible to calculate the great circle distance with quite high accuracy.

In Appendix C, it is shown that the propagation time, t_p, from the master clock to a given slave clock, is given, in terms of two measurements Δt and δt, thus:

$$t_p = \left(\frac{\Delta t - \delta t}{2} \right) \tag{3}$$

where: δt is the delay time a time pulse undergoes from its reception at the slave clock until its retransmission, and Δt is the delay time, measured at the master clock, between a transmitted pulse and its return to the master clock. From this, the path length is, obviously:

$$d_p = t_p c \tag{4}$$

where: d_p is the apparent path length, at the time of the measurement, from the master to the slave clock, and c is the velocity of light. Also, it may be easily shown, Equation (13), Appendix B, that the great circle distance, d_g, for an n-hop mode, is given in terms of d_p, as:

$$d_g = \left[\frac{d_p}{\left(\frac{\sin(\theta/2)}{\theta/2} \right) \sqrt{(1+\gamma)\left(\frac{\gamma}{2 \sin \theta/2} \right)^2}} \right] \tag{5}$$

where: the symbols are as defined in Appendix B. Obviously, it is not precisely known a priori what the mode of propagation will be at the time of a given measurement. For a given great circle distance and a given ionospheric layer height there is, in general, a definite minimum number of hops possible and a practical maximum number that would furnish a usable signal. In practice, the most consistently received pulse with the least delay time, as observed on an oscilloscope or similar timing device, would be the mode to use in the measurements indicated above. A brief discussion of ionospheric propagation is given below.

V. ONE-WAY TRANSMISSION OF TIME SIGNALS

a. Ground Wave Timing Signals

There are at present some LF radio navigation systems which use sharp pulses and, therefore, have very good time resolution. These systems transmit ground wave as well as sky wave propagated signals, but they can be used to obtain very stable timing signals. By gating to accept the ground wave part of the signal at the receiving end, it is possible to synchronize clocks to a high precision, even when they are separated by distances of several hundred miles.

Figure 1, Section XV, may be used to determine the ground wave propagation delay time. Of course, it is necessary to know, quite precisely, the distances from the master to each slave clock. This may be obtained as shown in Appendix A.

Usually, only one-way propagation of the ground wave signals would be necessary because their path length is very nearly constant. However, if higher accuracies are desired, two-way propagation of the signals would be necessary.

b. Ground Wave Propagation Delay

Although it is relatively easy to calculate the ground wave propagation delay time for a given great circle distance, a chart is included (Fig. 1) to simplify this work.

The distance is given, for convenience, in paralleled scales in miles and kilometers versus propagation delay time.

c. HF Propagation

This method of synchronizing widely separated clocks requires a precise knowledge of the signal path lengths between the master and each slave clock, which may be calculated from other known data. The longitude and latitude at the master and each slave clock must be accurately known along with the average ionospheric layer height at the time of day at which the calculation is to be made. In addition, the mode of the signal being propagated between the master and each slave clock is required.

This method first requires a calculation of the great circle distance between the two points on the earth for which the signal propagation time is desired. Although the calculations are, in principle, rather simple they are quite lengthy, and are best explained by means of an example. This is done in Appendix A. After the great circle distance, d_g, between the master and each slave clock is calculated, the propagation time for a ground wave signal may then be easily determined from Figure 1 or by the well-known relationship:

$$t_g = d_g / c \tag{6}$$

where: d_g is the great circle distance and c is the velocity of the ground wave and may be taken equal to that of light. The next step is to calculate the difference in propagation times between a given mode sky wave signal and a ground wave signal over the path in question. In Appendix B, this is shown to be given by the expression:

$$\Delta t = \left(t_p - t_g \right) = t_g \left(\frac{\sin \theta/2}{\theta/2} \right) \sqrt{\left[(1+\gamma) + \left(\frac{\gamma}{2 \sin \theta/2} \right)^2 \right]} - 1 \tag{7}$$

where: $\theta = \frac{d_g}{4r}$ and $\gamma = h'/r$. This may be further simplified (e. g., Appendix B) to:

$$\Delta t = t_g \left(y \sqrt{h + p^2} - 1 \right) \tag{8}$$

where: $y = \left(\frac{\sin \theta/2}{\theta/2}\right)$, $h = (1+\gamma)$ and $p = \left(\frac{\gamma}{2 \sin \theta/2}\right)$.

Either equation (7) or (8) would normally be evaluated for the given path in terms of various ionospheric layer heights, h', and angles $(\theta/2)$, and the mode of the signal, n. This involves considerable calculations for even one path.

To reduce this labor, a set of curves, Figures 10 through 15, are included in Section XV, from which values of Δt may be taken directly. Each chart has a given fixed layer height, h', and the curves are in terms of the great circle distances, d_g, the modes of propagation, from $n = 1$ to $n = 4$, and the differential propagation time, Δt.

After the differential time, Δt, is determined from the charts in Figs. 10-15, for a given path and mode, it is added to the ground wave propagation time, t_g, to obtain the total propagation time, t_p, for the path.

VI. ACCURACY OF METHODS GIVEN

a. Transporting a Master Clock

The limits of the accuracy of this method lie in two factors: (a) long term stability or constancy of the master clock, and (b) resolution of the system[1, 2] used to compare the master and slave clocks. If the master clock is controlled by atomic or molecular resonances, the error due to the long term clock stability is negligible. In case the master clock is controlled by a quartz oscillator of high quality, the error would be quite small: from 10 to 100 μsec/day is easily attainable.

The resolution of the time comparison system can be made very good: from 0.001 μsec to 0.01 μsec is not too difficult to achieve with good signal-to-noise ratio.

Thus, the overall accuracy of this method would be from 0.001 μsec to 0.1 μsec, depending on the clocks and the time comparison system used. With care and very high quality apparatus and rapid transportation of the master or reference clock, it is conceivable that the accuracy could be extended one to two orders better.

b. Transmission of Time Signals (one- and two-way)

(1) General

The systems of synchronizing clocks by either one- or two-way transmission of timing signals is not as accurate as the one described above, but they are obviously more convenient, especially when the clocks are many thousands of miles apart. Because of the different nature of the problems, the hf and the ground wave timing signals will be considered separately.

(2) HF Timing Signals

(a) One Way Transmissions

Results of measurements, described in Appendix D, indicate that the stability of the HF delay time is dependent on several factors. Among these the following are susceptible to control by the observer: (a) time of day that the measurements are made, (b) method of observing the received timing signals, i. e., whether a zero crossing or a peak of the signal is used, (c) the extent to which the antenna pattern favors the optimum mode of propagation, (d) whether or not the propagation mode with the least delay time is used, (e) whether or not the measurements are related to a precise slave clock at the receiving site so that advantage may be taken of the average (over several days), (f) the actual frequency used, i. e., in general, the higher the frequency (but still below the MUF) the more stable the delay time, and (g) accuracy of knowledge of path lengths involved.

Other factors which may affect the stability of the relative delay times of the signals are: (h) epoch of the sunspot cycle; (i) whether or not the radio path passes near or through one of the auroral zones, (j) whether or not the path is all in darkness or all in daylight at the time of the measurement, (k) signal-to-noise ratio at the receiving site, and (l) number of hops involved in the propagation of the signal.

Ideally, the following should prevail:

	Factor	Ideal Condition
(a)	time of day for observation	near noon or midnight at the center of the path.
(b)	part of signal to measure	zero crossing time of the least delayed pulses.
(c)	directivity of antenna	favorable to mode with the least delay time.
(d)	propagation mode to use	the one with least, consistent delay time.
(e)	measurements	be relative to a very precise slave clock at the receiving site.
(f)	choice of frequency	highest available at which a consistent signal can be obtained at the time of the measurements (see item (a) above).
(g)	accuracy of knowledge of path lengths	as accurate as possible.
(h)	epoch of sunspot cycle	near minimum.
(i)	location of radio path	well away from both auroral zones.
(j)	length of longest ideal path	short enough to be at least 2 hours all in daylight or all in darkness.
(k)	signal-to-noise	as high as possible.
(l)	number of hops of path	equal to next integer greater than the quotient of the length of the great circle path expressed in km divided by 4000.

The accuracy of the synchronization of the slave clock using this method obviously then depends on how near to ideal conditions the actual conditions are. Under conditions approaching the ideal, the accuracy may be better than ± 0.1 millisecond, even for very long paths; however, it will usually be less than this by a factor of from 2 to 5, or more.

At the other extreme, for example if the path passed near or through an auroral zone, or was so long that it was never in all darkness or all daylight, etc., the accuracy might be only about ± 10 milliseconds, or worse.

More definite statements of accuracy are not possible at this time, but it is hoped that further measurements in the future will make this possible.

Further data on this may be obtained by reference to Appendix D.

(b) Two-Way Transmissions

If a transponder, or similar device, is used at the slave clock, it is possible to:

1) determine the apparent length of the path, the long term average of which may be quite precise.

2) increase the knowledge of the accuracy of each measurement over that obtained with one-way transmissions.

The method of calculating the path delay time from the measurement data is given in Fig. 4. From this, and other considerations, it is quite apparent that the actual conditions of the measurements will not need to be as near the ideal, as in one-way transmissions, to produce quite good results. This is also apparent from the data given in Appendix D.

Using this method, and under quite favorable conditions, as defined in paragraph (a) above, an accuracy of time synchronization of slave clocks, at distances approaching half-way around the world, might be about 10 microseconds; however, a more conservative estimate would be from 0.1 to 1 millisecond.

(3) Ground-Wave Timing Signals

(a) One-Way

Because the delay times of ground wave signals are relatively constant, it is possible to achieve. with them, a high accuracy in synchronization of widely separated clocks. Within the distance range wherein the signal-to-noise ratio is sufficiently good, an accuracy of from 0.01 to 0.1 μsec is possible. This requires a narrow-band receiver, gating circuits to accept the ground wave component of the timing pulses, and associated circuitry to permit pulse-locking of a highly precise local oscillator, which drives the clock to be synchronized. It is assumed, of course, that the timing pulses are sufficiently sharp to meet the accuracy quoted.

(b) Two-Way

One distinct advantage of using a transponder, or similar device, with ground wave timing signals is that it is possible to determine the great circle path length between the master and slave clock to better than 10 feet up to distances of several thousand miles by using the results of the measurements. This fact may be of considerable importance in geodetic or similar work and in missile tracking systems.

Of course the above implies: (1) a method of signal gating so as to receive only the ground-wave component of the timing pulses; (2) precise measurements of the signal delay in the transponder; and (3) use of timing signals with very fast rise-time to permit (1) above.

The accuracy of this method of time synchronization of the slave clocks using the latest techniques, appears to be limited only by the signal-to-noise ratio in the receiving systems.

VII. MODES OF PROPAGATION OF HF SIGNALS

a. Introduction

High frequency signals are propagated between two points on the earth's surface by means of one or more reflections from one of the ionospheric layers and the surface of the earth. One reflection from the ionosphere is called a one-hop mode, two reflections, with a single reflection from the earth's surface, is called a two-hop mode, and so forth.

Only two of the reflecting layers will be considered here, the F-2 and E-layers, because other layers (F-1 and Es) may usually be neglected in considering the propagation of hf timing signals over long periods of time. Signals reflected by the F-2 or E-layer are called F-2 and E-layer modes, respectively.

b. Maximum Usable Frequencies (MUF)

In the hf range, where signals are propagated by ionospheric reflection, there is an upper limit of frequency which may be transmitted over a path of a given length. At frequencies above this limit, which is called the "maximum usable frequency" (MUF), the wave is said to "skip," or pass over the receiving end of the path.

c. One-Hop Modes

Over relatively short distances, sky wave signals are propagated by means of a single reflection from an ionosphere layer; either F-2 or E-layer. Remembering that both the earth's surface and the ionosphere are curved, it is quite clear that there will be a limiting distance for a one-hop mode. For frequencies equal to, or lower than, the MUF (explained above) for the path, the limiting distance may range from 3500 to 5000 km (2200 to 3000 mi.) for the F-2 layer mode, depending on the reflecting layer height, and is an approximately constant distance of about 2400 km (1500 mi.) for the E-layer mode. However, it has been found in practice that the average limiting distance for the F-2 layer mode is about 4000 km (2500 mi.), which will be the value used here.

Thus, for distances less than 4000 km (2500 mi.) but greater than 2400 km (1500 mi.), the F-2 layer single-hop mode would be predominant. For shorter distances, either the E- or F-2 layer mode could exist, if the frequency used were equal to, or below, the MUF for that layer.

d. Multihop Modes

For great circle paths longer than about 4000 km, it is obvious that more than one reflection from the ionospheric layer usually must occur. The minimum number of hops is approximately the next integer greater than the quotient of the total great circle distance expressed in kilometers divided by 4000. This mode will be observed during the time the signal frequency is below the MUF for the path.

As an example, assume the path length is 9,000 km (6800 mi.). It is quite obvious that two hops of maximum limit (4000 km each), laid end-to-end, would not cover this distance. Therefore, the simple geometric picture of the minimum mode of propagation would be three-hops, each of 3000 km length, laid end-to-end.

e. Control Points (see Fig. 9)

Over very long paths, the MUF for the signal may be estimated by considering the condition of the reflecting layer at only two reflecting or control points, i.e., one a distance "d" from the transmitter, call it point "a", and the other a distance "d" from the receiver, which will be called point "k". The distance "d" is one-half the limiting distance for a one-hop mode (i.e., one-half of 4000 km), or 2000 km (1250 mi.) for F-2 propagation. The MUF for the path is taken to be the lower of the 4000-km MUF's for the points "a" and "k".

Unfortunately, the MUF for a given path of any length, varies from hour-to-hour, day-to-day and year-to-year. Diurnal variations in the reflecting layers are a consequence of the rotation of the earth, and seasonal variations are associated with the movement of the earth in its orbit around the sun. In addition, there are longer period variations that have been correlated quite well with the sunspot cycle.

Further information on the above may be found in standard treatises in the subject, such as Reference 5, Section XIII.

VIII. VARIATIONS IN PROPAGATION DELAY OF HF SIGNALS

a. Introduction

The propagation delay time of hf timing signals over a given path are not constant from hour-to-hour or day-to-day, but undergo some variations. In many timing problems, such as the ones under discussion, it is necessary either to measure the variations or determine the maximum variations that are likely to occur at the given time over the given path.

b. Factors Affecting the Propagation Delay

This subject can only be very sketchily treated here because of its relatively complex nature. Among the many factors affecting the velocity of propagation of an hf signal, only one will be discussed here. This is the variation in the virtual height of the F-2 ionospheric layer. Besides its quite regular diurnal and seasonal variations, the F-2 layer also undergoes severe changes during magnetic or ionospheric disturbances. The ion density of the F-2 layer usually decreases and then slowly returns to normal over a period of 1 to 3 days. The virtual height ($h'F_2$) suffers rapid changes during such periods, sometimes by a factor of nearly two.

This, of course, will introduce unknown but relatively large variations in the propagation time of the signals and thus seriously affect high precision work. Data taken during such periods are, obviously, useless in the precision timing problems under consideration here, and should be discarded; only data taken during normal conditions should be used.

c. Estimation of Effect of Normal Variations of Virtual Height

The effect of diurnal changes in the virtual height of the F-2 layer may be minimized by: (a) measuring the timing signals at the same time every day, and (b) selecting this time of day so that the whole path, from the master clock to the slave clock, is all in daylight or all in darkness. In other words, neither the sunrise nor sunset period should occur anywhere over the path during the time the measurements are being made. On fairly short paths, for instance, the measuring period should be selected when it is approximately noon at the center of the path.

There remains, however, some small and irregular changes in the F-2 layer height which are unpredictable. To obtain an accurate value for these changes would require some detailed measurements of the ionospheric height versus time at the reflecting points of the given path. This is obviously not a practical undertaking for those engaged in timing problems.

Alternatively, a knowledge of the effect of changes of the F-2 virtual height on the propagation delay time, for a given path, might be useful. To make this information available in an easily usable form, Fig. 5 was devised. This is a plot of virtual height, h', (in km) versus ($\Delta t/n$) (μsec), (where n is the number of hops in the path in question), along with a third parameter which is the great circle path length, d_g divided by n. To simplify its use, only five discrete path lengths are plotted; 500, 1000, 1500, 2000 and 3000 km; the latter curve being suitable also for the 4000 km path.

To use the chart, determine the desired stability, δt, in propagation delay time (Δt) and the nominal virtual height (h') for the path. For example, say h' is 300 km, the desired propagation delay time stability ($\Delta t/n$) is 10 μsec., the great circle path length (d_g/n) is 3000 km and n = 1 (one-hop mode) (see Fig. 5). Then from the curve, the variations in virtual height, $\delta h'$, must be less than 5 km.

IX. CHART FOR DETERMINING TOTAL HF PROPAGATION DELAY

a. Introduction

In many timing problems, the use of the detailed curves given in Fig. 10 through Fig. 15 along with the chart in Fig. 1 is not necessary, if a lower precision than given in these charts is acceptable. In this case, the chart given in Fig. 6 may be used.

b. Use of Chart

This chart gives the total propagation delay time, per hop, for a given path. That is, the delay time in Fig. 6 includes the ground wave delay time in Fig. 1 plus the additional delay time in Figs. 10-15 caused by one reflection of the signal from the ionospheric layer.

It is assumed, in preparing the chart, that the total propagation time was that of one hop multiplied by the number of hops. Thus, the distance is given as (d_g/n), where n is the number of hops, and the path delay time as (t_p/n).

For example, if the great circle distance, d_g, is 9,900 km, then assuming a three-hop path, (d_g/n) would be 3300 km, and the path delay time, as read on the chart, assuming a layer height of 350 km, would be $(t_p/n) = 11.5$ ms, which when multiplied by $n = 3$, is 34.5 ms.

X. ASTRONOMICAL AND "ATOMIC" TIME

a. Introduction

Until very recently the only unit of time available was that derived from the movements of the earth and it was called solar time. This is the time obtained by an observer on a given meridian marking the passages of the sun. Because of variations in the length of a "solar day" throughout the year, the mean solar day was used as the time standard; it is the mean of all the solar days throughout a given solar year.

In 1955 a more uniform astronomical time was adopted as the international time standard and is called Ephemeris Time. It is defined by the orbital motion of the earth about the sun but in practice it is determined from the orbital motion of the moon about the earth.

The latest uniform time, as yet officially undefined, is based on the motions of the atoms or molecules subject to electrical and nuclear forces. Several standards are now in use employing either the cesium atom or the ammonia molecule.

Definitions of a few quantities of interest to those using time signals or engaged in timing problems are given below.

b. Mean Solar Time[8]

(1) Mean Solar Second

The unit of mean solar time is the mean solar second and it is defined as the (1/86, 400) part of a mean solar day.

(2) Mean Solar Day

A mean solar day is defined as the mean of all the solar days in a solar year; there are 365 solar days in one solar year.

(3) Uniform Solar Time UT

(a) UT-0

This was the same as mean solar time; it was uncorrected for perturbations that are known to occur from year to year.

(b) UT-1

Is mean solar time corrected for polar variations.

(c) UT-2

Is mean solar time corrected for polar variations and estimated annual fluctuations.

c. Ephemeris Time[8]

The Ephemeris Second is defined as the fraction 1/31, 556, 925. 9747 of the tropical year for 1900 January 0 at 1200 hours Ephemeris Time.

d. Atomic Time[8]

The atomic second is at present not defined. However, if one maintains a frequency standard whose frequency is derived from the atom, then a properly geared clock driven by some submultiple of this frequency would keep "atomic time", its main merit being its uniformity.

e. Standard HF Time Signals

There are at present many nations broadcasting hf standard time signals, including the two USA stations (WWV, Washington, D.C., and WWVH, Maui, Hawaii), JJY (Tokyo, Japan), and MSF (Rugby, England). All known stations, as listed by the CCIR are given in Appendix E.

XI. USE OF TIME SIGNALS FROM MORE THAN ONE STATION

If all the world's hf standard time signals, as broadcast, were precisely synchronized, it would be possible to use any one of them in any timing problems and then when necessary switch to another one without loss of timing accuracy. Unfortunately, this is not the case at present. Except for WWV and WWVH,[6] none of the transmitted standard hf time signals are synchronized.

This should be taken into account, for instance, when a missile tracking and timing problem arises wherein, say, the time signals from WWVH are used initially and then later a switch is made to the JJY standard time signals. Errors up to several milliseconds may occur, in this case, if suitable corrections are not made.

XII. ACKNOWLEDGMENTS

The data used in plotting the curves in Fig. 10 to Fig. 15, inclusive, was obtained from Mr. A. G. Jean, who kindly consented to their use in this paper. Also, many helpful discussions were held with Mr. T. N. Gautier.

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XIV. APPENDICES

APPENDIX A

GREAT CIRCLE DISTANCE CALCULATIONS

XIV. APPENDICES

APPENDIX A

GREAT CIRCLE DISTANCE CALCULATIONS

1. Data Given (see Fig. 7)

Site	Latitude	Longitude
B	38° 59' 33.16" N.	76° 50' 52.35" W.
A	34° 56' 43.19" N.	117° 55' 01.57" W.

2. Great Circle Distance (Z) Calculations[4]

a. Equations Used

$$\tan\left(\frac{y-x}{2}\right) = \cot(c/2)\left[\frac{\sin\left(\frac{L_B - L_A}{2}\right)}{\cos\left(\frac{L_B + L_A}{2}\right)}\right] \tag{1}$$

$$\tan\left(\frac{y+x}{2}\right) = \cot(c/2)\left[\frac{\cos\left(\frac{L_B - L_A}{2}\right)}{\sin\left(\frac{L_B + L_A}{2}\right)}\right] \tag{2}$$

$$\tan\left(\frac{Z}{2}\right) = \tan\left(\frac{L_B - L_A}{2}\right)\left[\frac{\sin\frac{y+x}{2}}{\sin\frac{y-x}{2}}\right] \tag{3}$$

$$\text{Angle C} = \text{longitude A} - \text{longitude B} \tag{4}$$

b. Calculation of $\left(\frac{L_B - L_A}{2}\right)$ and $\left(\frac{L_B + L_A}{2}\right)$

Sites	$\left(L_B - L_A\right)$	$\left(\frac{L_B - L_A}{2}\right)$	$\left(L_B + L_A\right)$	$\left(\frac{L_B + L_A}{2}\right)$
B, A	4° 02' 49.97"	2° 01' 24.98" 2° 01.416'	73° 56' 16.35"	36° 58' 08.17" 36° 58.14'

c. Determining Angle (c) and (c/2) and cot (c/2)

Sites	c	c/2	cot c/2
B, A	41° 04' 09.22"	20° 32' 04.61" 20° 32.077'	2.6698'

d. Tabulation of Trigonometric Functions Needed

Angles	cos	sin	tan
$\left(\frac{L_B - L_A}{2}\right) = 2^\circ$ 01.416'	0.99931	0.03531	0.03531
$\left(\frac{L_B + L_A}{2}\right) = 36^\circ$ 58.14'	0.79896	0.60138	

e. Calculation of (Z) in degrees

$$\tan\left(\frac{y - x}{2}\right) = 2.6699\left(\frac{0.03531}{0.79896}\right) = (0.11799) \tag{5}$$

$$\left(\frac{y - x}{2}\right) = \tan^{-1}(0.11799) = 6^\circ\ 43.76' \tag{6}$$

$$\sin\left(\frac{y - x}{2}\right) = \sin 6^\circ\ 43.76' = 0.11718 \tag{7}$$

$$\tan\left(\frac{y + x}{2}\right) = 2.6699\left(\frac{0.99938}{0.60138}\right) = 4.436 \tag{8}$$

$$\left(\frac{y+x}{2}\right) = \tan^{-1}(4.4369) = 77^{\circ}\ 17.93' \tag{9}$$

$$\sin\left(\frac{y+x}{2}\right) = \sin 77^{\circ}\ 17.93' = 0.9755 \tag{10}$$

$$\tan\left(\frac{Z^{\circ}}{2}\right) = 0.03533\left(\frac{0.97552}{0.11703}\right) = 0.2942 \tag{11}$$

$$\left(\frac{Z^{\circ}}{2}\right) = \tan^{-1}(0.29429) = 16^{\circ}\ 23.6' \tag{12}$$

$$Z^{\circ} = 32^{\circ}\ 47.2' = 32.79^{\circ} \tag{13}$$

f. <u>Converting $(Z)^{\circ}$ to (Z) kilometers (or miles)</u>

$$Z_{km} = Z^{\circ} \times 111.195 \text{ km/deg} = 32.79 \times 111.195 = \underline{\underline{3642 \text{ km}}} \tag{14}$$

$$Z_{mi} = 0.6214 \text{ mi/km} \times \text{km} = 0.6214 \times 3642 = \underline{\underline{2262 \text{ mi.}}} \tag{15}$$

g. <u>Determining the Mode of Propagation</u>

For a great circle distance of 3650 miles, the minimum mode of propagation would be one-hop, i. e., the distance is less than the maximum of 4000 kms for the one-hop mode.

APPENDIX B

HIGH FREQUENCY PROPAGATION DELAY TIMES

APPENDIX B

HIGH FREQUENCY PROPAGATION DELAY TIMES

1. Definitions (Figure 8)

   (a) d'_g = ac = ce, great circle distance of one-hop path.

   (b) $d_g = nd'_g$ = aceg . . ., great circle distance of n-hop path.

   (c) d'_p = abc = cde, sky wave distance of one-hop path.

   (d) $d_p = nd'_p$ = abcde, . . ., sky wave distance of n-hop path.

2. Sky Wave Propagation Time (Based on Geometric Paths)

From Figure 8, it is easily shown that, for path abc,

$$d'_p = 2\sqrt{2r\,(r + h')\,(1 - \cos\theta) + (h')^2} \tag{1}$$

For simplification, let:

$$h'/r = \gamma, \text{ and } 2\,(1 - \cos\theta) = (2\sin\theta/2)^2 \tag{2}$$

Then, using (2) in (1), and factoring:

$$d'_p = 4r\sin(\theta/2)\sqrt{(1 + \gamma) + \left(\frac{\gamma}{2\sin\theta/2}\right)^2} \tag{3}$$

Then, the propagation delay time, for the path d'_p, is:

$$t'_p = \frac{d'_p}{c} = \frac{4r\sin\theta/2}{c}\sqrt{(1 + \gamma) + \left(\frac{\gamma}{2\sin\theta/2}\right)^2} \tag{4}$$

Now, the ground wave propagation delay time for the great circle distance, d'_g = ac, is:

$$t'_g = \frac{d'_g}{c} \tag{5}$$

Now, from trigonometry, we know that:

$$d'_g = 2r\theta \tag{6}$$

so,

$$t'_g = \frac{2r\theta}{c} \tag{7}$$

Then, using (7) in (4), we get:

$$t'_p = t'_g \left[\left(\frac{\sin \theta/2}{\theta/2} \right) \sqrt{(1 + \gamma) + \left(\frac{\gamma}{2 \sin \theta/2} \right)^2} \right] \tag{8}$$

This is the sky wave propagation delay time in terms of the great circle distance delay time. For more than one hop, say n-hops both sides of equation (8) are multiplied by n.

3. Difference in Sky Wave and Ground Wave Propagation Times

From (8) and (2):

$$\Delta t = \left(t_p - t_g \right) = t_g \left(\frac{\sin \theta/2}{\theta/2} \right) \sqrt{\left[(1 + \gamma) + \left(\frac{\gamma}{2 \sin \theta/2} \right)^2 \right]} - 1 \tag{9}$$

If we let:

$$(1 + \gamma) = h, \left(\frac{\gamma}{2 \sin \theta/2} \right) = p, \text{ and } \left(\frac{\sin \theta/2}{\theta/2} \right) = y, \tag{10}$$

we may write (9) as:

$$\Delta t = t_g \left(y \sqrt{h + p^2} - 1 \right) \tag{11}$$

4. Great Circle Distance Determined from Path Distance

Multiply equation (8) by n, and solve for t_g:

$$t_g = \left[\frac{t_p \,(\theta/2)}{\sin \theta/2 \sqrt{(1+\gamma) + \left(\frac{\gamma}{\sin \theta/2} \right)^2}} \right] \tag{12}$$

from which it is obvious that:

$$d_g = t_g c = \left[\frac{d_p}{\left(\frac{\sin (\theta/2)}{\theta/2} \right) \sqrt{(1+\gamma) + \left(\frac{\gamma}{2 \sin \theta/2} \right)^2}} \right] \tag{13}$$

5. Tables for Use with Equations (9) and (11)

TABLE I

(See Fig. 2)

$\left(\frac{d_g}{n} \text{ versus } \theta/2 \right)$

$\left(\frac{d_g}{n} \right)$ km	$(\theta/2)$ deg.	$(\theta/2)$ rad.
444.4	1	0.01745
888.9	2	0.03491
1333.3	3	0.05236
1777.8	4	0.06981
2222.2	5	0.08727
2666.7	6	0.10472
3111.5	7	0.12217
3555.5	8	0.13963
4000.0	9	0.15708

TABLE II

(y versus $\theta/2$)

$(\theta/2)^{o}$	$(\theta/2)$ rad.	$\sin\theta/2$	y
1	0.01745	0.01745	1.0000
3	0.05236	0.05234	0.99962
6	0.10472	0.10453	0.99818
9	0.15708	0.15643	0.99586

where:

$$y = \frac{\sin\theta/2}{\theta/2}$$

$$\theta/2 = \frac{\phi}{4n}$$

TABLE III

(See Fig. 3)

(h versus h')

h'	γ	h
100	0.0157	1.0157
150	0.0236	1.0236
200	0.0314	1.0314
250	0.0392	1.0392
300	0.0471	1.0471
350	0.0550	1.0550
400	0.0628	1.0628

where:

$$h = (1 + r)$$

$$\gamma = \frac{h'}{r}$$

$$r = 6368 \text{ km}$$

APPENDIX C

TWO-WAY TRANSMISSION OF TIME SIGNALS

APPENDIX C

TWO-WAY TRANSMISSION OF TIME SIGNALS

1. Definition of Symbols Used (Fig. 9)

(a) d_g = great circle path, A bd . . . B.

(b) d_p = n-hop path, Aabcd . . . kB.

(c) δt = delay time signal suffers, at clock B, from its reception to its retransmission.

(d) Δt = time difference between a transmitted and received pulse at A.

(e) t_o = time first pulse leaves A.

(f) t_1 = time first pulse arrives at B.

(g) t_2 = time retransmitted pulse leaves B.

(h) t_3 = time retransmitted pulse reaches A.

(i) t_p = apparent path length.

Now:

$$t_1 = t_o + \left(\frac{d_p}{c}\right) = t_o + t_p \tag{1}$$

and:

$$t_2 = t_1 + \delta t = t_o + 2t_p + \delta t \tag{2}$$

so:

$$\Delta t = t_3 - t_o = t_o + 2t_p + \delta t - t_o = 2t_p + \delta t \tag{3}$$

Therefore:

$$t_p = \left(\frac{\Delta t - \delta t}{2}\right) \tag{4}$$

and:

$$d_p = ct_p \tag{5}$$

APPENDIX D

MEASUREMENT OF HF TIMING SIGNALS

APPENDIX D

MEASUREMENT OF HF TIMING SIGNALS

1. A Note on the Measurements

(a) Acknowledgments

The measurements of the propagation delay time of the hf signals[6] from WWV to WWVH and return, and at Boulder, Colorado, were made by personnel of the National Bureau of Standards Boulder Laboratories at Maui, Hawaii, and Boulder, Colorado, of the Naval Research Laboratory,[3] and of the U. S. Naval Observatory. In all cases, the measurement precision was at least 0.1 millisecond for each individual observation.

(b) Definitions of Symbols Used in Appendix D

d_g = great circle path length.

t_g = propagation delay time of a signal over a given d_g.

r = mean radius of earth; taken as 6368 km.

c = velocity of light in free space; taken as 300,000 km/sec.

d_p = length of path of sky wave (ionospherically reflected) signal.

t_p = propagation delay time of a signal over a given d_p.

σ = standard deviation of a single observation.

h' = virtual height of the reflecting ionospheric layer.

n = number of hops (mode) of the signal propagated over a given d_p.

2. Measurement of Propagation Delay Times

(a) Data on Following Paths

		d_g	t_g	min. n	d_g/r
(1)	WWV to WWVH	7687 km	25.62 ms	2	1.205
(2)	WWV to Boulder	2430 km	8.10 ms	1	0.382
(3)	WWVH to Boulder	5270 km	17.56 ms	2	0.825

(b) Calculated Propagation Delay Times (on above paths) versus Number of Hops (n)

(1) Figure 16 - WWV to WWVH

(2) Figure 17 - WWV to Boulder

(3) Figure 18 - WWVH to Boulder

(c) Measurements Made on Above Paths (September 1955)

	Signals	Meas. Made at	Freq. Mc	t_p ms	σ ms
(1)	WWV vs. WWVH	Maui	5	27.77	0.26
			10	27.32	0.24
			15	27.13	0.22
			ave.	27.40	0.22
(2)	WWVH vs. WWV	Wash. D. C.	5	-----	----
			10	27.16	0.19
			15	-----	----
			ave.	27.16	0.19
(3)	WWV vs. WWVH	Boulder	5	10.95	0.27
			10	11.31	0.28
			15	11.67	0.37
			ave.	11.31	0.31

(4) Actual Propagation Delay Time, Based on a and b, above (round trip)

Path			
WWV to WWVH	5	-----	----
(or vice versa)	10	27.24	0.20
(at 10 Mc)	15	-----	----
		27.24	0.20

(5) Calculations of Actual Propagation Delay Times, Based on Above Measurements, at 10 Mc

Path	Freq., Mc	t_p, ms
WWV to Boulder	10	8.14
WWVH to Boulder	10	19.10
WWV to WWVH	10	27.24

(6) Use of Charts in Figure 16, 17 and 18 (with h' = 350 km) to Determine Propagation Delay of Signals on

Path	d_g km	t_g	t_p ms	n
WWV to WWVH	7687	25.62	27.19	3
WWV to Boulder	2430	8.10	8.63	1
WWVH to Boulder	5270	17.57	19.00	3

(7) Summary of Above Data at 10 Mc

Path	Meas. t_p, ms	Calc. t_p, ms	Difference ms	n
WWV to WWVH	27.24	27.19	0.05	3
WWV to Boulder	8.14	8.63	0.49	1
WWVH to Boulder	19.10	19.00	0.10	3

(d) Second-to-Second Measurements, at 10 Mc, of Actual Delay Times of Path, WWV to WWVH (Based on Round Trip)

(1) Figure 20 - WWV to WWVH, December 19, 1956

(2) Figure 19 - WWV to WWVH, December 21, 1956

(3) Summary of Above

Fig.	Date	UT Time	No. Observ.	Ave. t_p, ms	Max. dev. from ave., ms	Min. t_p, ms	σ
20	12-19	0200	6	27.50	0.1	27.10	0.10
		0500	30	27.70	0.3	27.45	0.15
		1100	92	27.40	0.5	27.00	0.15
21	12-20	0200	26	27.50	0.4	27.20	0.17
		0500	18	27.30	0.2	27.10	0.11
		0800	33	27.25	0.3	27.05	0.10
	Significant Values			27.41	0.5	27.00	0.17

(4) Conclusions Based on Above and Figure 16

Ave. of ave. t_p, ms	Max. σ, ms	Min. t_p, ms	Probable n	Ave. d_p, km	Probable h', km
27.41	0.17	27.00	3	8223	350

(e) Hourly Measurements, at 10 Mc, on Path WWV to WWVH, June 1956

(1) Measurements, June 20

Signals	Meas. made at	UT Time	No. Observ.	Ave. Diff., ms	σ ms
WWV vs. WWVH	Wash. D. C.	0600	16	29.01	0.43
		0900	9	28.72	0.24
		1200	8	29.15	0.21
WWVH vs. WWV	Maui	0600	8	26.43	0.13
		0900	10	27.08	0.15
		1200	--	-----	----

(2) Average, Actual Propagation Delay Times (June 20)

Path	UT Time	Ave. t_p, ms	Ave. σ, ms
WWV to WWVH	0600	27.72	0.28
	0900	27.90	0.22
	1200	-----	----
Average Values		27.81	0.25

(3) Measurements, June 22

Signals	Meas. made at	UT Time	No. Observ.	Ave. Diff., ms	σ ms
WWV vs. WWVH	Wash.	0600	11	27.83	0.25
	D. C.	0900	20	28.09	0.32
		1200	4	28.68	0.53
WWVH vs. WWV	Maui	0600	10	26.94	0.21
		0900	10	27.12	0.10
		1200	9	27.22	0.10

(4) Average Actual Propagation Delay Times (June 22)

Path	UT Time	Ave. t_p, ms	Ave. σ, ms
WWV to WWVH	0600	27.38	0.23
	0900	27.60	0.21
	1200	27.95	0.31
Average Values		27.64	0.25

(5) Summary of Above (Based on Round Trip Measurements)

Day	Path	(3 hour) Ave. t_p, ms	(3 hour) Ave. σ, ms	(3 hour) ave. path length, km
June 20	WWV to WWVH	27.81	0.25	8343
June 22	WWV to WWVH	27.64	0.25	8292

(6) Conclusions from Above and Figure 16

Ave. t_p, ms	Ave. σ, ms	Probable n	Ave. d_p, km	Approx. h', km
27.75	0.25	4	8317	350

(b) Daily Measured Values of Actual Delay Times on Path WWV to WWVH, 1958

(1) Figure 21 - January 1958 to September 1958 (Based on Round Trip)

(2) Summary of Figure 21 (WWV to WWVH)

Month	Min. t_p, ms	Ave. t_p, ms	Max. t_p, ms	No. days Observ.	Ave. d_p, km
Jan.	27.20	27.52	27.90	17	8256
Feb.	26.85	27.24	27.80	11	8172
Mar.	27.10	27.46	27.85	18	8238
Apr.	27.35	27.63	27.90	9	8289
May	27.25	28.12	28.85	12	8436
Jun.	27.45	27.85	28.75	9	8355
Jul.	27.20	27.65	28.10	14	8295
Aug.	27.20	27.59	27.85	16	8277
Sep.	27.30	27.67	28.10	11	8291

(3) Considerations from Above and Figure 16

a) Estimated Values of n and h'

Month	Min. t_p, ms	Prob. n	Approx. h', km	Ave. t_p, ms	Prob. n	Approx. h', km
Jan.	27.20	3	350	27.52	4	325
Feb.	26.85	2	400	27.24	3	355
Mar.	27.10	3	335	27.46	3	380
Apr.	27.35	3	365	27.63	4	330
May	27.25	3	355	28.12	4	375
Jun.	27.45	3	380	27.85	4	350
Jul.	27.20	3	350	27.65	4	330
Aug.	27.20	3	350	27.59	4	330
Sep.	27.30	3	360	27.67	4	330

b) Summary of Data (Over a 9-Month Period)

Ave. t_p, ms	Max. t_p, ms	Min. t_p, ms	Max. Dev. from Ave., ms	Prob. n	Ave. h', km	Ave. d_p, km
27.63	28.85	26.85	1.22	4	345	8289

c) Conclusions (10 Mc)

1) The actual delay time variations from day-to-day (data taken same time every day) were taken about the average, about 1 millisecond or less.

2) In general, the mode of propagation was 4 hops.

3) Average virtual height of the layer was near 350 km.

(g) Summary of Above (Items c through f)

Item	Ave. t_p, ms	Min. t_p, ms	Prob. n	Ave. d_p, km	Ave. or Prob. h', km	Ave. σ
(c)	27.24	-----	3	8172	350	----
(d)	27.41	27.00	3	8223	350	0.17
(e)	27.75	-----	4	8317	350	0.25
(f)	27.63	26.85	4	8289	345	----
Average	27.50	26.92		8250	350	0.21

APPENDIX E

STANDARD FREQUENCY AND TIME STATIONS

APPENDIX E

MAIN CHARACTERISTICS OF STANDARD FREQUENCY AND TIME SIGNAL STATIONS

			BUENOS AIRES Argentine	HAWAIIAN ISLANDS USA	JOHANNESBURG South Africa	LOWER HUTT N. Zealand	MOSKVA U.S.S.R.	RUGBY England	TOKYO Japan	TORINO Italy	UCCLE Belgium	WASHINGTON U.S.A.
1	Stations											
2	Latitude and Longitude		34° 37' S 58° 21' W	20° 46' N 156° 28' W	26° 11' S 28° 04' E	41° 14' S 174° 55' E	- -	52° 22' N 1° 11' W	35° 42' N 139° 31' E	45° 03' N 7° 40' E	50° 48' N 4° 21' E	39° 00' N 76° 51' W
3	Call Signs		LOL	WWVH	ZUO	ZLFS	-	MSF	JJY	IBF	-	WWV
4	Carrier Power to antenna (kW)		2	2	0.1	0.03	20	0.5	1	0.3	0.02	0.1 - 10
5	Type of antenna		-	Vert. dip.	Hor. dip. [9]	-	Hor. dip. [9]	Vert. dip.	Vert. [24]	Hor. dip. [31]	Vert.	Vert. dip.
6	Number of simultaneous transmiss.		6	3	1	1	1	3	2-3	1	1	6
7	Number of carrier frequencies used		6	3	1	1	2	3	4	1	1	6
8	Trans- missions	Days per week	6 [1]	7	7	1 [13]	6 [1]	7	7-1 [25]	6 [1]	7	7
9		Hours per day	5 [2]	23 [5]	24 [10]	3 [14]	1/2 [15]	24 [20]	24 [25]	1 [32]	22 [34]	24 [35]
10	Standard frequencies used	Carriers (Mc/s)	2.5; 5; 10; 15; 20; 25	5; 10; 15	5	2.5	10 [16] 15 [17]	2.5; 5; 10;	2.5 [26]; 5 [27]; 10 [28]; 15 [29]	5	2.5	2.5; 5; 10; 15; 20; 25
11		Modulation (c/s)	1 [3]; 440; 1000	1 [6]; 440 600	1 [11]	None	1 [18]	1 [21]; 1000	1 [30]; 1000	1 [33]; 440 1000	None	1 [36]; 440; 600
12	Duration of audio modulation (minutes)		4 in each 5 [4]	3 in each 5 [7]	Nil	Nil	Nil	5 in each 15	4 in each 5	5 in each 10 [4]	Nil	3 in each 5 [7]
13	Frequency accuracy (10^{-9})		± 20	± 10	± 20	± 100	± 20	± 5 [22]	± 20	± 20	± 10	± 10
14	Max. val. of steps of freq. adjust. in parts in 10^9		-	5	10	-	20	10	20	20	-	1
15	Duration of time signal transmissions (minutes)		4 in each 60	continuous	continuous	Nil	5 in each 30 (19)	5 in each 15	continuous	5 in each 10	Nil	continuous
16	Accuracy of time intervals		$\pm 20 \times 10^{-9}$ ± 1 micro-second	$\pm 10 \times 10^{-9}$ ± 1 micro-second	$\pm 20 \times 10^{-9}$ ± 1 micro-second	None	-	$\pm 5 \times 10^{-9}$ ± 1 micro-second	$\pm 20 \times 10^{-9}$ ± 1 micro-second	$\pm 20 \times 10^{-9}$ ± 1 micro-second	None	$\pm 10 \times 10^{-9}$ ± 1 micro-second
17	Method of time signal adjustment		20 ms approx.	By steps of 20 ms [8]	By steps of 20 ms [12]	None	-	By steps of 50 ms [23]	By steps of 10 ms	By steps	None	By steps of 20 ms [8]

DAILY SCHEDULES

1. Weekdays.
2. From 11.00 to 12.00, from 14.00 to 15.00, from 17.00 to 18.00, from 20.00 to 21.00, and from 23.00 to 24.00 U. T.
3. Pulses of 5 cycles of 1000 c/s tone; no 59th pulse of each minute.
4. Alternately 440 or 1000 c/s.
5. Interruptions from minute 0 to minute 4, and from minute 30 to minute 34 of each hour, as well as from 19.00 to 19.34 U. T.
6. Pulses of 6 cycles of 1200 c/s tone; no 59th pulse of each minute.
7. Alternately 440 and 600 c/s.
8. Adjustments are made on Wednesdays at 19.00 U. T. when necessary.
9. Maximum radiation N - S.
10. Interruption from 06.30 to 07.00 U. T.
11. Pulses of 10 cycles of 1000 c/s tone; the first pulse of each minute is prolonged (500 ms).
12. If required, the first Monday of each month.
13. Tuesdays.
14. From 01.00 to 04.00 U. T.
15. From 07.15 to 07.45 U. T.
16. Even days.
17. Odd days.
18. Signals A1 keyed. Duration of each signal 100 ms; the first signal of each minute is prolonged.
19. From 07.15 to 07.18, and from 07.43 to 07.45 U. T.
20. Interruption from minute 15 to minute 20 of each hour.
21. Pulses of 5 cycles of 1000 c/s tone; the first pulse of each minute is prolonged (100 ms).
22. Relative to an atomic standard.
23. Adjustments are made the first day of the month, when necessary.

24. Two half-wave dipoles on 15 Mc/s; one half-dipole on 2.5 and 5 Mc/s.

25. See (26) to (29).

26. From 07.00 to 23.00 U. T.; interruption from minute 29 to minute 39 of each hour.

27. Mondays. Interruption from minute 9 to minute 19, from minute 29 to minute 39, and from minute 49 to minute 59 of each hour.

28. Wednesdays. Interruptions as for (27).

29. From 21.00 to 11.00 U. T. Interruptions as for (26).

30. Transmission suspended for 20 ms; the suppression before second 0 lasts 200 ms.

31. Maximum radiation NW-SE.

32. From 07.00 to 07.30 and from 11.00 to 11.30 U. T.

33. Pulses of 5 cycles of 1000 c/s tone; the first pulse of each minute is repeated 7 times at intervals of 10 ms.

34. Interruptions from 11.30 to 12.30, and from 20.30 to 21.30 U. T.

35. Interruption from minute 45 to minute 49 of each hour.

36. Pulses of 5 cycles of 1000 c/s tone; no 59th pulse of each minute. The first pulse of each minute is repeated 100 ms later.

XV. ILLUSTRATIONS AND FIGURES

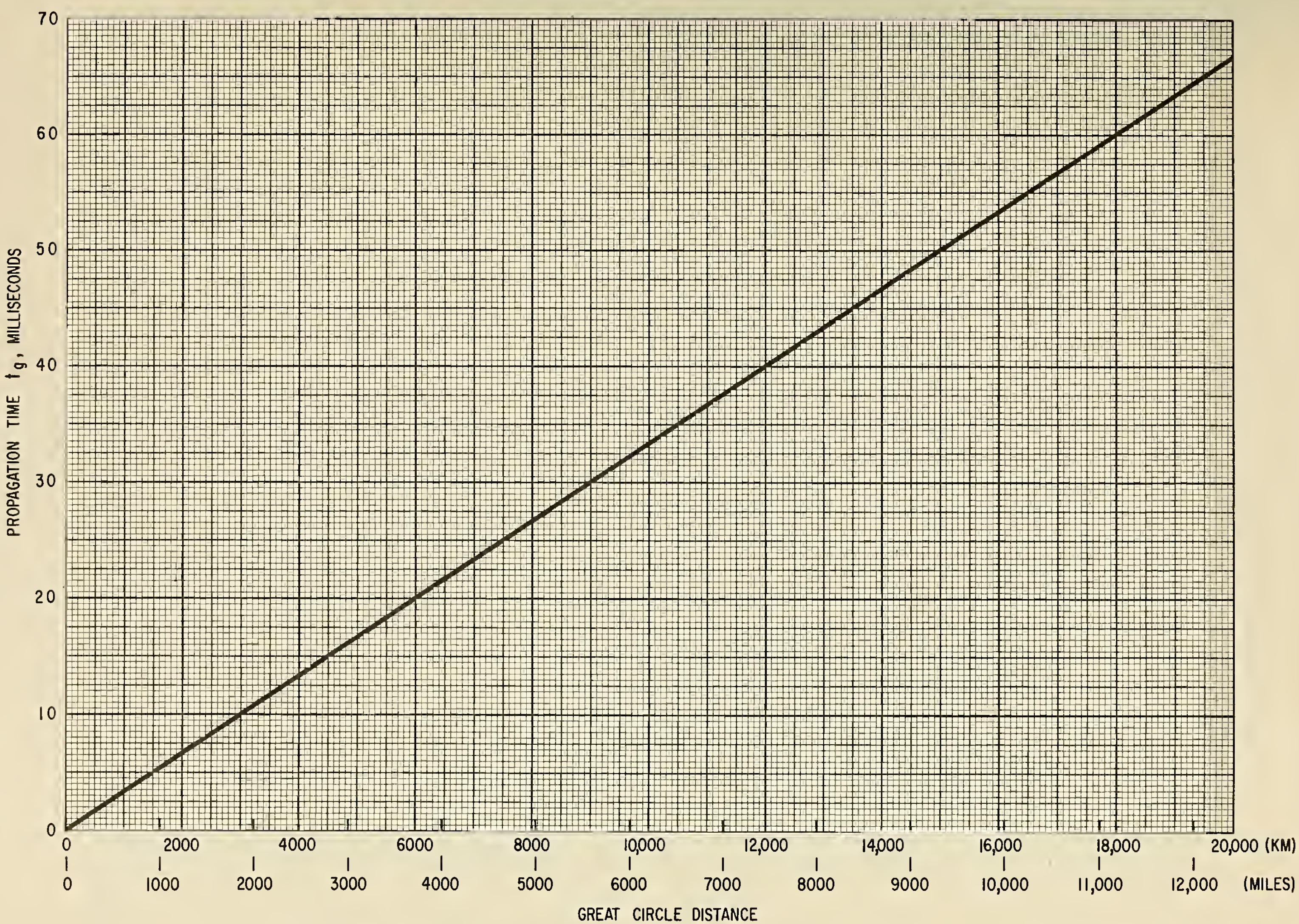


FIG. I GREAT CIRCLE DISTANCE VS PROPAGATION TIME

NOTE: 1000 MILES = 1609 KM

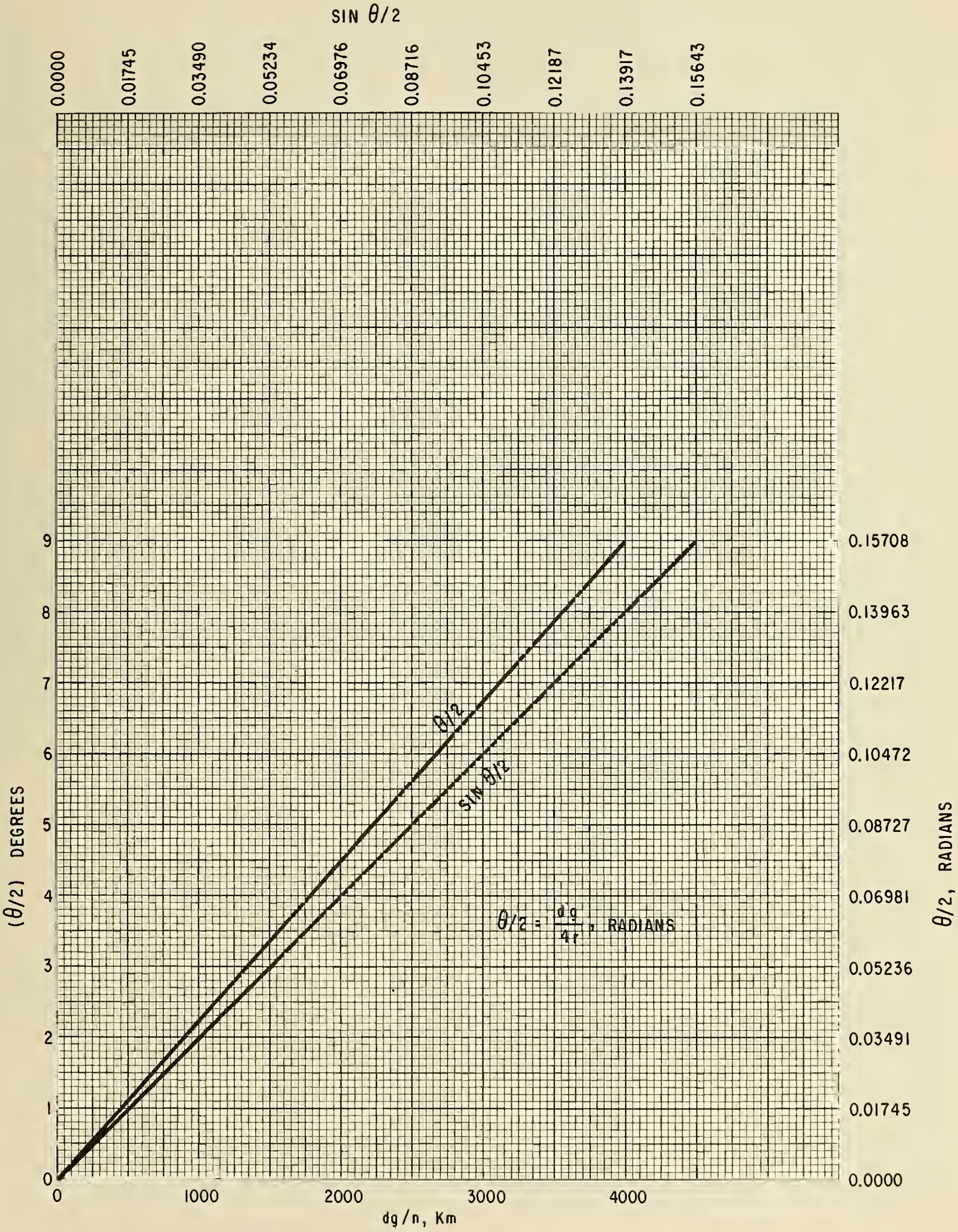
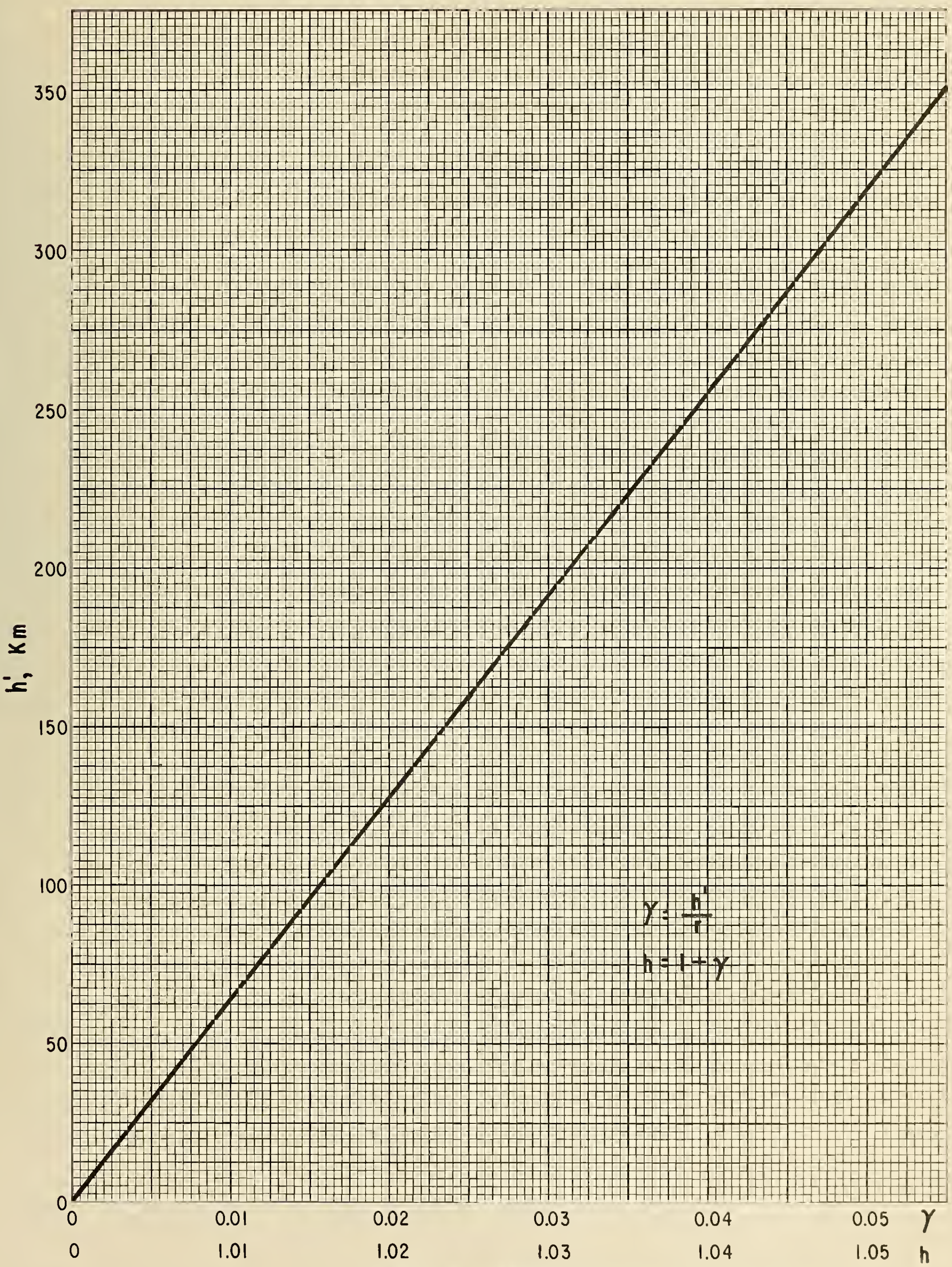


FIG. 2 PLOT OF TABLE I



PLOT OF TABLE III

FIG. 3

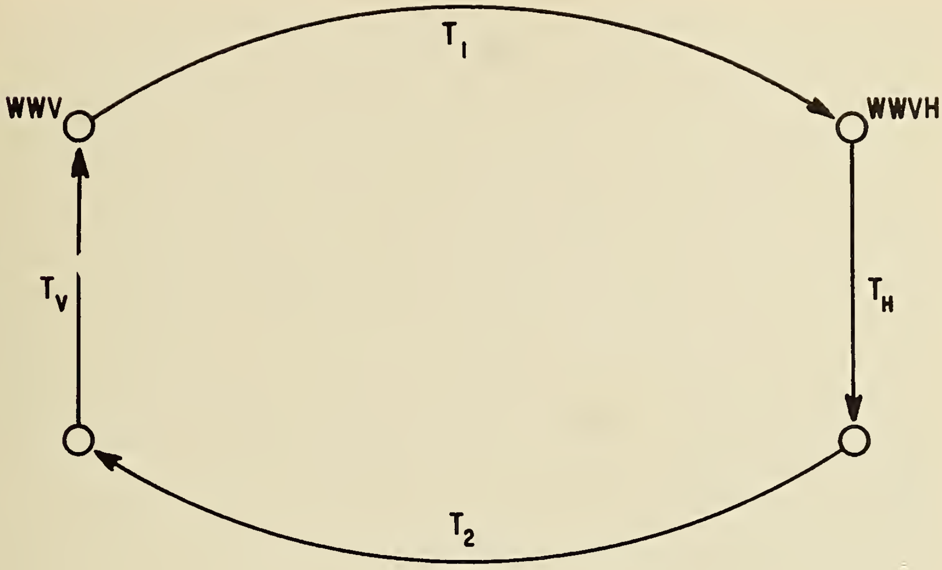


FIG. 4 ILLUSTRATION OF DELAY TIMES, WWV AND WWVH

T_1 = delay time of signal from WWV to WWVH

T_2 = delay time of signal from WWVH to WWV

T_H = delay time introduced into pulses transmitted from WWVH

T_v = delay time measured at WWV of received WWVH time signals

Then:

(1) $(T_1 - T_H + T_2) = T_v$,

and

(2) $T_1 + T_2 = T_v \; T_H$,

therefore,

(3) $T_p = \dfrac{T_1 + T_2}{2} = \dfrac{T_v + T_H}{2}$.

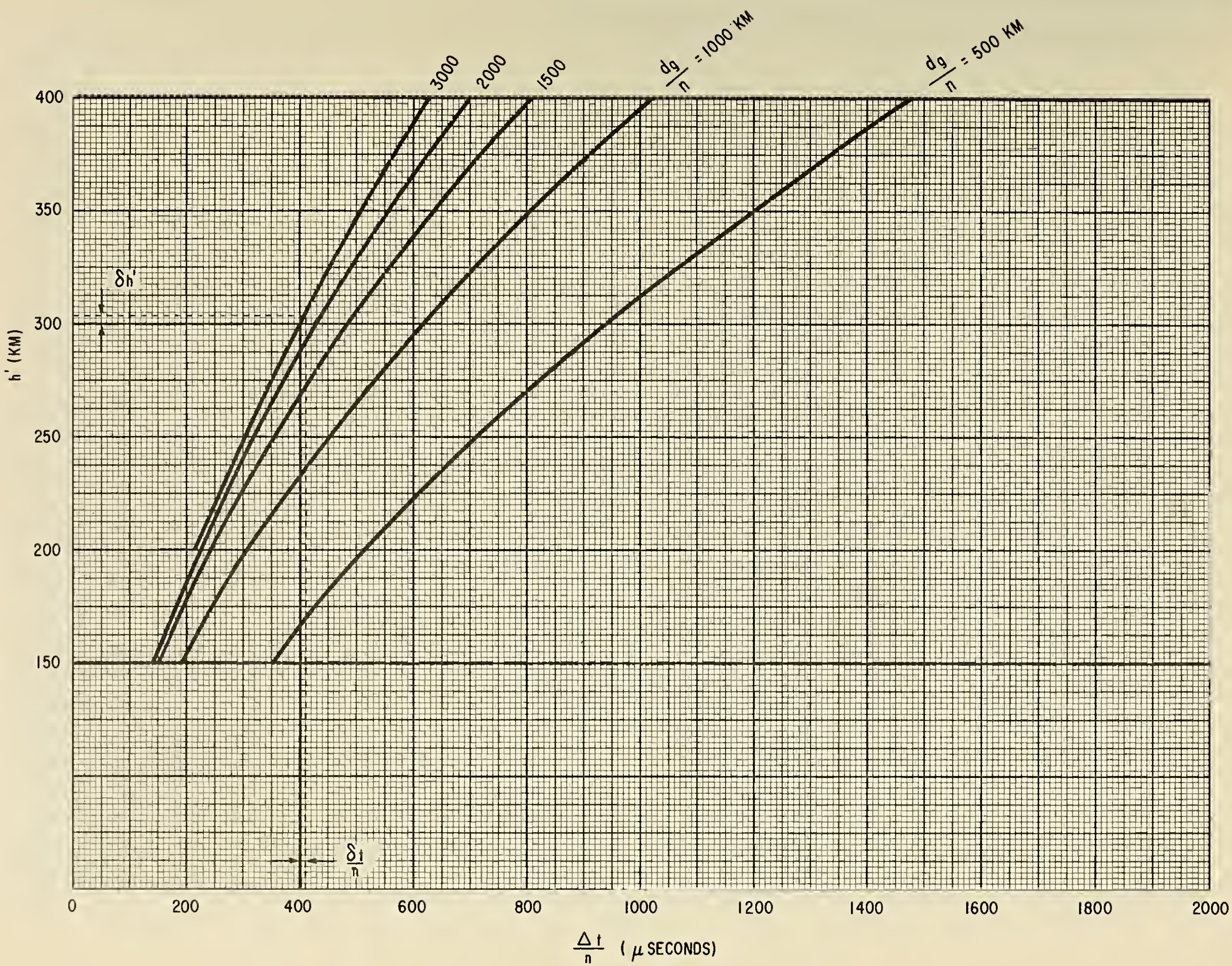

3000
2000
1500
$\frac{d_g}{n}$ = 1000 KM
$\frac{d_g}{n}$ = 500 KM
400
350
300
250
200
150
h' (KM)
$\delta h'$
$\frac{\delta t}{n}$
0
200
400
600
800
1000
1200
1400
1600
1800
2000
$\frac{\Delta t}{n}$ (μ SECONDS)

FIG. 5

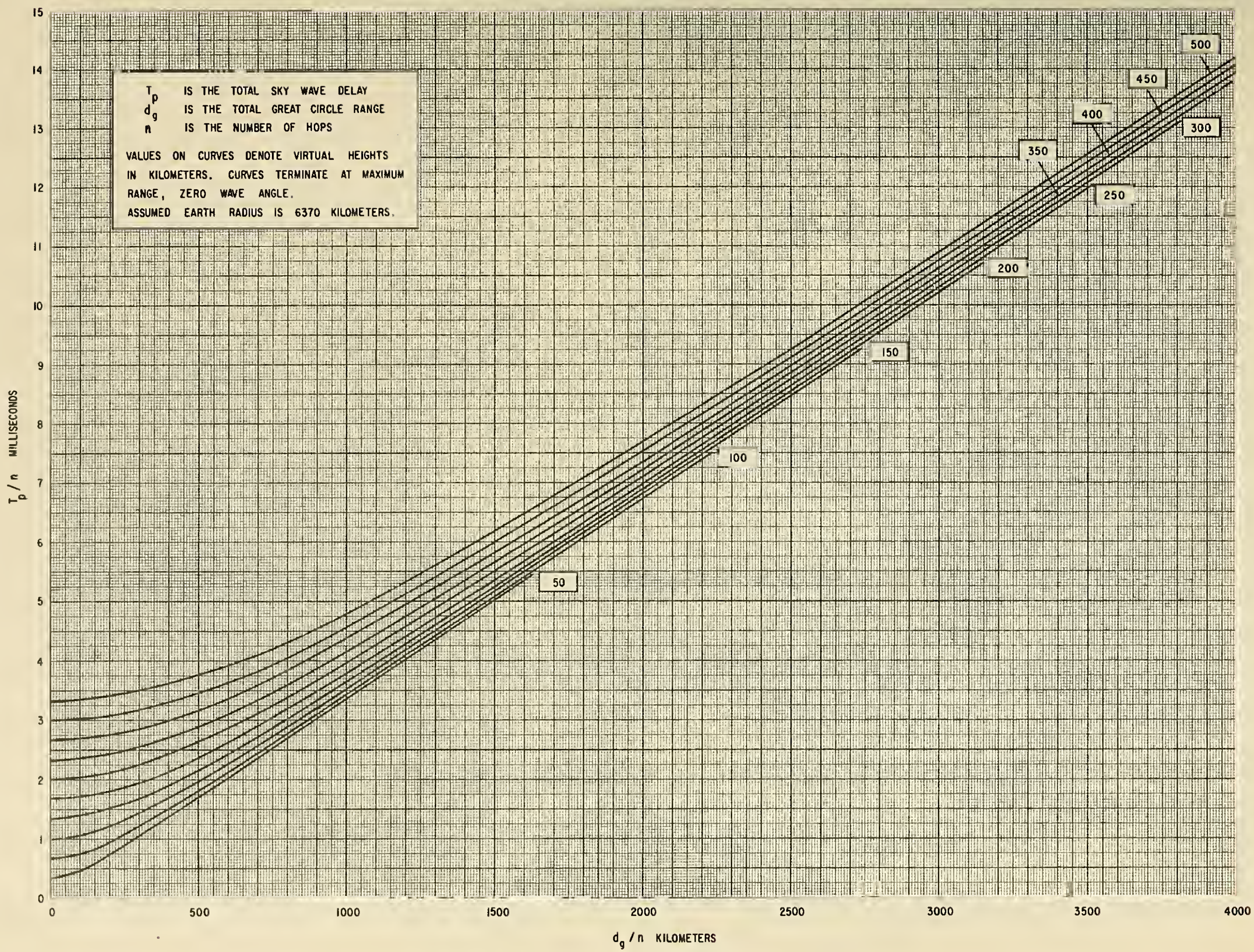


FIG. 6 SKY WAVE PROPAGATION TIME

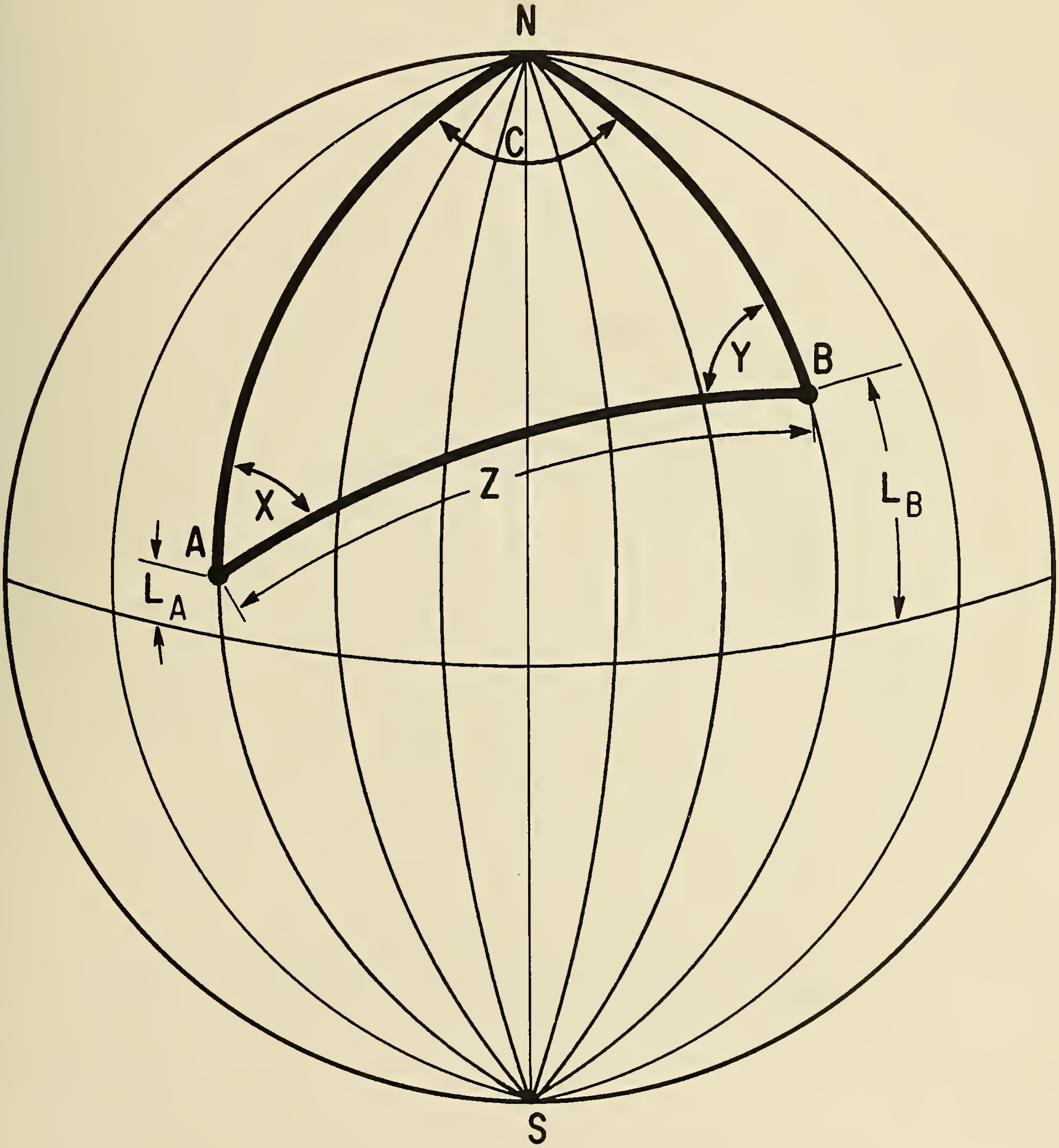


FIG. 7 GREAT CIRCLE DISTANCE CALCULATIONS

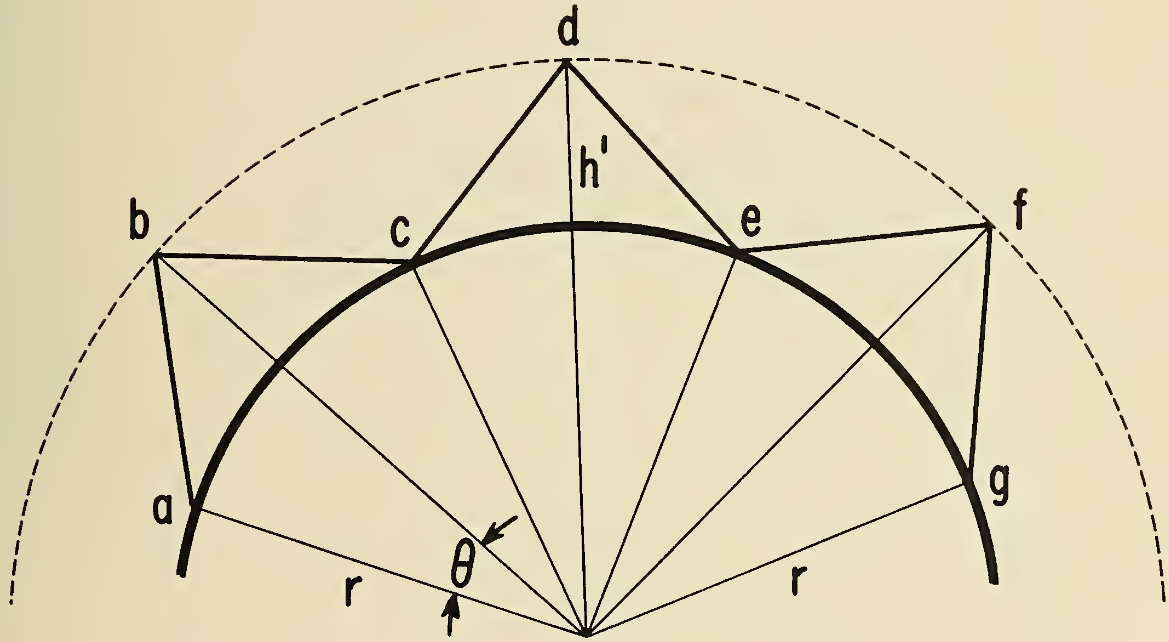


FIG.8 GEOMETRY OF THREE-HOP MODE

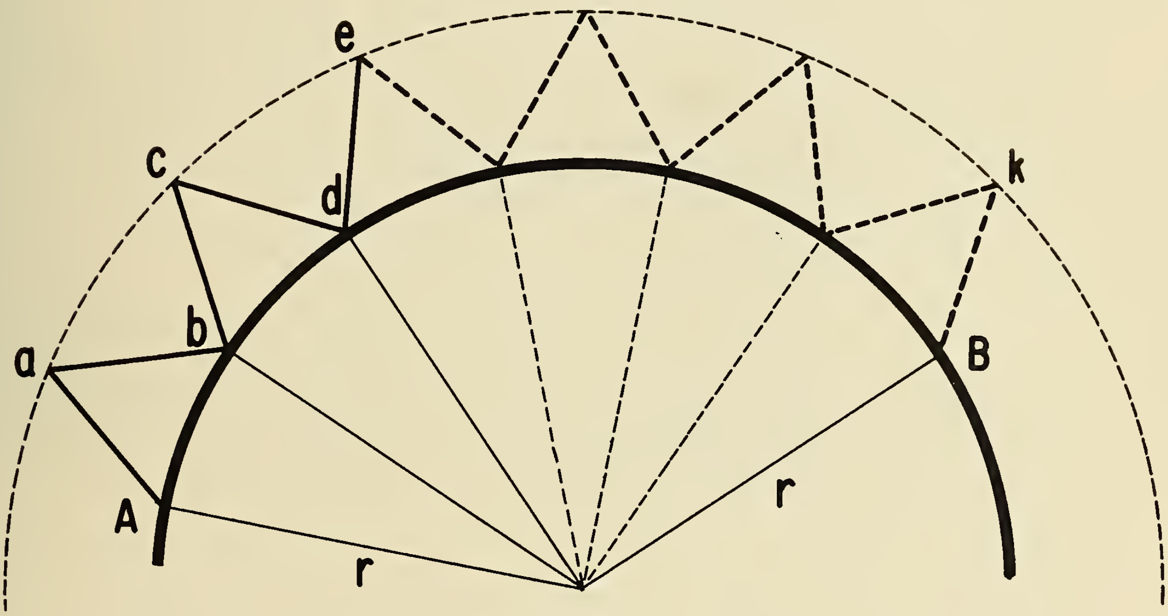


FIG. 9 GEOMETRY OF n-HOP MODE

APPENDIX IV

CURVES OF FIXED h': WITH Δt (μSEC) VERSUS DISTANCE d_g (KM)

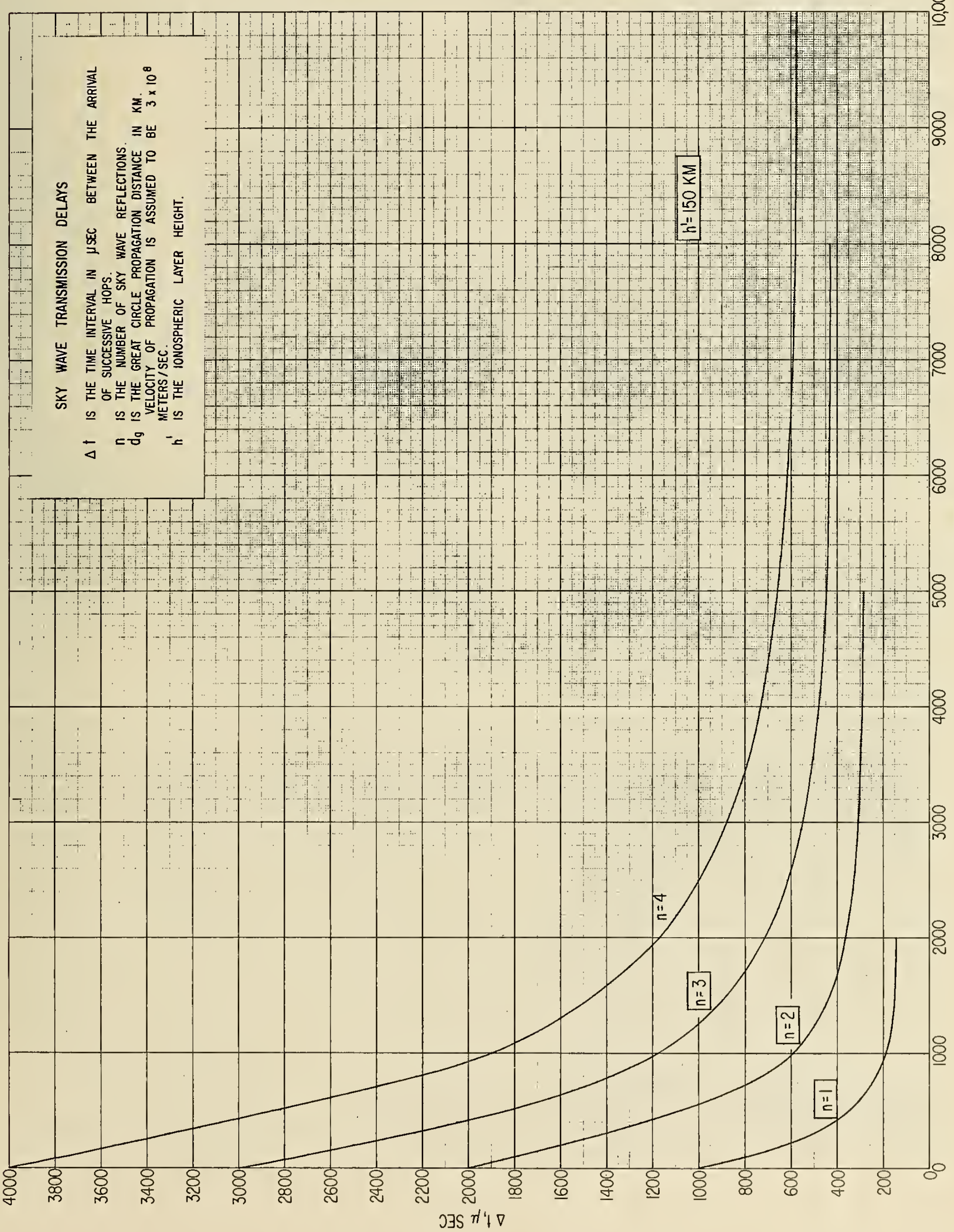


FIG. 10

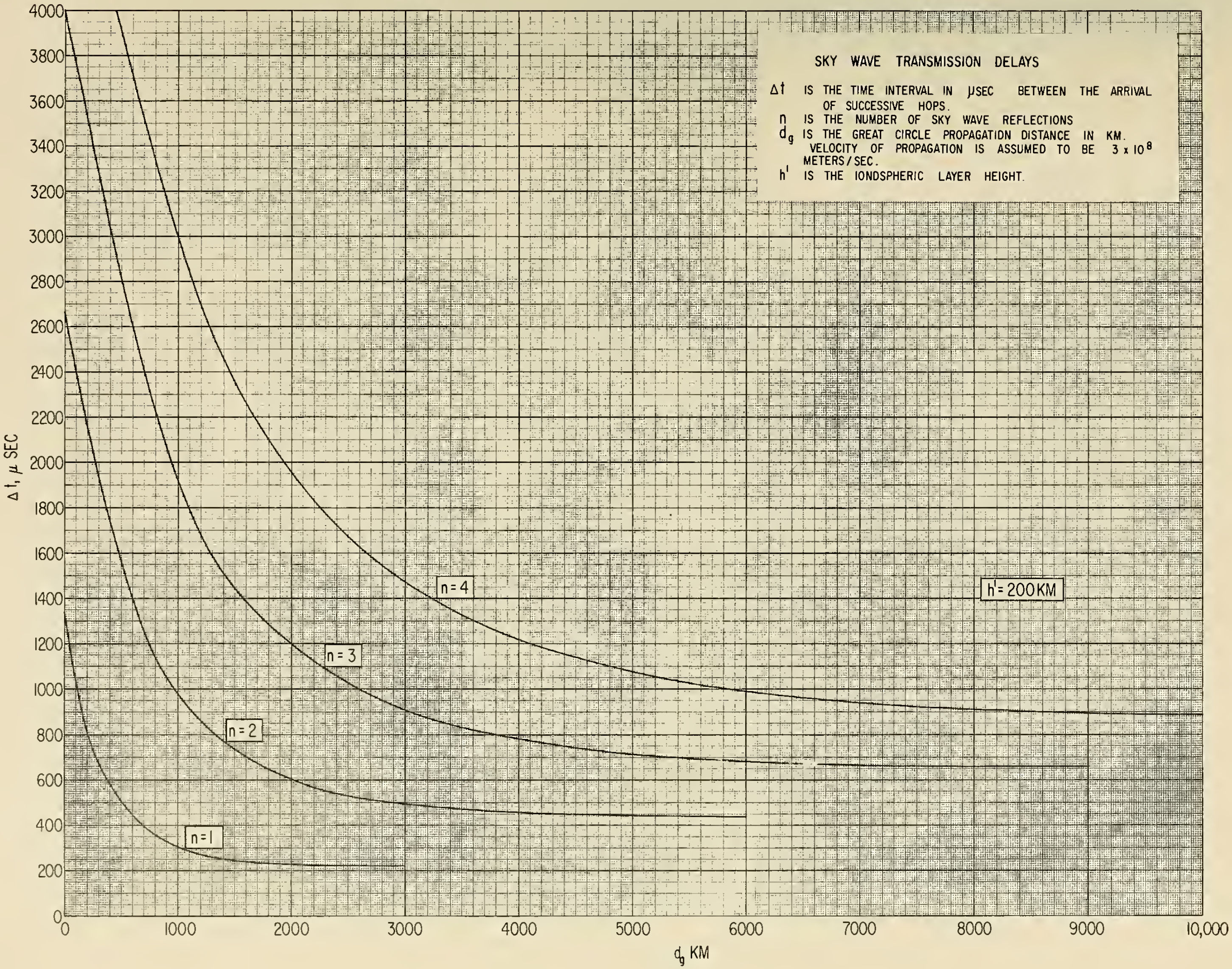

SKY WAVE TRANSMISSION DELAYS
Δt IS THE TIME INTERVAL IN μSEC BETWEEN THE ARRIVAL OF SUCCESSIVE HOPS.
n IS THE NUMBER OF SKY WAVE REFLECTIONS
dg IS THE GREAT CIRCLE PROPAGATION DISTANCE IN KM.
VELOCITY OF PROPAGATION IS ASSUMED TO BE 3 x 10^8 METERS/SEC.
h' IS THE IONOSPHERIC LAYER HEIGHT.
h' = 200 KM
n = 1
n = 2
n = 3
n = 4
Δt, μ SEC
dg KM
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800
3000
3200
3400
3600
3800
4000
1000
2000
3000
4000
5000
6000
7000
8000
9000
10,000

FIG. 11

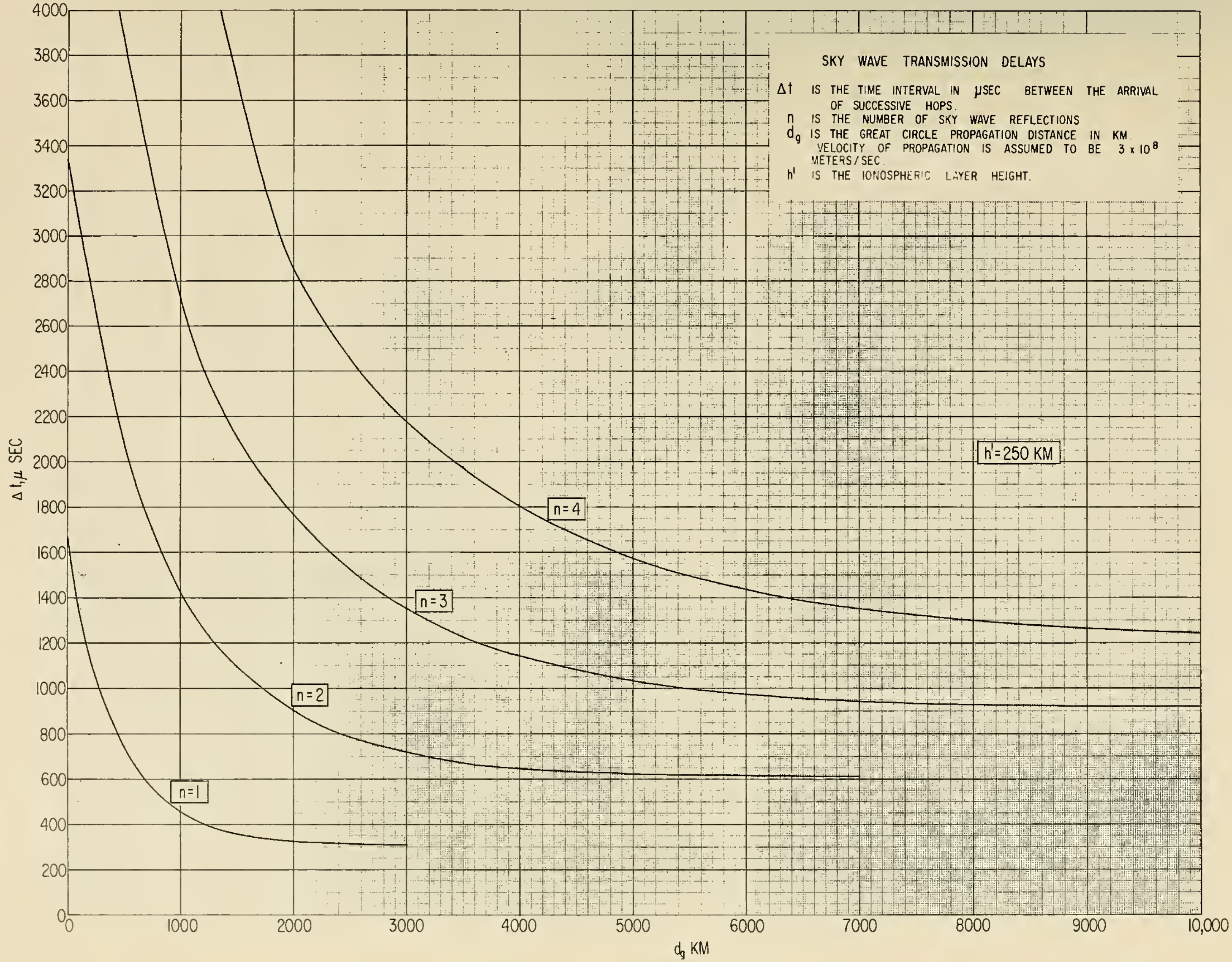
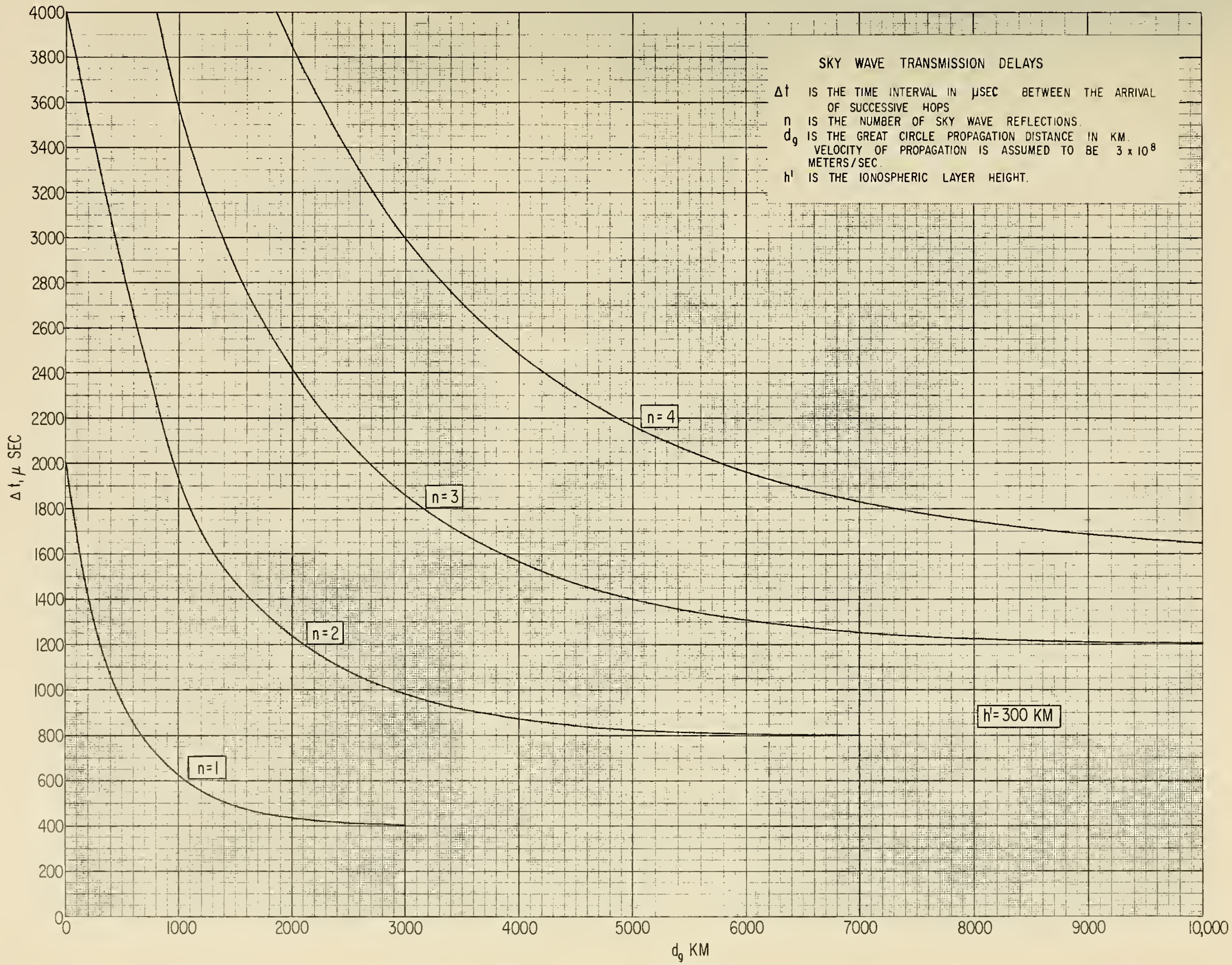

SKY WAVE TRANSMISSION DELAYS
Δt IS THE TIME INTERVAL IN μSEC BETWEEN THE ARRIVAL OF SUCCESSIVE HOPS.
n IS THE NUMBER OF SKY WAVE REFLECTIONS
d_g IS THE GREAT CIRCLE PROPAGATION DISTANCE IN KM. VELOCITY OF PROPAGATION IS ASSUMED TO BE 3×10^8 METERS/SEC.
h' IS THE IONOSPHERIC LAYER HEIGHT.
h'=250 KM
n=1
n=2
n=3
n=4
Δt, μ SEC
d_g KM
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800
3000
3200
3400
3600
3800
4000
1000
2000
3000
4000
5000
6000
7000
8000
9000
10,000

FIG. 12



SKY WAVE TRANSMISSION DELAYS
Δt IS THE TIME INTERVAL IN μSEC BETWEEN THE ARRIVAL OF SUCCESSIVE HOPS
n IS THE NUMBER OF SKY WAVE REFLECTIONS.
d_g IS THE GREAT CIRCLE PROPAGATION DISTANCE IN KM. VELOCITY OF PROPAGATION IS ASSUMED TO BE 3×10^8 METERS/SEC.
h' IS THE IONOSPHERIC LAYER HEIGHT.
n=1
n=2
n=3
n=4
h'=300 KM
Δt, μ SEC
d_g KM
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800
3000
3200
3400
3600
3800
4000
1000
2000
3000
4000
5000
6000
7000
8000
9000
10,000

FIG. 13

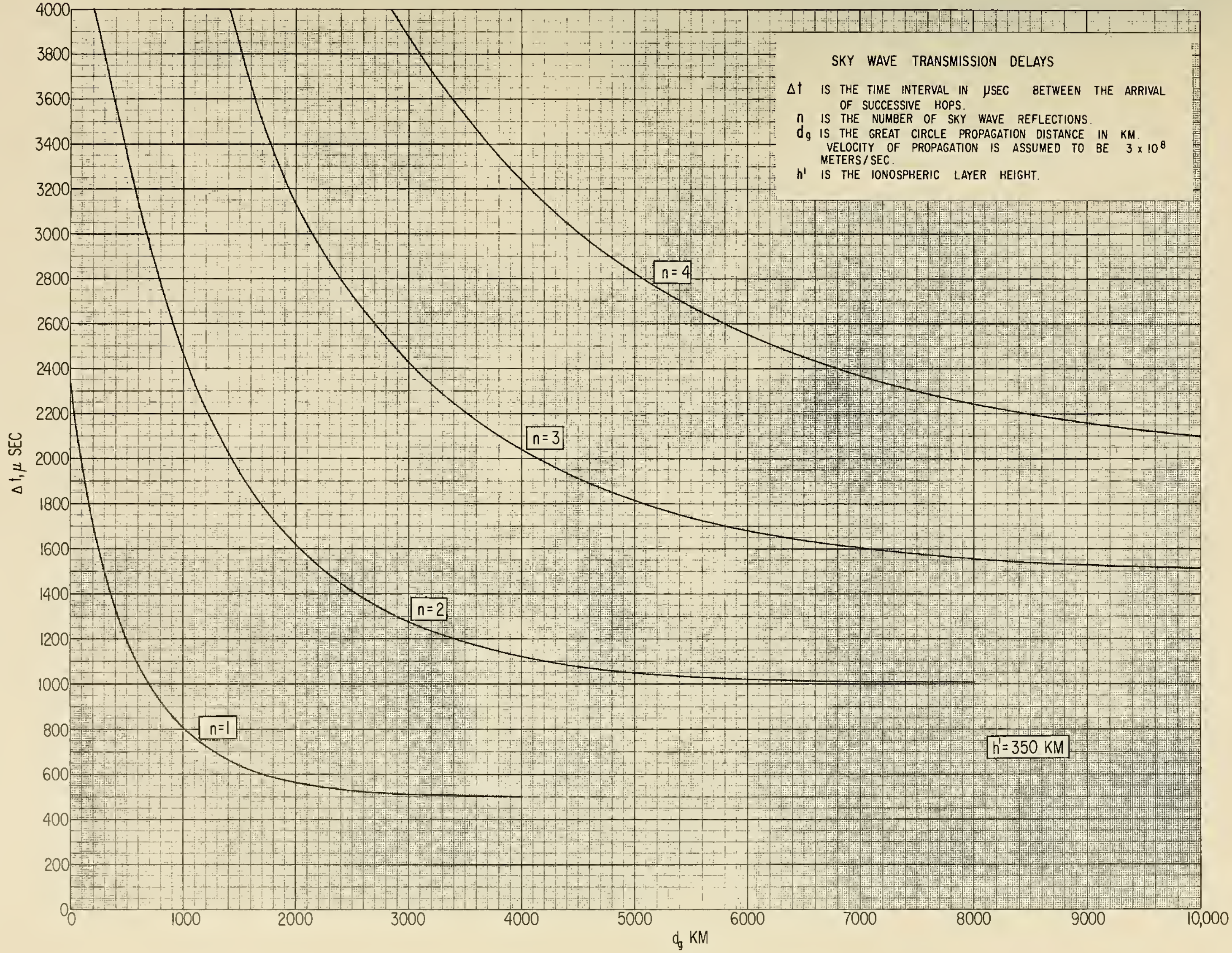

SKY WAVE TRANSMISSION DELAYS
Δt IS THE TIME INTERVAL IN μSEC BETWEEN THE ARRIVAL OF SUCCESSIVE HOPS.
n IS THE NUMBER OF SKY WAVE REFLECTIONS.
dg IS THE GREAT CIRCLE PROPAGATION DISTANCE IN KM.
VELOCITY OF PROPAGATION IS ASSUMED TO BE 3 x 10⁸ METERS/SEC.
h' IS THE IONOSPHERIC LAYER HEIGHT.
n=1
n=2
n=3
n=4
h'=350 KM
Δt, μ SEC
dg KM
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800
3000
3200
3400
3600
3800
4000
1000
2000
3000
4000
5000
6000
7000
8000
9000
10,000

FIG. 14

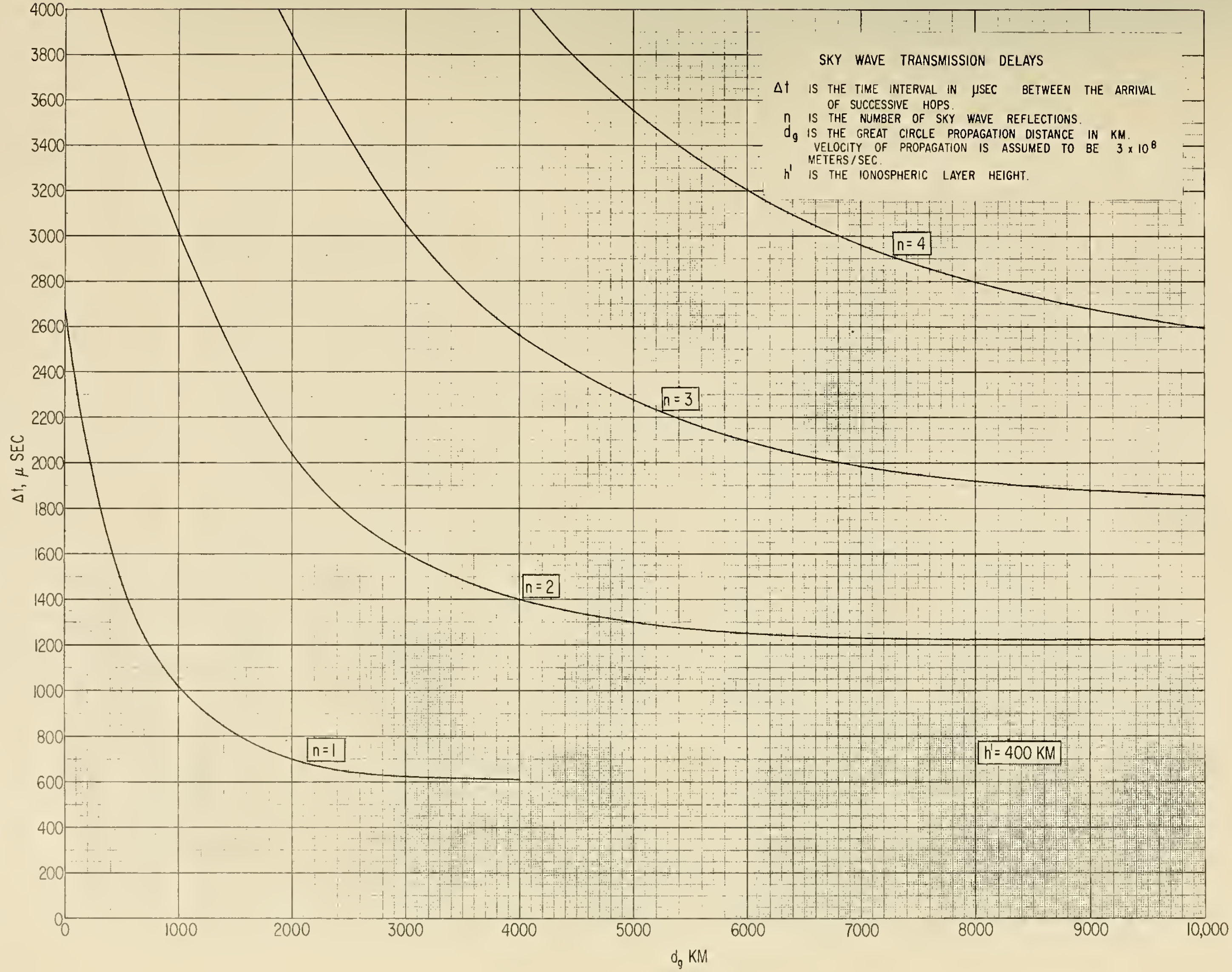

SKY WAVE TRANSMISSION DELAYS
Δt IS THE TIME INTERVAL IN μSEC BETWEEN THE ARRIVAL OF SUCCESSIVE HOPS.
n IS THE NUMBER OF SKY WAVE REFLECTIONS.
d_g IS THE GREAT CIRCLE PROPAGATION DISTANCE IN KM. VELOCITY OF PROPAGATION IS ASSUMED TO BE 3×10^8 METERS/SEC.
h' IS THE IONOSPHERIC LAYER HEIGHT.
n = 1
n = 2
n = 3
n = 4
h' = 400 KM
Δt, μ SEC
d_g KM
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800
3000
3200
3400
3600
3800
4000
5000
6000
7000
8000
9000
10,000

FIG. 15

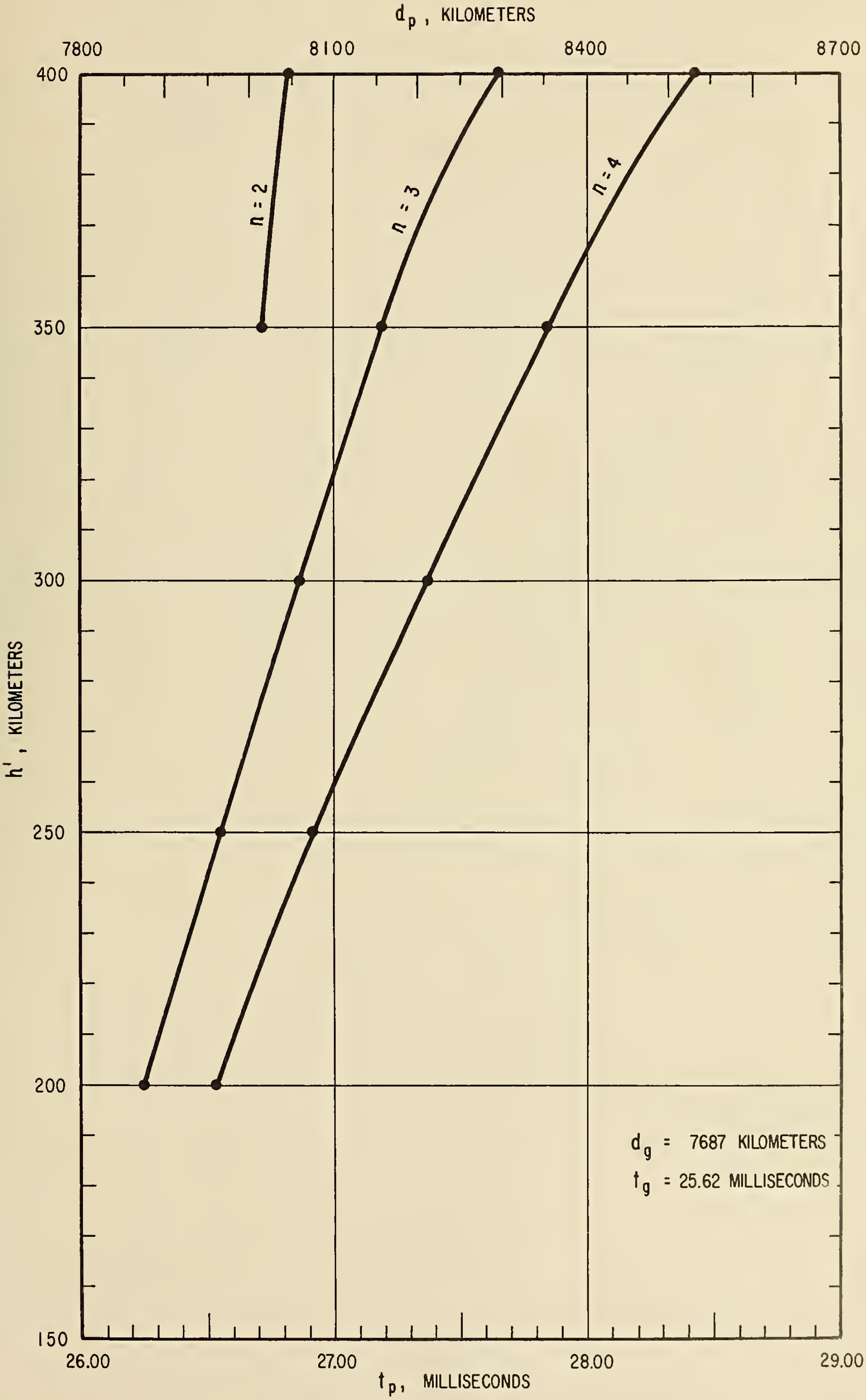


FIG. 16 CALCULATED PROPAGATION DELAY TIME WWV TO WWVH

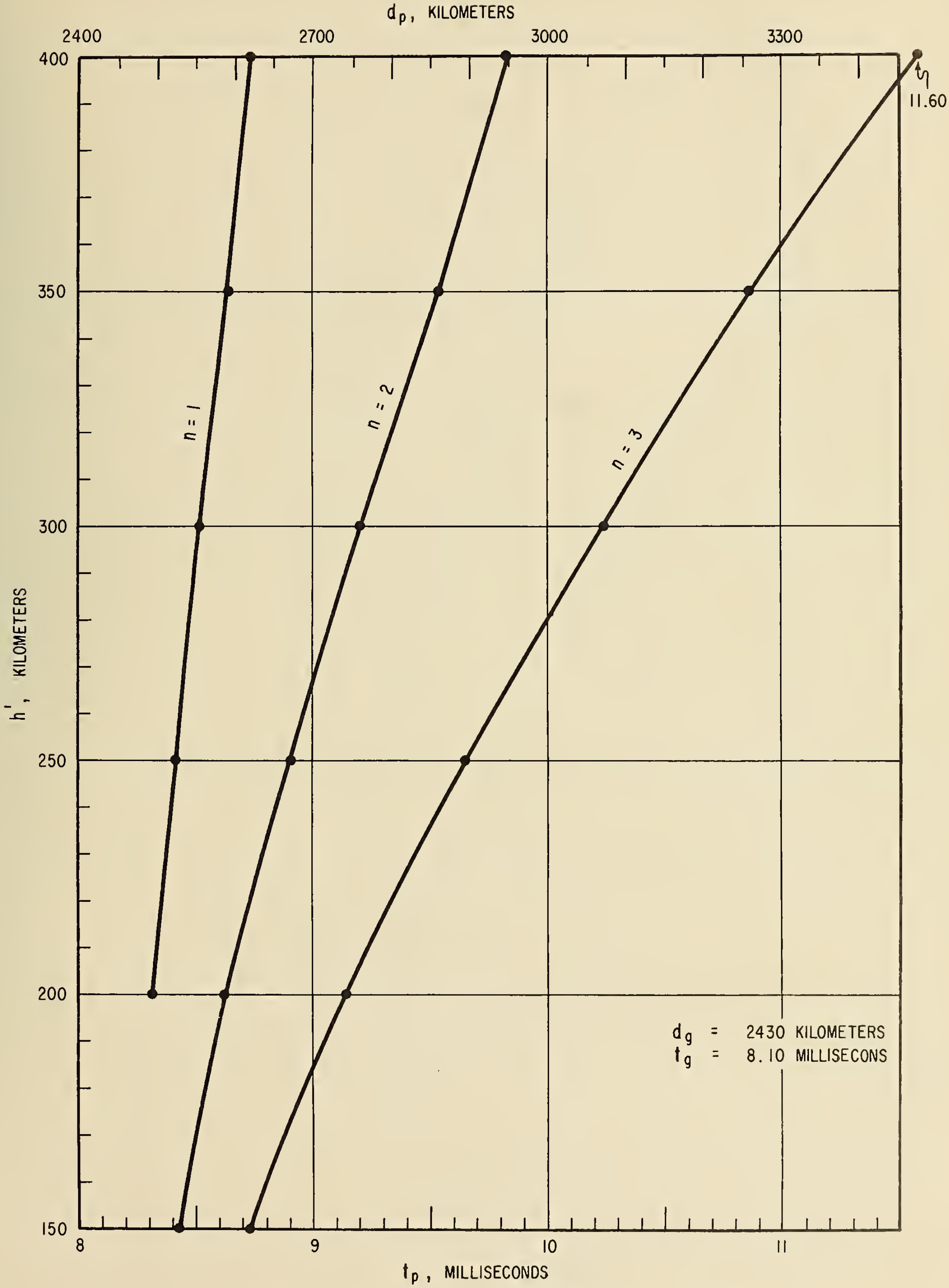


FIG. 17 CALCULATED PROPAGATION DELAY TIME WWV TO BOULDER

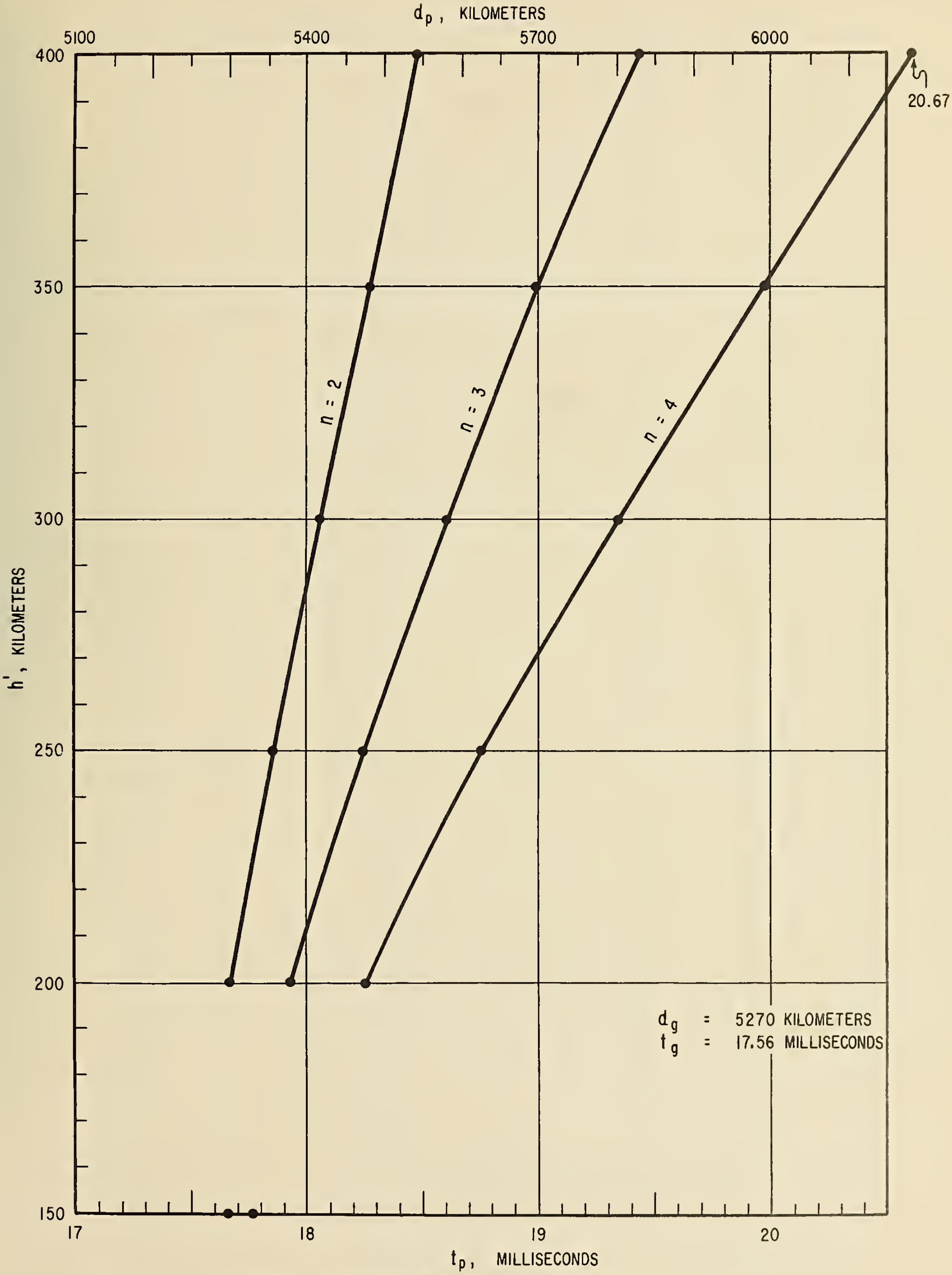


FIG. 18 CALCULATED PROPAGATION DELAY TIME WWVH TO BOULDER

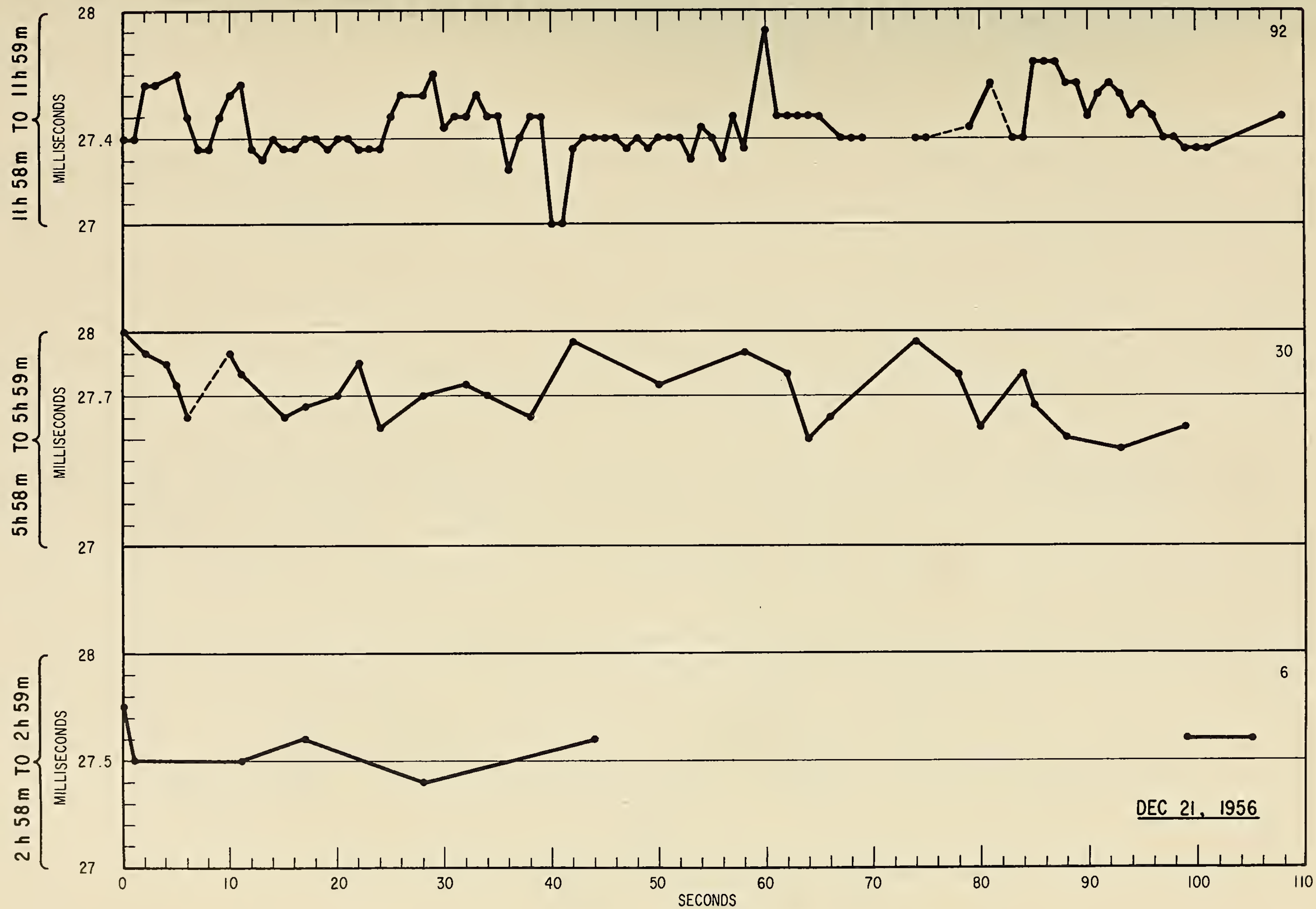


FIG. 19 SECOND TO SECOND MEASUREMENT OF DELAY TIME WWV TO WWVH

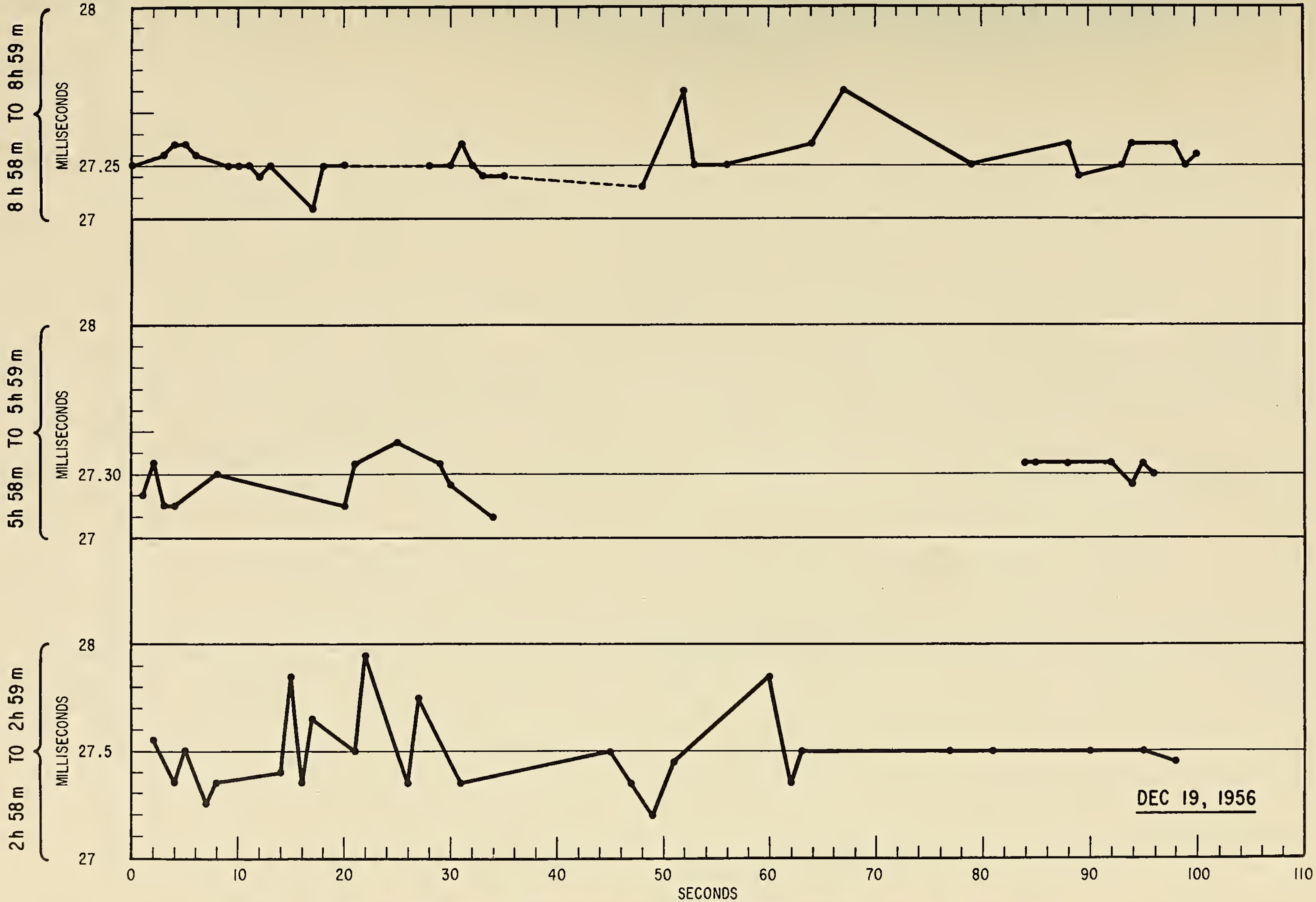


FIG. 20 SECOND TO SECOND MEASUREMENT OF DELAY TIME WWV TO WWVH

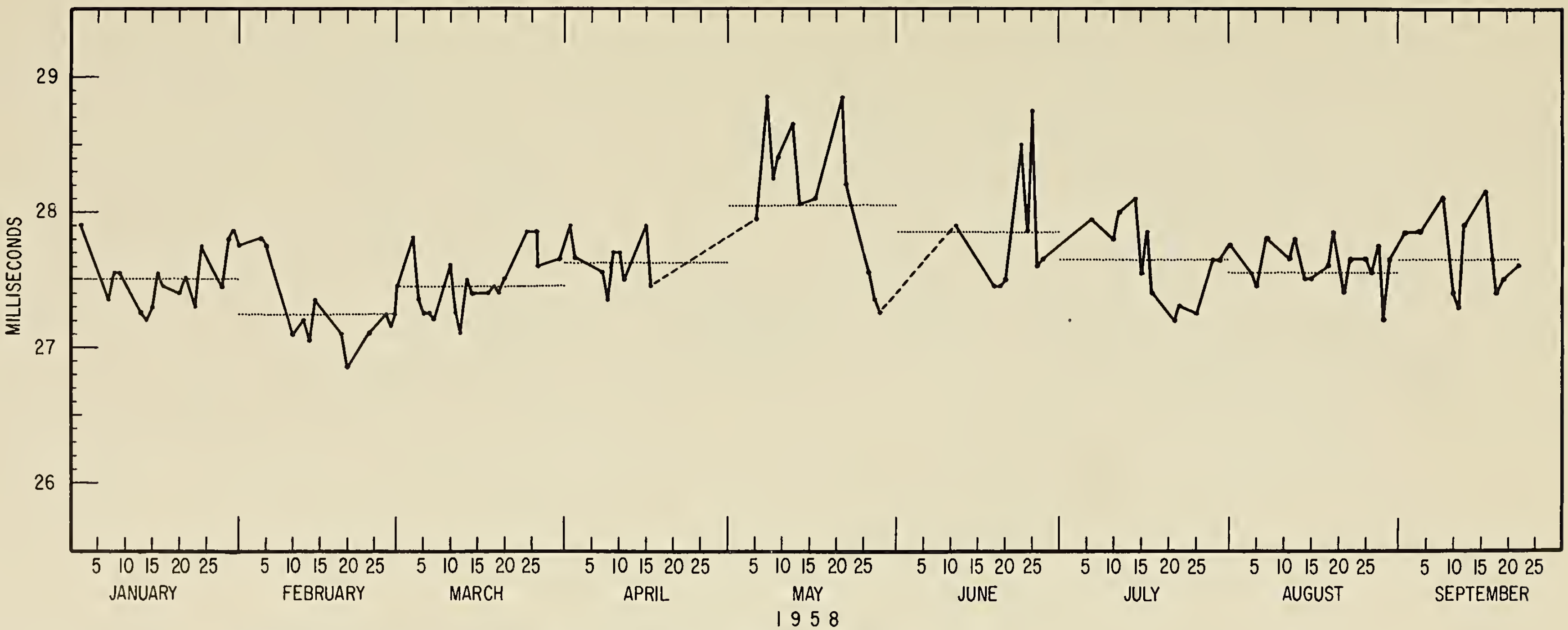


FIG. 21 MEASURED DELAYED TIME ON THE PATH WWV TO WWVH

NBS