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TM 5-235

WAR DEPARTMENT

TECHNICAL MANUAL

SURVEYING

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TECHNICAL MANUAL

SURVEYING

CHANGES No. 1

WAR DEPARTMENT,
WASHINGTON 25, D. C., 18 September 1944.

TM 5–235, 1 October, 1940, is changed as follows:

52. Handling level.

* * * * * * *

c. Steps in leveling.

* * * * * * *

(2) *Example.*—Assume the level * * * or 259 feet. This sight taken on a rod held upon a point of **known** elevation is termed a "backsight" (B. S.), so the equation can be written: $El+B.\ S.=H.\ I.$ The line of * * * $H.\ I.-F.\ S.=El.$

* * * * * * *

55. Profile leveling.

* * * * * * *

e. Profile and grade lines.—Profile and grade * * * on the plot. For a detailed discussion of setting grade stakes and slope stakes, etc., see paragraph **188.**

* * * * * * *

57. Barometric elevations.

* * * * * * *

d. Procedure (Superseded).—With a calibrated aneroid barometer, take the aneroid barometer readings at two stations whose heights are to be compared. Also take the temperature (Fahrenheit) at the two stations. Assume that the following data have been obtained:

	Station	*Barometer*	*Temperature*
△	Beach (at sea level)	29.98	78° F.
△	Top	23.66	70° F.

To compute the difference of elevation between these two stations, obtain from table VII, TM 5–236, the difference of elevation between △ Beach and △ Top and interpolate to the nearest 0.1 foot, as shown here:

△	Beach	Barometer 29.98	Elevation	18
△	Top	Barometer 23.66	Elevation	6,469
			First difference	6,451

Next obtain coefficient (C) from table VIII, TM 5–236, for $(t+t')$ the sum of the thermometer readings, $(78°+70°)=148°$. Interpolating from the table, $C=0.0554$. This multiplied by the first difference

(6,451×0.0554)=357. The latter added to the first difference (6,451+357) gives a final corrected difference of 6,808 feet. Assuming that the true elevation of △ beach is 14.0 feet, the elevation of △ top is 14.0+6,808=6,822 feet.

57.1 Surveying altimeter (Added).—*a. Description.*—The surveying altimeter (fig. 20.1) is an aneroid-barometer type instrument used to determine differences in elevation. There are two classes of altimeters, one with 10-foot divisions and a maximum range of 6,000 feet of elevation, the other with 20-foot divisions and a maximum range of 15,000 feet.

b. Use.—Altimeter leveling is more accurate than barometric leveling (par. 57) and less accurate than differential leveling (par. 54). With the proper corrections applied, altimeter leveling gives results accurate enough for vertical control of multiplex topography (TM 5–240 and 5–244). It is a particularly useful method of providing vertical control over terrain areas within a radius of 8 miles of a starting point when differential-leveling accuracy is not required.

c. Reference.—For detailed descriptions and instructions for using the surveying altimeters, see TM 5–9420 for the 6,000-foot altimeter, and TM 5–9418 for the 15,000-foot altimeter.

65. Transit.

* * * * * * *

b. Verniers.

* * * * * * *

(3) Style *C* represents * * * =20 seconds. *The figure reads 351°30′+5′40″=351°35′40″ from* ***right*** *to* ***left*** *and 8°15′+9′20″=8°24′20″ from* ***left*** *to* ***right.***

(4) Style *D* represents * * * =20 seconds. *The figures read 341°40′+9′20″ from* ***right*** *to* ***left*** *and 358°20′+10′40″=358°30′40″ from* ***left*** *to* ***right.***

* * * * * * *

167. Astronomic triangle.

* * * * * * *

e. Astronomic formulas.—(1) The following formulas * * * the general case.

* * * * * * *

(*h*) *To find local hour angle of a star at elongation.*

$$\cos t = \tan \varphi \cot \delta \qquad (10)$$

* * * * * * *

170. Observation on stars.

* * * * * * *

d. Observation methods for * * * on the star. Usually the movement in **altitude** will be so rapid that it is better to keep the vertical hair on the star and wait a few moments until the star moves to the intersection of the two hairs. This is similar * * * of the sun.

FIGURE 20.1.—Surveying altimeter, 6,000 feet, 10-foot divisions.

[A. G. 300.7 (2 Aug 44).]

BY ORDER OF THE SECRETARY OF WAR:

G. C. MARSHALL,
Chief of Staff.

OFFICIAL:
J. A. ULIO,
Major General,
The Adjutant General.

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IC 5: T/O 5–87; 5–88; 5–327; 5–627; 5–186; 5–187; 5–189; 5–188; 5–167; 5–400, Hq Plat Hq & Sv C AB, Hq Plat Photomapping Co AC, Hq Plat Reprod AD, HQ Plat Engr Avn AE, Opn Sec, Topo CoAF, Photoprinting Sec, Topo CoAG, Geodetic Control Plat AM; 5–500, Plat Hq Sep AA, Co Hq AB, Survey Liaison Det OA.

IC 6: T/O 6–12; 6–20–1; 6–50–1; 6–212S; 6–78S; 6–277T.

For explanation of symbols, see FM 21–6.

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TECHNICAL MANUAL
No. 5–235

WAR DEPARTMENT,
WASHINGTON, *October 1, 1940.*

SURVEYING

Prepared under direction of the
Chief of Engineers

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*The matter contained herein supersedes TR 190–35, March 16, 1925; TM 2180–30, February 18, 1929; TM 2180–35, January 3, 1928; TM 2180–45, April 16, 1928; TM 2180–60, March 1, 1928; and TM 2180–61, July 15, 1927.

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SECTION I

GENERAL

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Purpose and scope	1
Aim	2

1. Purpose and scope.—*a. Purpose.*—This technical manual is intended to provide, in single text form, detailed directions for the care, use, and adjustment of surveying instruments, together with step-by-step descriptions of practical and modern methods applicable to military survey activities. It is also intended to provide instruction in the utilization and exploitation of aerial photographs in connection with military mapping.

b. Scope.—The manual embraces all subjects relating to military surveying, including incidental computations, and contains all relevant matter of the Training Regulations and Training Manuals which it replaces. In addition, a number of new approved methods have been incorporated.

2. Aim.—The aim of this manual is to train personnel as surveyors and to guide such training along efficient lines. Men who are to be trained for the duties of surveyors, especially topographers, should complete exercises 1, 2, 5, 6, 11, 14, 15, 16, and 17 (outlined in TM 5-230) at an early date.

SECTION II

MILITARY SURVEYS

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3. Purpose.—Military surveys are conducted to obtain—

a. Control data (horizontal and vertical) for—

(1) Mapping to any required scale, meeting the battle needs of artillery, infantry, etc.

(2) Construction plots for engineer activities.

(3) Hydrographic surveys.

b. Topographic detail (including relief, names, etc., when necessary) for—

(1) Mapping to scales of from 1:20,000 to 1:62,500.

(2) Construction plots, scales 1:600 to 1:2,400.

(3) Hydrographic surveys of various scales.

c. Special military information for—

(1) Overlays and overprints.

(2) Intelligence purposes.

4. Methods.—*a. General.*—Most military maps are prepared in two distinctly different and yet closely related stages, as follows:

(1) *Field work*, to include establishment of ground control stations; measuring angles and distances between stations; determining azimuths where required; plotting of topography, including relief and names where necessary; and gathering and recording of special military information.

(2) *Office work*, including computations of positions and elevations from data obtained in the field and from other sources; preparation of projections on which the ground control points are plotted; selection and adjustment of supplementary control from aerial photographs; plotting of planimetric detail from photographs; procurement of relief from photographs by photogrammetric methods and by interpolation; compilation of final map sheets including the checking and editing; preparation of progress and other data plots and sheets; tracing for overlays and overprints; the preparation of special charts, drawings, and photomaps; and the filing of all data, records, and originals. (These are important functions and must be provided for regardless of whether many are later referred to higher authority or other disposition made.)

b. Control.—The *horizontal control*, consisting mainly in positions of triangulation or traverse stations, is plotted on a polyconic projection, or on a superimposed grid if the coordinates have been converted.

The *vertical control* consists of bench marks, their elevations being found by differential leveling, and also the horizontal control points, of which the elevation is usually determined by trigonometric leveling. For military surveys, the elevations of all horizontal control points are generally determined. Supplementary control for fixing the position of additional ground control points may be obtained by intersections, resections, and similar methods described in sections XX to XXIII, inclusive. These methods include both triangulation and traversing. Further supplementary control for the horizontal location of points may be obtained by graphic methods. Points so obtained are often called photographic control points, their positions being adjusted to the ground control points during the plotting of graphic triangulation by one of the photographic control and adjustment methods described in section XVI, TM 5–230.

c. Planimetry and relief.—Planimetry or planimetric detail on a map includes the natural and cultural features usually represented by conventional signs (FM 21–30). Maps showing only planimetric detail are *planimetric maps* or *line maps*. Such maps may be of considerable military value in emergencies when more complete maps are not available. A complete topographic map for all around military use shows relief, that is, variation in elevations of the ground surface. This is indicated by hachures, shading, or more accurately by contours lines. The latter method of showing relief is preferred and used on most military maps. The draftsman performs the bulk of work pertaining to planimetry and relief, obtaining such detail, so far as possible, from aerial photographs.

5. Classifications.—Military surveys include all field work and computations for the purposes described below:

a. Topographical surveys.—These are conducted for the purpose of preparing accurate and detailed maps of particular areas, with reference to features of terrain.

b. Fire control data.—The 1:20,000 contoured battle map is used for target designation and tactical and technical purposes by various arms of the service. When such maps are not available, provisional maps, battle maps without contours, and others must be used. In any case, it is necessary to obtain and supply additional data by extending the control from the nearest known point or points in order to establish, within the field artillery battery and observation point areas, the necessary number of control points with their interrelated directions, positions, and elevations. Establishment of control is usually by triangulation, but may be accomplished by traverses, intersection, resection, or other methods described herein.

c. Location surveys.—(1) Location surveys (sec. XXXI) are conducted for the purpose of determining and laying out on the ground the final location of roads, railroads, and other large construction projects, such as semipermanent mobilization centers, training camps, water reservoirs, etc. When no suitable map of the area is available, location surveys include the preparation of a sketch map, etc., requiring the establishment of some hasty control. When photographs are available, a more or less accurate topographic map may be developed from them.

(2) Often several alternate routes or sites are selected from an existing map. It is the mission of the location survey to obtain, when necessary, additional data to establish the projected route or site in its most suitable and economic location.

(3) Traverses, either with tape or stadia, together with level lines, are generally used to establish the preliminary as well as the final location of routes or construction sites.

d. Construction surveys.—Construction surveys (sec. XXXII) entail the establishment by instrumental methods of all stations and points required before and during the progress of an engineering project. This includes the setting of center and grade stakes.

e. Hydrographic surveys.—These include soundings for navigation and construction purposes or for the determination of discharge of streams etc.

f. Utilities maps.—These maps are usually prepared to a large scale and are necessary for the proper operation and maintenance of buildings, lines of communication, and pipe lines. Topographic detail thereon is plotted to a minimum degree.

SECTION III

AERIAL PHOTOGRAPHS

Paragraph
Importance 6
Use 7

6. Importance.—A most important contribution to surveying and mapping has been the development of instruments and methods for the preparation of maps from aerial photographs. Present-day mapping practice dispenses with practically all field work except that necessary for establishing ground control and for plans or plots drawn to large scales.

7. Use.—*a. General.*—Aerial photographs of various types (sec. V, TM 5-230) are used whenever available to supply the planimetric data necessary for complete topographical maps, including relief.

The transformation of such photographic data into map data is exclusively the function of the draftsman and photogrammetrist, and since clear, distinct, and perfect photographs supply all data except ground control, the surveyor's principal task is mainly that of establishing adequate ground control and collecting such auxiliary information as may be needed in any particular case. When details on photographs are obscured or indistinct, their interpretation becomes uncertain or impossible. At other times, due to unsatisfactory weather conditions, mechanical imperfections of the equipment used, or other causes, certain portions of a photographed area are not shown on the photographs. In such cases the surveyor must obtain the missing data by field methods. See section XXVI.

b. Planimetric detail and contours.—Aerial photographs are the principal source for obtaining planimetric detail and data for the representation of relief by either contours, form lines, or spot elevations. For hasty maps, planimetric detail is obtained by tracing the desired features from the photographs, while form lines, contours, etc., are interpreted with the simple stereoscope, preferably the mirror type, as explained in section VII, TM 5–230. For more accurate maps these data may be obtained by the draftsman as described in section XVIII, TM 5–230.

c. Hasty guide maps.—Aerial photographs, single verticals, composites, or mosaics (sec. V, TM 5–230), or rough tracings from them (with or without contours, form lines, or spot elevations), serve as excellent guide maps. Very little data, if any, need be obtained in the field to supplement the topographic detail for such maps. Sometimes it may be desirable to determine the positions or elevations of a few important points by one of the time-saving methods described in paragraph 158 in combination with one of the rapid instrumental methods explained in paragraphs 80, 81, 121, and 124.

d. Supplementary and vertical control.—When existing ground control is insufficient, aerial photographs may still provide horizontal and vertical control. The transformation of photographic data into supplementary control is made by the drafting room personnel using one or more of the methods described in section XVI, TM 5–230. At times, the surveyor may have to spot some of these supplementary control points on the ground or plot some ground control points on the photographs. For detailed instructions, see section XXVII.

e. Maps of enemy territory.—Successful operations in time of war depend to a great extent on the availability of up-to-date maps of the enemy's territory. Such maps should show, besides topographic features, the position of the enemy's forces and all other military in-

formation that can be obtained. Aerial photographs can be depended upon more than any other means to furnish the desired data. Here again, the draftsman will prepare the complete map from the photographs supplied him, while the surveyor will obtain the data for a few advanced ground control points usually located in the enemy's territory. These points together with those suitably located in friendly territory will constitute the ground control on which the map is based. Advanced ground control points in enemy territory must be objects that can be distinguished on the photographs. Their position and elevation are determined by one of the intersection methods described in section XX.

f. Quick reconnaissance.—Aerial photographs, if the area in question is adequately covered by them, are excellent as a basis for quick reconnaissance purposes. They may serve for the preliminary location and planning of engineering projects and, on an enlarged scale, for final location plans for construction projects.

g. Map substitute.—Aerial photographs will often supply needed information when a more complete map is not available. Such map substitutes may be single vertical photographs, composites, or mosaics, either with or without the additional information listed in section IX, TM 5–230.

h. Intelligence purposes.—Aerial photographs are indispensable for interpreting much detail of military value and are an important factor in determining the assignment and distribution of troop units. As a rule, no field work is required for aerial photographs that are to be used for intelligence purposes.

i. Revision of maps.—Periodic rephotographing of areas already mapped will generally supply all data necessary for a revision of existing maps. Since revision is usually confined to planimetric data, it is seldom necessary that field parties obtain data on the ground in connection with such work. Only when photographs show developments to such an extent that additional control is required will the surveyor have to cover the parts affected by the changes.

j. Mosaics.—A picture made by joining several adjacent photographs to make a continuous picture of a larger area is called a mosaic. Single vertical photographs, are normally used for making mosaics. The making or "laying" of mosaics is done in the drafting room. For a detailed discussion of mosaics, both controlled and uncontrolled, see sections VIII and XXII, TM 5–230.

SECTION IV

PROGRESSIVE STAGES OF SURVEYS—SURVEY PRINCIPLES

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8. General.—Surveying is the art of measuring distances and angles for locating points on the surface of the earth. When the area to be surveyed is small so that the effect of the earth's curvature may be neglected, it is called *plane surveying;* when the area is larger so that the effect of curvature of the earth must be considered, it is called *geodetic surveying.* On all maps or plans used for military purposes, distances between points on the earth's surface are plotted as *horizontal* distances. Consequently, all distances measured on slopes must be reduced to their horizontal equivalents before being plotted. (See pars. 80*c*, 105*d*(2), and 106*e*(4).) Surveys for military maps are to be performed in accordance with the survey specifications (sec. V), and must conform in all respects to established standards of surveying practice. A logical and orderly arrangement of the work is necessary to secure the best results within the time specified. In general, the survey work may be divided into three distinct stages: preparation (pars. 9 to 11); control (pars. 12 to 16); and topography (pars. 17 to 18). This section describes in general terms the various stages of an average survey, the main classifications of the work following each other in logical order.

9. Request for photographs.—A request through proper channels for photographs of the area to be mapped should be the first step in the preparatory stage of the survey. This request should include a detailed statement as outlined in section IX, TM 5-230. Whenever suitable photographs from other Government agencies are available they should be utilized.

10. Collection of data.—As soon as instructions for executing the survey of a certain area are received, arrangements should be made through the office of the Chief of Engineers, Washington, D. C., or through the unit engineer of the unit to which the survey organization is attached, to obtain all existing data that may speed the mapping project. Such data may include photographs, maps, and plans, plus control data from any governmental agency (Federal, State, county, or municipal) and from any public or private source, such as railroad companies, private mapping firms, and others. After these data are received, they are scrutinized and all suitable material listed. Utilization of such data not only avoids duplication but saves considerable time and effort.

11. Conversion of data.—*a.* All data received from any source whatsoever and listed as suitable must be checked, and any data found expressed in terms other than those used for the survey must be converted. In order to provide data conforming to the adopted grid system, existing tables, etc., all measurements and computations relating to grid triangulation and traverse are made in yards and fractions of yards, so that the plotting of points will conform to grid coordinates (in yards) and azimuths from grid north.

b. Triangulation of the third and higher orders is computed in geographic coordinates, and some agencies, as the United States Coast and Geodetic Survey, the United States Geological Survey, and others, will often furnish control data for triangulation points, giving their position and location by geographic coordinates, geographic azimuths, and metric distances. These data should be converted into grid coordinates, grid azimuths, and yard distances so that the information will be readily available to field parties and office personnel alike (par. 117*g* and *h*).

12. Reconnaissance for control.—Reconnaissance preliminary to the control will be chiefly for triangulation as described in section XV. It should be thorough enough to serve for selecting the most practical locations for triangulation points, for locating signals and scaffolds, for clearing lines of sight, for obtaining consent of the owner if on private land, and for pointing out available camp sites.

13. Control by triangulation and trigonometric leveling.—The main control for maps of extended areas is generally established by triangulation. Determination of elevations of triangulation stations is made by trigonometric leveling, a method which uses the computed distance and the vertical angle between the points observed. (See pars. 110, 117*h*, and 118*n*.)

14. Supplementary control.—*a. Traverses* —When the area to be mapped is small or where triangulation would prove highly uneconomical, some main control points may be established by chained traverses as described in paragraph 79. In general, elevations for these traverse stations are obtained by differential leveling (par. 54), but vertical angles may be read to obtain their elevations and adjust them to conform to those of the initial and terminal points of the traverse. Such traverse must always start from an adjusted triangulation point or traverse point of equal or higher order and be tied to another such point. Traverses for supplementary control are usually of the type described in paragraph 80. They are called azimuth traverses with transit and stadia. Elevations for stations of stadia traverses are, as a rule, obtained by trigonometric leveling (vertical angles).

b. Level.—Elevations for supplementary control stations are sometimes established by differential leveling. This is done especially when points for which accurate elevations are required are far apart, or when determining elevations for bench marks.

c. Special and miscellaneous methods.—Very often some supplementary control, horizontal and vertical, of a more or less accurate nature must be established for various purposes, especially for hasty or provisional maps and for fire control data. Depending on local conditions, one or more of the different methods described in sections XX to XXIII, inclusive, will be applicable.

15. Computation and adjustment of horizontal and vertical control.—As soon as sufficient control has been established and its accuracy verified in the field, the data are turned over in original field notebooks or records to the computing section for adjustment. Methods for computing and adjusting control data will be found in the sections describing various cases.

16. Compilation and transmission of control data.—After the control is computed, adjusted, and checked, the resultant data are compiled (pars. 117*i* and 118*p*) and the transferred figures are again checked. Photographic copies of the record sheets are then made for use of the field and office force. Compilation and transmission of control data must conform to instructions in the specification of the survey.

17. Topography.—*a. Supplementing detail from photographs.*— Field observations are sometimes required to provide topographic details lacking in aerial photographs. This is usually a relatively simple task.

b. Instrumental methods.—Topographic detail for construction maps, hydrographic charts, utilities maps, etc., especially when their scale is 1 : 2,500 or larger, will in general be plotted from measurements in the field. Ordinarily this would be done with the plane table as explained in sections XXV and XXVI.

18. Compiling topographic detail.—*a. New maps.*—This is the exclusive function of the draftsman. See section XX, TM 5–230.

b. Revision of maps.—This, too, is a duty of the draftsman who may employ one or another of the restitution methods (sec. XXI, TM 5–230) in conjunction with the procedure of section XX, TM 5–230.

19. Cooperative efforts.—*a. Advance sheets.*—It is often necessary to provide some sort of map with latest information before the final map can be even partially completed. Surveyors, therefore, should arrange their work so that each phase of it will form a continuous part, adjacent to similar work previously completed or worked on by other parties. The idea is to get a map with at least the minimum requirements and without delay into the hands of those who have an immediate use for it.

b. Suitability of field sheets for immediate reproduction.—Field sheets or sketches that show topographic detail and other data which in itself would make an acceptable map for emergency use may be reproduced photographically or lithographically. Field sheets on transparent vellum may be reproduced by blue print or a similar process when quick reproductions are needed in an emergency.

c. Ready reference file for control data, etc.—A reference file for control data, all properly indexed and diagramed, should be kept at the headquarters of each mapping unit. Such file should include all previously established and all new control stations.

20. Survey principles.—*a. Surveying logic.*—The field work of surveying consists wholly of measuring distances and angles, which, after certain computations and corrections have been applied, will serve as a structural base for the required map or plot.

b. Consistent accuracy.—Since no measurement can ever be made exactly, an allowable error is usually specified. Variations allowed are proportional to the magnitude of measurements and depend, also, upon the scale of the finished map or plan. In general, the allowable error should be such that any point affected by it, after being plotted on a map or plan, will appear to be in its accurate position. Consistent accuracy is a relative term embracing the observation, computation, and plotting of related phases of the survey. For instance, the primary control of the survey may have to be accurate to within 1 in 50,000, while the measurements pertaining to topographic detail

need only be accurate to within 1 in 1,000. Speed is inversely proportional to accuracy. Allowable error must not be confused with probable error, which involves a mathematical predetermination of the suitability of observed values for inclusion in calculating final results, the latter usually not to exceed a certain specified allowable error. Determination of probable error is chiefly confined to work of great precision, and is described in paragraph 105*d*(12).

c. Factors influencing selection of most suitable methods.—Time is usually the principal factor to be considered in selecting methods of prosecuting a survey. Other main factors are the availability of essential equipment and properly trained men.

d. Independent check of all work.—In order to avoid errors and eliminate a great deal of unnecessary time-consuming work, the surveyor must take every precaution and have his work checked at each step.

e. Foreseeing revision.—Surveyors should provide for future revisions by establishing all principal control points so they can be readily found, and by preparing and transmitting all pertinent data in the form of a permanent, accessible record.

SECTION V

SURVEY SPECIFICATIONS

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Scale, name, and size of maps (or photomaps)	25
Marginal data	26
Control methods	27
Permanent and semipermanent monuments and bench marks	28
Topography	29
Special military information	30

21. General.—*a. Uniformity.*—All military surveys are conducted according to specifications based on AR 300–15, on specifications approved by the Federal Board of Surveys and Maps, and such special instructions as may be published from time to time by the office of the Chief of Engineers or other competent authority. AR 300–15 designates what data are to appear on the finished map or photomap. Technical specifications describing methods to be used in obtaining these data are contained in the directions issued by the other agencies named above. Every surveyor should be familiar with the general requirements of AR 300–15.

b. Purpose of specifications.—Specifications for surveys have the effect of orders and are issued to coordinate the various phases of the work. Compliance with specifications avoids duplication of effort and waste of time, and results can be accepted with confidence by the various survey agencies of the Federal Government for incorporation into later surveys.

c. Contents.—Survey specifications should contain specific directions for performing every step of the work relating to the project, and the surveyor should study them carefully before commencing work. He should look especially for instructions pertaining to the items mentioned in this section and strictly conform thereto.

22. Purpose of survey.—The purpose of the survey should be clearly stated so that the surveyor knows from the beginning just what is required. This will enable him to arrange his work for efficient prosecution and timely completion.

23. Type of projection and grid.—*a. Projection.*—Unless otherwise stated, all survey data will be based on the polyconic projection (sec. XI, TM 5–230). (A large amount of geographic data for triangulation points were observed and computed by the United States Coast and Geodetic Survey and other mapping agencies prior to 1927. Additional observations and recomputations have been made and adjusted to what is known as the "North American Datum of 1927.")

b. Grid.—For reference purposes and for locating any point quickly and accurately it is necessary to have a system of rectangular coordinates on every military map. A uniform system called the "Grid System of the United States" has been adopted for military use. (See sec. XI, TM 5–230, and Special Publication No. 59, U. S. C. & G. S.) The rectangular lines of the grid system are often referred to simply as "the grid." In emergencies, when the standard grid cannot be determined and used (or on photomaps, mosaics, etc.), a temporary or other grid, as outlined in AR 300–15, may be specified and used.

24. Geographic index.—For ready reference and filing purposes each map is marked with its geographic index symbol. This symbol consists of letters and numbers indicating the geographic coordinates of the southeast corner of the map and the extent in minutes of latitude and longitude from that corner. For example, for a 15-minute quadrangle whose southeast corner is the intersection of 38°15′00″ N. latitude and 79°30′00″ W. longitude, the index symbol would be N3815–W7930/15. See AR 300–15.

25. Scale, name, and size of maps (or photomaps).—The scale and size of finished maps are specified in AR 300–15.

26. Marginal data.—Marginal data will also conform to directions in AR 300–15. Specifications for marginal data on maps will normally be separate from that of photomaps. Section IX, TM 5–230, illustrates a standard form for marginal data on military maps.

27. Control methods.—*a. Horizontal.*—Specifications for horizontal control usually state to what extent triangulation or traversing is to be used, and prescribe the allowable error of closure for each method. Surveys are frequently made with the object of establishing principal control points for a large area by triangulation of a higher order. From these principal control points, additional control by minor triangulation methods or traverse is generally established so that for a given area a certain number of control points are available.

b. Vertical.—Elevations of principal points and bench marks are determined by methods prescribed in the specifications (par. 50). The surveyor is responsible that the allowable error for the method described is not exceeded. Detailed instructions for the various control methods are found in section IX, and pars. 110 and 117*i*.

28. Permanent and semipermanent monuments and bench marks.—*a. Frequency of monuments.*—All monuments and bench marks are to be constructed in accordance with the specifications of the survey, and instructions relating to the placing of them in definite locations must be strictly observed. Frequency of monuments differs with the required number of control points in a given area and is determined from the purpose for which the map or survey is made. For detailed descriptions concerning monuments and bench marks see paragraphs 50*b* and 98*j*.

b. Record of location.—A detailed record of the location for every new permanent monument or bench mark must be prepared by those engaged in establishing them and determining their locations. Such record will include a detailed word description of the monument or bench mark and its reference marks, together with directions to reach it, whether or not a diagram or sketch showing its position in relation to its immediate surroundings is also included. This record must also include a list of accurate directions and distances from each monument to its reference marks to aid in the recovery of the monument or bench mark if its location should become obscured or lost. Records of location for permanent monuments or bench marks are transcribed into a permanent control record.

c. Secondary control points.—All supplementary or temporary control points, such as traverse stations, intermediate (or temporary) bench marks, intersected points from minor triangulation, etc., are

considered secondary or semipermanent control points. With few exceptions, they are for use of the personnel engaged in the current survey only and are not included in the permanent control record. However, supplementary control points must be established as directed in the specifications and should be of sufficient stability to serve during the progress of the survey without the necessity of reestablishing them. Complete descriptions of supplementary control points are usually made in the field notebooks and transcribed therefrom as specified.

d. Reference and azimuth marks.—Reference and azimuth marks serve as an aid in identifying permanent control points and in their relocation or recovery. Reference points or marks (which should never be less than two in number for any triangulation control point) must be of a permanent character as specified and are usually set within a few hundred feet of the station and in well-distributed directions therefrom. The accurate azimuth and distance from the station to each reference point are determined and recorded with the permanent control record. It is of great advantage to determine and record additional azimuths from the control point to other prominent objects, such as church steeples, water towers, etc., which can be used as azimuth marks for starting traverses, etc., without the necessity of reerecting signals over other control stations. Reference points for traverse stations that serve as control points should never be less than two in number. They are established principally to facilitate certain recognition of the traverse station, and, while perhaps of a less permanent character than those for triangulation points, should nevertheless be substantial enough to last until the usefulness of the traverse station, of which they are a part, has come to an end. Data pertaining to reference points for traverse stations are recorded and sketched in the field notebook and transcribed therefrom as specified.

29. Topography.—*a. Requirements as to detail.*—While the directions in the survey specifications pertaining to topographic detail in most cases concern the draftsman rather than the surveyor, the latter should familiarize himself with the requirements as to the amount and character of topographic information on the final map, in order to execute his part of the work in the field to the best advantage of all those engaged in the execution of the project. Detailed instruction for obtaining topographic detail by usual methods will be found in sections XVIII, XIX, and XX, TM 5–230.

b. Contour interval.—The specifications will prescribe the contour interval to be shown on the finished map. This, too, is usually the work of the draftsman. If contours have to be obtained in the field,

the surveyor will comply with the specifications and follow the methods explained in paragraph 154*c*.

c. Special requirements.—The surveyor will carefully scrutinize the specifications as to special requirements for topographic detail or other data relating thereto that he may have to obtain in the field. He will have to perform such work quite frequently, especially on large scale maps, and should be ready at all times to do this methodically and efficiently.

30. Special military information.—*a. Preparation of overlays and overprints.*—Specifications for military surveys often contain directions for producing maps, or overlays for them, showing features or detail of special military value. The type of detail usually required is of such character that data for some of the features must be obtained by field observations. These data together with data from aerial photographs are then compiled by the draftsman and shown on the map itself or on overlays.

b. Collecting data for overlays and overprints.—Specifications will prescribe the data to be collected in the field.

c. Records of special information.—Frequently, in connection with other field work, the surveyor must collect and record special information of military value that cannot otherwise be shown, either on a map or an overlay. In such cases the surveyor must keep an accurate record of all information to be obtained as directed in the specifications.

SECTION VI

GENERAL INSTRUCTIONS FOR FIELD WORK

	Paragraph
General	31
Code of signals	32
Field notes	33
Sketching and filling-in topography	34

31. General.—*a. Speed and accuracy.*—Best work is accomplished by properly combining these two generally opposing factors. In war, emphasis is upon speed while in peace there may be greater need for accuracy. In any case proper balance should be effected in accordance with the situation.

b. Studying of specifications.—A mission to be accomplished must be understood. It is therefore necessary to read and reread the specifications, and to study each small requirement in detail.

c. Inferior instruments.—Poor instruments are no excuse for poor work.

d. Assignment of duties.—In training, the members of each party should alternate in discharging the several kinds of service involved in field problems, unless otherwise instructed. Training in the subordinate positions is essential whether the beginner is to occupy them in actual practice or not, for intelligent direction of work demands thorough knowledge of all its details.

e. Care of equipment.—(1) *Responsibility.*—Each man is responsible for the proper use and safe return of his equipment. *All cases of breakage, damage, loss, or misplacement must be reported promptly.* Equipment should be examined when assigned and a report made at once of any injury or deficiency found.

(2) *Range poles.*—Range poles should never be unduly strained, and their points should be properly protected.

(3) *Chains and tapes.*—Chains should not be jerked, nor tapes stepped upon. Kinks in steel tapes must be avoided, particularly in cold weather. Band tapes will be done up in 5-foot loops, figure-of-eight form. Etched tapes should be wiped clean and dry at the end of the day's work, and should be oiled.

(4) *Axes and hatchets.*—Axes and hatchets will be employed for their legitimate purposes only. Their wanton use in clearing survey lines is forbidden, and their use at all for such purpose on private property must be governed strictly by the rights of the owner.

(5) *Plumb bobs.*—Care must be taken to protect the point of the bob and to avoid needless knots in the string.

(6) *Stakes.*—The consumption of stakes should be controlled.

32. Code of signals.—In order to transmit information between the instrument man or observer and his rodmen, a maximum use will be made of hand signals. Figure 1 shows a code of signals used almost universally. The illustrations clearly indicate the position and movement of arms for the various signals.

33. Field notes.—*a. General.*—In addition to the title of the job and the record of the data observed, the field notes should include the date, organization of party, equipment used, time devoted, and any other information which is at all likely to be of service in connection with the work. No item properly belonging to the notes should be trusted to memory. Data relating to different jobs will not be recorded on the same pages. The notes should be arranged systematically and neatly. They should be perfectly legible and easily understood. Each recorder must keep complete notes of each job. Field notes must not be taken on loose slips or sheets of paper or in other notebooks, but the original record must be put in the field notebook during the progress of the field work.

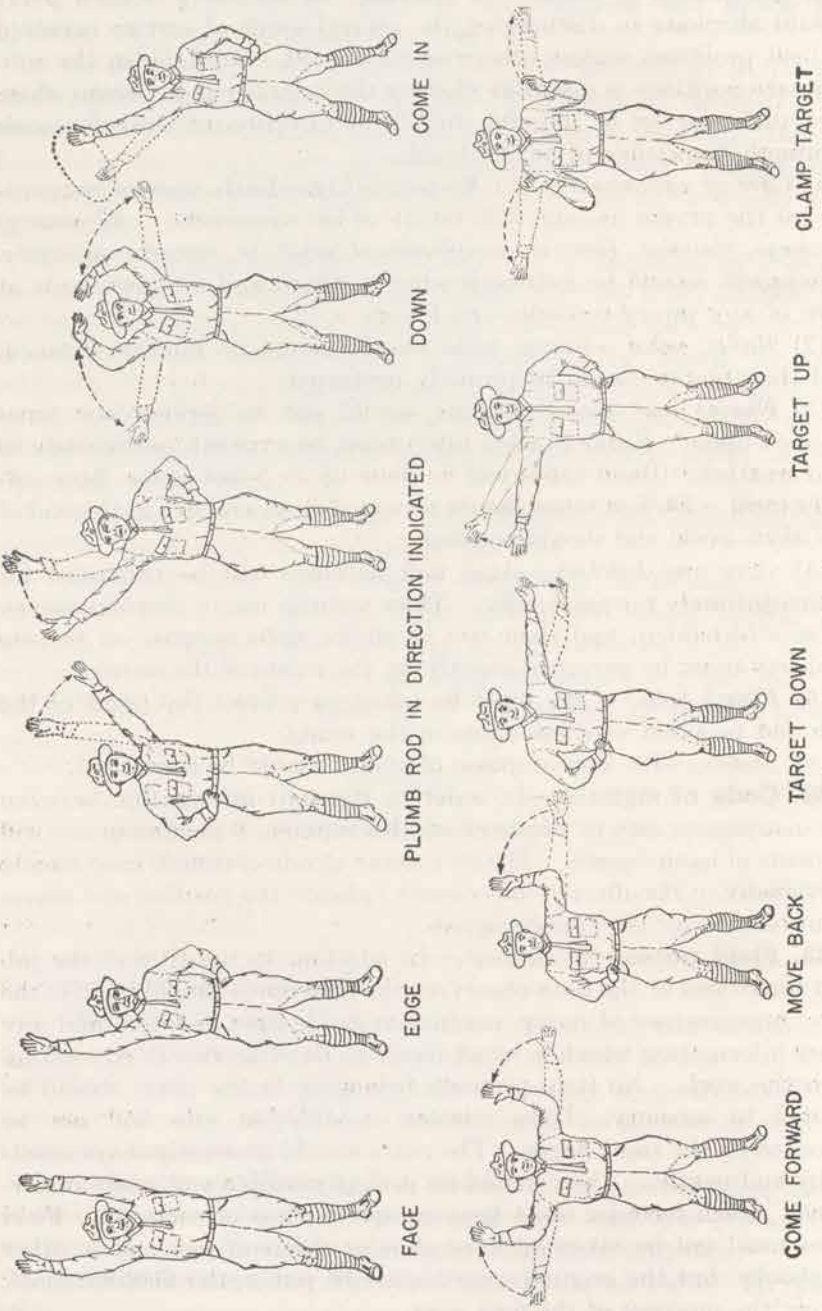

FIGURE 1.—Code of signals.

b. Field notebook.—The field record must be kept in the *prescribed* field notebook. For purposes of identification the name of the recorder will be printed in bold letters at the top of the front cover. The name of the project, type of field work, and serial number of the book should be plainly marked on the front cover and on the outside of the binding edge.

c. Pencil.—To insure permanency, all notes will be kept with a pencil, preferably a 4H or 5H. The pencil should be kept well sharpened and used with sufficient pressure to indent the surface of the paper somewhat in order to insure permanence of record.

d. Title page.—An appropriate title page, as specified, will be printed on the first page of the field notebook.

e. Indexing and cross referencing.—A systematic index of the field notes will be kept on the two pages following the title page. Related notes on different pages will be liberally and plainly cross-referenced. The pages of the notebook will be numbered to facilitate indexing, two pages facing each other to constitute one page.

f. Methods of recording.—There are three general methods of recording field notes, as follows:

(1) By sketch.
(2) By description or narration.
(3) By tabulation.

It is not uncommon to combine two or perhaps all three of these methods.

g. Form of notes.—The chief of party or recorder will devise a form in cases where a certain form is not prescribed.

h. Lettering.—Field notes will be printed in the style of freehand lettering. Lower case letters will be used in general, capitals being employed for initials and important works, as required.

i. Field note sketches.—Sketches will be used liberally in the notes and will be made in the field. If desired, a ruler may be used in drawing straight lines, but the recorder should develop skill in making good, plain freehand sketches. The field sketches should be bold and clear, in fair proportion, and of liberal size so as to avoid confusion of detail. The exaggeration of certain details in a separate sketch sometimes adds greatly to the clearness of the notes. The sketches should be supplemented by descriptive statements, and important points of the sketch should be lettered for reference. The precise scaling of sketches in the field notebook, while sometimes necessary, is usually undesirable owing to the time consumed.

j. Numerical data.—The record of numerical data should be consistent with the precision of the survey. In observations of the

same class a uniform number of decimal places should be recorded. When the dropped fraction in a result is exactly one-half the smallest unit or decimal place observed, the *nearest even unit* (divisible by 2) is recorded. Careful attention should be given to the legibility of numerals.

k. Erasures.—Erasures in the field notes are not permitted. In case a figure is incorrectly recorded, it should be crossed out and the correct entry made above the wrong figure. The neat cancelation of an item in the notes inspires confidence, but evidence of an erasure or alteration casts doubt upon their genuineness. Rejection of a page of notes should be indicated by neatly crossing the entire page, and cross reference should be made between the rejected and substituted pages.

l. Transcript of notes and results.—Office copies of field notes or results therefrom will be submitted promptly as required. These copies must be actual transcripts from the original record contained in the field notebook of the individual submitting the copy.

m. Completed notes.—The field notes must be kept in shape for inspection at any time, and be submitted on call. All calculations and reductions must be kept up to date. The recorder is expected to criticize his own notes and submit them in as nearly perfect condition as possible. The notes are incomplete until each page has been checked and initialed by the observer.

n. Calculations.—All field calculations and reductions of a permanent character must be shown in the field notebook in the specified form. Cross references between field data and calculations must be shown. Consistency between the precision of the computed results and that of the observed data should be maintained. Computed results should be verified habitually, and the verified results indicated by a check mark. Since most men are prone to repeat the same error, it is desirable in checking calculations to employ independent methods and to follow a different order. A fruitful source of trouble is in the transcript of data, and this should be checked first when reviewing doubtful calculations. Skilled computers give much attention to methodical arrangement and to contracted methods of computing and verifying results. Familiarity with the slide rule and other labor-saving devices is important.

34. Sketching and filling-in topography.—Modern mapping methods require very little if any sketching and filling-in of topography in the field. It will probably be resorted to only when aerial photographs are indistinct or not available.

SECTION VII

GENERAL INSTRUCTIONS IN USE AND CARE OF INSTRUMENTS

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Tools and supplies	36
Cleaning and oiling	37
Minor repairs	38
Replacing cross hairs	39
Telescopes and verniers	40
Parallax	41
Level vials	42
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35. Packing and shipping.—*a. Instruments cases.*—Surveying instruments should always be carried or transported in the special cases provided for them. These cases are for carrying purposes only and when shipped should be enclosed in another box. Since instruments always come in rigid contact with some part of their special cases, a cushion of elastic material should be placed on all sides between the case and the shipping box to prevent the direct transmission of shocks to the instrument from rough or careless handling in transit. As an additional precaution against injury, particularly from loose parts, the instrument case should be filled with soft, dustless material.

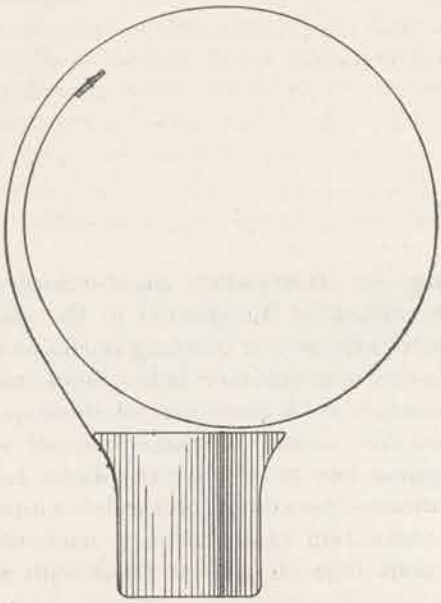
b. Packing boxes.—Packing boxes should be made of light, sound lumber, securely and rigidly put together. Half-inch boards for the top, sides, and bottom and 1-inch boards for the ends are generally sufficient for single instruments. For more than one instrument, or for large instruments, 1-inch lumber should be used. Transits, levels, and other delicate instruments will be less liable to injury if the packing box is provided with a carrying strap or rope to facilitate gentle handling in transit.

c. Shipping directions.—Packages should be marked "Delicate Instruments, This Side Up, Handle With Care," and shipments should be made by express rather than by ordinary freight.

d. Pack transportation.—(1) In the mountains where pack trains are the sole means of conveyance, the animal to carry surveying equipment should be well-trained, sure-footed, and under constant supervision.

(2) The transit and level outfit is most conveniently carried in a pair of canvas pack bags, which must be properly balanced on the animal. The tripod, level rod, flagpole and stadia rod, and similar

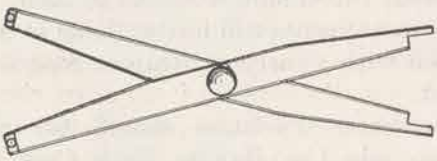
equipment should be placed lengthwise and on top of the animal, lashed to the saddle, and further balanced by properly disposing them on either side of the center. A canvas pack cover should be thrown over the whole, tucked in on all sides, and tied in place.



① Jeweler's glass with hairspring.

② Special pin wrench.

③ Jeweler's screw driver (swiveled).



④ Special adjustable pin wrench.

FIGURE 2.—Special tools.

(3) The plane table outfit is best packed by hanging the table in its case flat against one side of the animal and balancing it with the alidade and smaller articles in a pack bag on the other side. The tripod and stadia rods are lashed lengthwise to the saddle along the top, with the heavier parts, such as the head of the tripod, toward the head of the animal. The arrangement must be such that pro-

jecting ends will not strike the animal. A canvas pack cover should cover the whole.

36. Tools and supplies.—Each survey company should be provided with a few simple tools and supplies that may be used for minor or emergency repairs to equipment and instruments. Besides the usual accessories provided for each instrument, including some spare parts, the following should be available at all times: small and large pair of pliers with side wire cutter, screw drivers for large and small screws, jeweler's glass, small flat and round files, a spool of soft copper or brass wire, a few assorted brass nails and screws, a bottle of instrument (watch) oil, beeswax, concentrated liquid shellac (orange), spider web, plaster of paris, and other special appliances illustrated in figure 2.

37. Cleaning and oiling.—*a.* Too much emphasis cannot be laid upon the importance of care in the handling of instruments. Tapes and rods should receive the same care as transits and levels, and are to be considered included in all instructions relating to instruments. Everyone entrusted with instruments in the field will keep them clean and in adjustment, to protect them from undue wear and from unnecessary exposure to adverse weather.

b. Instruments having working parts exposed to air and dust require more frequent cleaning and oiling than others. Such exposed parts as the threads of tangent screws are particularly likely to collect dust and grit and should be wiped frequently with an oily rag, then rubbed dry. Only the *best quality of watch oil* should be used for this purpose.

c. Metal tapes should be cleaned and oiled with a good grade light lubrication oil immediately after use. Moisture and grit must be wiped off or they will deteriorate rapidly.

d. Neither the object glass nor the eyepiece of a telescope should ever be rubbed with rough cloth or touched with the fingers, as the glass may be permanently scratched. Special cleansing tissue or a soft, lintless cloth should be used to clean them after they are dusted with a clean camel's-hair brush. Lenses should never be removed from the cell that holds them nor separated from one another.

e. Sometimes friction develops in the telescope slides when working during low temperatures. This is usually due to an excess of oil. The parts affected should be taken down, all old oil wiped off, porpoise oil applied and again wiped off, and the parts assembled.

f. Range poles, rods, and their targets should be cleaned and oiled where necessary and must not be subjected to treatment that will unduly mar their painted markings. When not in actual use they

must be hung vertically or be kept supported throughout their entire length to avoid being bent or warped. An occasional coat of best grade clear varnish for exterior use will preserve instrument cases, tripods, etc., and the markings on rods and range poles, and prolong their usefulness.

38. Minor repairs.—Field work should not be delayed by sending an instrument away for repair if a member of the organization can possibly repair it himself. Repair of broken parts is largely a matter of ingenuity, combined with knowledge of what is essential to the proper working of an instrument. Stripped threads on screws may sometimes be held temporarily with gum or sealing wax provided they are not such as require moving in adjusting, or a pin of hardwood may be used in place of a broken screw. Broken plate-level mountings have been temporarily replaced with sealing wax, and even a broken micrometer microscope bracket has been made sufficiently rigid with properly shaped pieces of wood wrapped with cord or wire which was then stretched with wedges. Resourcefulness is a necessary quality of every surveyor in infrequented regions or in other emergencies.

39. Replacing cross hairs.—The repair job most frequently encountered in the field is replacing one or more cross hairs or "wires," either in the telescope or the reading microscopes. This requires care and patience but is not difficult if the proper materials and appliances are at hand. A description of the method of installing cross hairs follows:

a. For mounting cross hairs, a jeweler's glass, a small bottle containing orange shellac dissolved in alcohol to the consistency of sirup, or better still, some undissolved orange shellac and a candle, a pinch of beeswax, and a pair of dividers or a piece of wire bent into the form of a V with the points turned down are needed. Spider web freshly spun from a small spider is best, but as this is not always available, it is best to keep on hand a spider cocoon.

b. To mount the cross hairs, draw the reticle (see fig. 3 ①) from the instrument. Unscrew and remove the eyepiece slide. Take out two opposite capstan-headed screws and loosen the other two. Revolve the cross wire ring 90°, insert a pointed stick through the end of the telescope tube into a screw hole of the ring, and, using it as a handle, remove the other capstan screws and draw out the ring. To replace it in the telescope, reverse the procedure. When in place, the cross hairs should be on the side of the ring toward the eyepiece.

c. To install new cross hairs, press a bit of beeswax to each prong of the dividers and let a small web fall from the end of one of the prongs, pressing the thread into the beeswax. Stretch the thread

moderately, and attach to the wax on the other prong. If an old web must be used, it should first be dampened by dipping in water for a few seconds. Place the web across the reticle, using a magnifier or jeweler's glass to insure its exact coincidence with the marked lines. Put a small drop of shellac on each end and leave until dry. (See fig. 3 ②.) Natural (flake) orange shellac heated with a hot nail is easily applied and drys almost instantly.

40. Telescopes and verniers.—*a. Principle.*—(1) Instruments, as levels, transits, theodolites, and plane table alidades, are equipped with telescopes to facilitate observations over more or less long distances. A telescope consists, optically, of certain lenses which assist the eye in seeing distant objects; and, mechanically, of certain parts which facilitate the use of the optical parts. The mechanical parts will be discussed in connection with the instrument to which the telescope is applied; and the optical parts will be considered in turn, since they are of general application. The measuring telescope (to distinguish from telescopes like the opera glass) consists of three essential parts:

(*a*) A convex objective which collects the rays of light and forms a bright inverted image of the object.

(*b*) A convex eyepiece which is essentially a microscope, for viewing the image formed by the objective.

(*c*) Fine wires or spider webs placed in the plane of the image, the intersection of which indicates the precise point sighted. The objective collects the light, the eyepiece magnifies, and the cross hairs indicate the point at which the telescope is directed.

(2) A single lens used as an *objective* has the following defects:

(*a*) The rays of light which traverse a spherical lens near the edge are refracted to a point nearer the lens than the rays which pass through the central portion; consequently the image is blurred. This deviation of the rays from the focus is called *spherical aberration.*

(*b*) Rays of white light in passing a single lens are resolved into the colors of the spectrum, consequently the image will be disfigured by colored light. This deviation of the different colored rays is called *chromatic aberration.*

(*c*) The objective of a telescope is rendered almost free from these defects by substituting for the single lens a compound lens composed of a double-convex crown glass and a concave-convex flint glass lens. The two components have different refractive and dispersive powers, and by giving the four surfaces proper curvatures the above defects may be nearly eliminated. The image formed by the object glass at the cross hairs is inverted.

(3) A single lens used as an *eyepiece* has the same defects—spherical and chromatic aberration—as when used as an objective, but in a less degree. A single lens as an eyepiece has also the following defects:

(*a*) The image of a flat object formed by a lens does not lie in a plane but is concave toward the lens. This deviation of the image from a plane is termed aberration of sphericity, which is wholly separate and distinct from spherical aberration. The objective also has this defect, but in so slight a degree as to be inappreciable, while in an eyepiece of a single lens it is very serious.

(*b*) A telescope with a single lens for an eyepiece has a very limited field of view.

(*c*) The chromatic and spherical aberration, and also the aberration of sphericity of the eyepiece, are nearly eliminated by substituting two plano-convex lenses for the convex lens, which also increases the field of view.

(4) *Telescope slides.*—To assist in focusing the objective for different distances, the object glass is fastened in a tube which slides, with a rack and pinion, into one end of the main tube of the telescope. It is important that the slide be straight and fit snugly. To facilitate the focusing of the eyepiece upon the cross hairs, a pin moves in a helical slot and the eyepiece is moved in and out by hand, with a screwlike motion.

(5) The *cross wires* are filaments of platinum or spider web mounted on the face of a heavy brass ring which will always retain its shape. The vertical wire is fixed at right angles to the middle horizontal wire by the maker so as to divide the field of view into quadrants. These two wires are adjusted in the line of sight by means of four capstan head screws. Two fixed stadia wires, when required, are placed equally distant from the middle horizontal wire. The total distance between the stadia wires should be one one-hundredth of the focal length of the objective. Their diameter should not cover more than two one-hundredths of a foot on a rod held at a distance of 1,000 feet.

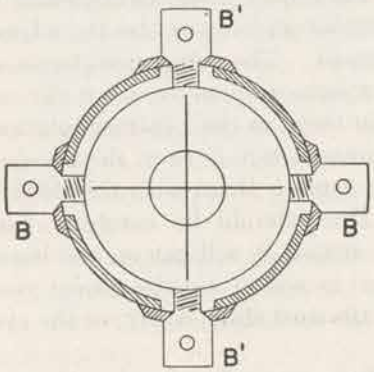
(6) So far as possible, all outside cleaning or dusting of instruments should be done with a soft camel's-hair brush. Care should be taken not to rub the lacquer from the finished surfaces, as the exposed metal will readily tarnish. Alcohol or similar solvents should never be used on the instruments. In cleaning the lenses, first remove the dust with a soft brush and then wipe lightly with a soft, dry cloth, free from lint. Rubbing the lenses dry sometimes causes small particles to stick to them, and, in this case, blowing the breath on the glass will furnish sufficient moisture so that they may be rubbed off easily. If any liquid is brought into contact with the lenses, it is likely to pene-

trate the mounting and give bad results. The interior face of the object glass and the interior lenses of the eyepiece will seldom need cleaning unless water should find access to the inside of the tube. Having removed the cell containing the object glass, care should be taken to screw it back exactly to its former position or else the adjustment of the line of sight may be destroyed. The component lenses of the objective should never be taken apart or removed from the cell containing them, since failure to return them to their former relations will disturb adjustment of the instrument; but if they should accidentally become separated, be sure to replace them with the double-convex, crown-glass lens outward. Dust should be carefully kept from the inside of the telescope tube; if not, it will get on the lenses and cross wires. When the instrument is not in use the object glass should be covered by its dust cap, and the dust slide, if any, in the eyepiece guard should be closed.

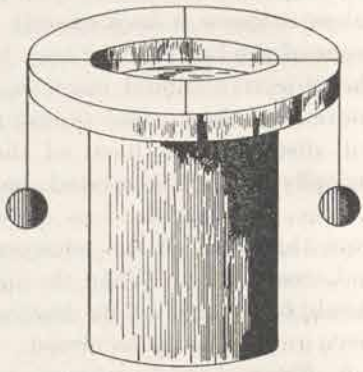
b. External and internal focus.—Telescopes for levels, transits, theodolites, etc., are of either the external or internal focusing type. The older instruments have an external focusing slide, while the later instruments are usually equipped with an internal focusing slide. The use of the latter leaves the position of the object glass unchanged in relation to the cross hairs, and the focusing for various distances is accomplished by moving an auxiliary lens, which is superior to a telescope with an external focusing slide, where the object glass is moved away from or toward the cross hairs to focus for various distances. Figure 3 ③ shows a sectional view of an internal focusing telescope. The removal of an interior focusing slide calls for a different procedure from the removal of the exterior type. First remove the objective, then the focusing pinion, after which the telescope should be held in a vertical position so that the weight of the slide will force it to come out. The hand or, still better, some soft material such as a clean linen towel should be held as a cushion under the slide to receive it and to prevent it from becoming damaged. In replacing the slide, one should be sure that the lens and the slide are perfectly clean.

c. Erecting and inverting eyepieces.—(1) The erecting or terrestrial eyepiece in its most elementary form consists of a convex lens placed between the eye and the inverting eyepiece of the measuring telescope; this inverts the inverted image formed by the objective and shows the object erect; but in its common form it consists of two pairs of plano-convex lenses instead of the single lens. The pair of lenses next to the eye is called the erecting eyepiece; the pair next to the objective is the ordinary positive eyepiece.

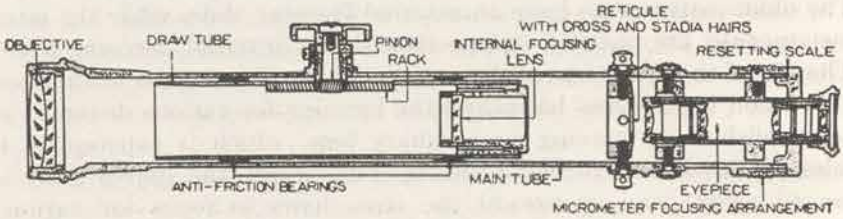
(2) The inverting eyepiece is superior to the erecting eyepiece, since loss of light is occasioned by the two extra lenses. The inconvenience of the inverted image is easily overcome with a little practice.



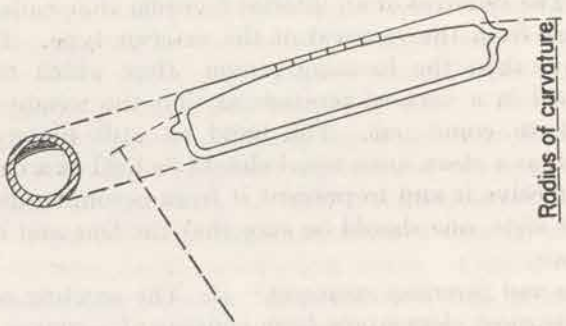
① Reticle in telescope.



② Fresh spider web in place.



③ Sectional view of internal focusing telescope.



④ Level vial showing radius of curvature.

FIGURE 3.—Telescope.

Furthermore, other things being equal, the telescope with the inverting eyepiece is shorter, which is quite an important advantage in the transit.

(3) To withdraw the eyepiece, remove the screw in the spiral slot to be found near the end of the main tube; take hold of the eyepiece and pull it out gently. On some instruments this screw is covered by a collar, guard, or sleeve which must be removed to expose the screw.

d. Verniers and graduations.—(1) The graduations on the arcs of the different instruments, including verniers, are usually cut on solid silver and lacquered and must be handled with great care.

(2) To remove dust from the graduations and verniers, use a fine camel's-hair brush. Do not rub the graduation, especially near its edge, as the slightest wear on this edge will make it difficult to read angles accurately. If it becomes necessary to clean the graduations, the greatest care must be taken not to scratch the silver surface nor to injure the edge. To clean the silver graduations, apply some fine watch oil and let it remain for several hours. Then take a piece of soft (old) linen and rub lightly until dry, but without touching the edge of the graduation. If stains still remain, moisten the finger with vaseline and apply it to the surface; then wipe the finger dry and rub it once or twice around the graduation. This should not be done, however, unless absolutely necessary, because it is likely to destroy the fineness of the graduation. See that the inner edges of the graduations and verniers are free from dirt and grease before replacing the inner center. The vernier should not be unscrewed from the upper plate nor the centering of the outer center altered, as their adjustment is too delicate for anyone but a maker.

(3) The glass covers protecting the verniers, as well as the compass cover glass, need to be brushed carefully and cleaned the same as the lenses, because any scratches or films will impair their transparency. If the reflector shades for the verniers become dirty, they should be taken from their frames and washed in soap and water. They should be kept clean so that they will illuminate the verniers and the graduations.

41. Parallax.—The eyepiece must be exactly focused on the cross hairs by each observer. This is most easily accomplished while the objective is out of focus or directed toward the sky. Then move the eyepiece slowly in and out until the cross hairs are most sharply defined. Test by focusing the objective on the rod or other distant object so that fine details are most clearly seen. If slight movement of the eye up and down, or to right and left, causes the least apparent movement of the cross hairs against the object observed, parallax of the eyepiece persists and no accurate observations can be taken. The eyepiece must be refocused as before until the parallax is eliminated. Some instruments have graduations by which the eyepiece may be

reset, but the absence of parallax must be insured by frequent tests. The eyepiece setting on an instrument remains practically constant for each individual, except that his eye may become so fatigued in a day of observing in unfavorable light that the change in the eye may necessitate a slight readjustment of the eyepiece.

42. Level vials.—Transits, theodolites, and levels are equipped with sensitive spirit levels, the best filling fluid being a mixture of two-thirds high-proof filtered grain alcohol and one-third best grade filtered ether. For use in extremely cold regions spirit level vials should be filled with pure ether which is nonfreezing at sub-zero temperatures. A spirit level or level vial is a round glass tube ground so that its inner upper surface is a circular arc on a longitudinal section. This is nearly filled, as described above, the remaining space being occupied with ether vapor which forms the bubble. (See fig. 3 ①.) The tube is usually graduated to assist in determining the position of the bubble which should be in the exact center of the tube when the instrument is level and in perfect adjustment. The radius of the arc on which the upper interior surface of the level vial is ground predetermines its sensitivity, which may range from 5 seconds to several minutes per division. This can be tested in a level trier or by taping the vial to a theodolite telescope and reading the vertical angle change caused by a movement of the bubble through several divisions of the vial, or by calculating the vertical angle from the difference in readings on a rod at a known distance.

43. Use and care of instruments in field.—*a. General.*—Careful attention to the suggestions given below will save needless wear on instruments and reduce the danger of accidents to a minimum, besides increasing the quality and speed of the work.

b. Tripod.—Inspect the tripod legs and shoes. The leg is of the proper tightness if, when lifted to an elevated position, it sinks gradually of its own weight. The tripod shoes should be tight and have reasonably sharp points.

c. Setting up indoors.—In setting up the instruments indoors, place the tripod shoes firmly into holes in the floor, or with each point in a crack. Avoid disturbing other instruments in the room.

d. Instrument case.—Handle the instrument gently in removing it from and returning it to the case. It is best to place the hand beneath the leveling base in handling the detached instrument. Considerable patience is sometimes required to close the lid after returning the instrument; if properly placed, the lid closes freely. Never force the lid; look for the obstruction and correct it.

e. Mounting instrument.—See that the instrument is securely attached to the tripod before shouldering it. Undue haste in this

particular may sometimes result in costly accidents. When screwing the instrument on the tripod head, it should first be turned in a reverse direction until a slight jar is felt, indicating that the threads are properly engaged.

f. Sunshade.—Always attach the sunshade regardless of the kind of weather. It is a part of the telescope tube. In attaching or removing the sunshade or object glass cap, hold the telescope tube firmly with one hand and with the other twist the shade or cap to the right to avoid unscrewing the object glass cell.

g. Carrying instrument.—Do not carry the instrument on the shoulder in passing through doors or in climbing fences. Before shouldering the instrument, the upper motion should be clamped; with the transit clamp the telescope on the line of centers; and with the level when the telescope is hanging down. In passing through timber with low branches, give special attention to the instrument. Before climbing a fence, set the instrument on the opposite side with tripod legs well spread.

h. Setting up in field.—When setting up in the field, bring the tripod legs to a firm bearing with the plates approximately level. Give the tripod legs additional spread in windy weather or in places where the instrument may be subject to vibration or other disturbance. On side-hill work, place one leg uphill. With the level, place two tripod shoes in the general direction of the line of levels.

i. Exposure of instrument.—Do not expose the instruments to rain or dampness. In threatening weather the waterproof bag should be taken to the field. Should the instrument get wet, wipe it thoroughly dry before returning it to the case. Protect the instrument from dust and dirt, and avoid undue exposure to the burning action of the sun. Avoid subjecting it to sudden changes of temperature. In cold weather when bringing an instrument indoors cover it with the bag. Always return the instrument to the case immediately when returning from work.

j. Guarding instrument.—Never leave an instrument unguarded when in the field nor standing on its tripod in a room.

k. Manipulation of instrument.—Cultivate from the very beginning the habit of delicate manipulation of the instrument. Many parts, when once impaired, can never be restored to their original condition. Rough and careless treatment of field instruments is characteristic of the unskilled observer. Should any screw or other part of the instrument work harshly, call immediate attention to it so that repairs can be made. Delay in such matters is very destructive to the instrument.

l. Foot screws.—In leveling the instrument the foot screws should be brought just to a snug bearing. If the screws are too loose the instrument rocks, and accurate work cannot be done; if too tight, the instrument is damaged and the delicacy and accuracy of the observations are reduced. Much needless wear of the foot screws may be avoided if the tripod head is brought about level when the instrument is set up. *With the engineer level, one pair of diagonally opposite foot screws should be shifted to the general direction of the back or fore sight before leveling up.*

m. Eyepiece.—Before beginning the observations, focus the eyepiece perfectly on the cross hairs. This is best done by holding the notebook page, handkerchief, or other white object a foot or so in front of the object glass so as to illuminate the hairs; and then, by means of the eyepiece slide, focus the microscope on a speck of dust on the cross hairs near the middle of the field. To have the focusing true for natural vision, the eye should be momentarily closed several times between observations in order to allow the lenses of the eye to assume their normal condition. The omission of this precaution strains the eye and is quite certain to cause parallax. *After the eyepiece is focused on the cross hairs, test for parallax by sighting at a well-defined object and observing whether the cross hairs seem to move as the eye is shifted slightly.*

n. Clamps.—Do not overstrain the clamps. In a well designed instrument the ears of the clamp screw are purposely made small to prevent such abuse. Find by experiment just how tight to clamp the instrument in order to prevent slipping.

o. Tangent screws.—Use the tangent screws for slight motions only. To secure even wear, the screws should be used equally in all parts of their length. The use of the wrong tangent movement is a fruitful source of error with beginners. In using the tangent screws always make the final half turn against the spring plunger.

p. Adjusting screws.—Unless the instrument is assigned expressly for adjustment, beginners should not disturb the adjusting screws. If adjustments have to be made in the field, the instrument men should be capable of making them.

q. Magnetic needle.—Always lift the needle before shouldering the instrument. Do not tamper with the needle. If possible, avoid subjecting the needle to magnetic influences. Should the needle become reversed in its polarity or require remagnetization, it may be removed from the instrument and brought into the magnetic field of a dynamo or electric motor for several minutes, the needle being jarred slightly during the exposure; or a good horseshoe magnet may be used for the same purpose. The wire coil counterbalance, usually

fastened to the south end of the needle, will in most cases require shifting after the foregoing process.

r. Lenses.—Do not remove or rub the lenses of the telescope. Should it be necessary to clean a lens, dust first with a soft, clean camel's-hair brush and use a very soft rag with caution, to avoid scratching or marring the polished surface. Protect the lenses from flying sand and dust.

s. Plumb bob.—Do not abuse the point of the plumb bob and avoid needless knots in the string.

t. Cleaning instruments and tripod.—Remove all soil from the tripod shoes before bringing the instrument indoors. Wipe all dust and grit from the instrument before returning it to the case and replace tripod head cover to protect it from possible damage.

44. Adjustments.—Directions for adjustment and calibration of surveying instruments are arranged in this manual in the various sections where their use is first described, and will be found as follows:

a. Levels.—See paragraph 53.

b. Transits.—See paragraph 71.

c. Theodolites.—See paragraphs 72, 73, and 74.

d. Plane table alidades.—See paragraph 138*d* and *e*.

e. Barometers.—See paragraph 57*c*.

f. Sextants.—See paragraph 196*h*.

g. Metal tapes.—See paragraphs 31*e*(3), 37*c*, 104*b*, and 106*b*.

SECTION VIII

DETERMINATION OF DISTANCES

	Paragraph
General	45
Steel tapes	46
Process of chaining	47
Distances by stadia	48
Distances by other methods	49

45. General.—*a. Methods of measuring distances.*—(1) Chaining or taping.

(2) Triangulation.

(3) Stadia.

b. Applicability of methods.—Calibrated steel tapes are used for measurements requiring a high degree of accuracy. For determining the lengths of base lines for triangulation systems, invar tapes are used. For determining the distance of a point from two known points, the distance between the latter two being known, triangulation is usually the quickest and easiest method. The quickest and perhaps the most common method of measuring distances is by stadia and

transit as described in paragraphs 71 and 80. This method is to be preferred for all work where the specified accuracy is to be within 1:750 or less.

46. Steel tapes.—*a. Kinds.*—Three types of tapes used for measuring distances are shown in figure 4. They are called steel tapes, metallic or woven tapes, and invar tapes.

(1) Steel tapes are flat steel wire tapes (fig. 4 ① and ③), band chains (fig. 4 ② and ④), and the ordinary steel tape in leather case (fig. 4 ⑤).

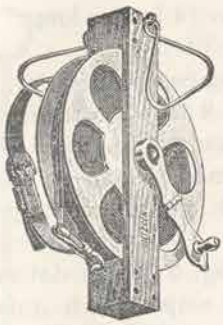
(2) Steel tapes come in various lengths up to 500 feet and 150 meters and are marked by etched graduations and numbers, or by clamped or soldered sleeves carrying the mark and number. The standard engineer tape is 100 feet long. Some tapes are divided decimally throughout their length into feet, tenths and hundredths, while on others the foot marks only are shown, with the last foot on each end (or on one end) divided into tenths and hundredths.

(3) The best method of graduating steel tapes is to have an extra foot added outside or before the zero mark of the tape and have this extra foot graduated to read to tenths and hundredths of a foot, thus permitting a distance to be read to the nearest full foot on the tape proper with remaining fractions on the extension. Steel tapes which give measurements in the metric system are subdivided into meters, decimeters, centimeters, and millimeters and are similar to those just described. They are marked in meters, the first meter being subdivided as mentioned above. Some steel tapes are graduated with the metric system on one side, and the English (foot) system on the other. In the Army most steel tapes are graduated in feet and fractions thereof.

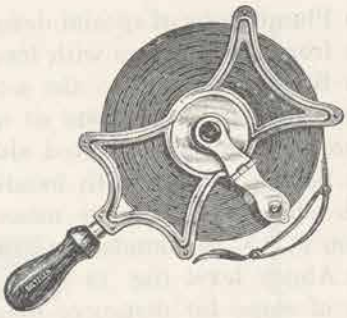
(4) Metallic or woven tapes (fig. 4 ⑥) are of cloth with fine brass wire woven into them to minimize stretching. They come in lengths from 25 to 100 feet and are used only in making measurements which do not require a high degree of accuracy, such as measuring buildings.

(5) *Invar tapes* (fig. 4 ⑦) are made of special nickel steel and are very little affected by extreme changes of temperatures. Their coefficient of expansion is only one-tenth or less than that of ordinary steel tapes (about 0.00000056 per degree of Fahrenheit). They are intended only for precise base line measurements and carry only the end marks and one or two intermediate marks. Invar tapes come in lengths of 50, 100, and 150 feet, and 30 and 50 meters. The shortest lengths are most used. They are usually well protected on iron or wooden reels. The use of invar tapes is described in paragraphs 104 and 105*a*.

b. Accessories.—For measuring distances with tapes, especially steel tapes, the following accessories (fig. 5) are usually employed:

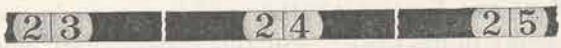


① Flat steel wire tape on reel.

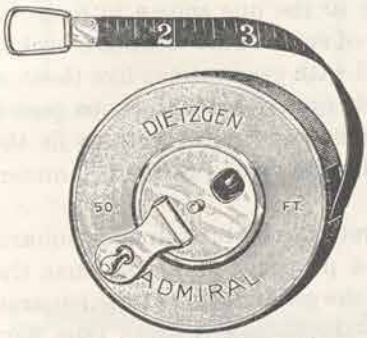


② Band chain on reel.

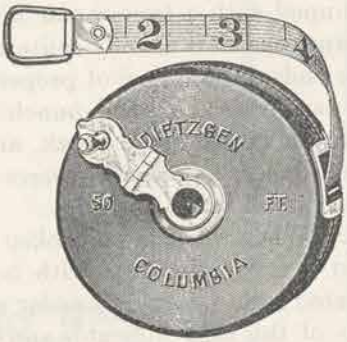
③ Flat steel wire tape.



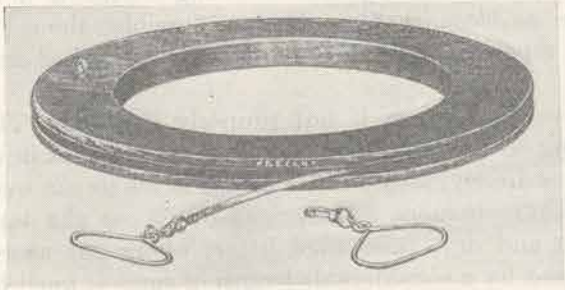
④ Band chain.



⑤ Ordinary steel tape in case.



⑥ Woven (metallic) tape.



⑦ Invar tape.

⑧ 100-foot steel tape done up in a figure-of-eight

FIGURE 4.—Tapes.

(1) Plumb bobs of special design, about 12 to 14 inches long. Best made from steel arrows with lead body (fig. 5 ①).

(2) Steel arrows, 11 to the set, with carrying case (fig. 5 ②).

(3) Range poles of wood or metal, usually 8 feet long, with foot divisions enameled or painted alternating red and white (fig. 5 ③).

(4) Spring balance with handle, to be fastened to end of tape to enable chainmen to make measurements with chain under specific tension (8, 12, 20 pounds, etc.) (fig. 5 ④).

(5) Abney level (fig. 13 ③) or hand level (fig. 5 ⑤), to determine angle of slope for distances measured on the slope which must be corrected to horizontal distances, and to insure that the tape is level when distances are measured horizontally.

c. Minor repairs, care, and calibration.—(1) Regardless of how well tapes are protected and cared for, the average surveyor will find that he must at one time or another repair a tape in order to avoid unnecessary delay in his work. For this reason every chaining party, especially those operating some distance from their base, should be equipped with a tape repair kit similar to the one shown in figure 6. A tape repair kit usually contains a pair of small snips to square broken tape ends, tape splices of proper size and with graduations like those of the tapes used, a hand punch or bench punch with block to punch holes for riveting the break, an assortment of small rivets to fit the punch holes, a pair of tweezers to handle the rivets, a small hammer, and a small taper file.

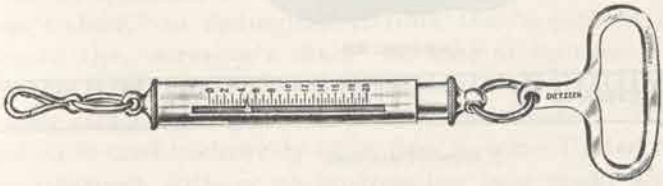
(2) When repairing a broken tape, first cut the broken ends square. Next arrange the tape with both parts properly alined so that the squared ends are exactly as far apart as the graduations at the squared ends of the tape indicate, and rivet to these ends a spare tape part long enough to span the distance between the ends of the tape and to overlap each end not less than 1 inch. Each overlap is riveted so that the edges of all parts of the joints are snug. This is done by placing the rivets as close to the edges as possible, about $\frac{1}{16}$ to $\frac{3}{32}$ of an inch. A repaired tape should always be checked and calibrated.

(3) Steel tapes are easily broken if not properly handled. The chief danger is in pulling on the tape when there is a loop or kink in it. Do not jerk the tape needlessly, step on it, allow vehicles to run over it, or bend it around sharp corners. After each day's use the tape should be wiped clean and dry, then oiled before being put away. If a tape is not to be used for a considerable length of time, it must be thoroughly cleaned and oiled before it is put in storage.

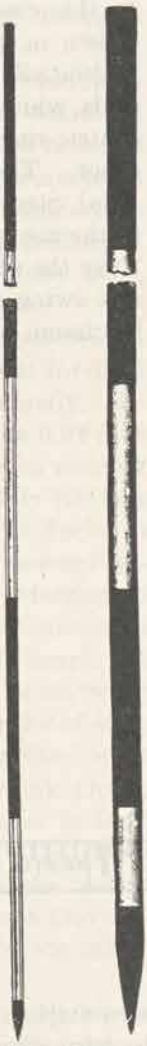
(4) The 100-foot steel tape is probably used more than any other and is more convenient to handle if removed from its box or reel.

① Plumb bob.

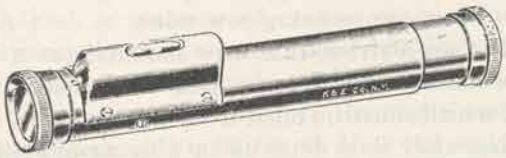
② Steel arrow.



④ Spring balance.



③ Range poles.

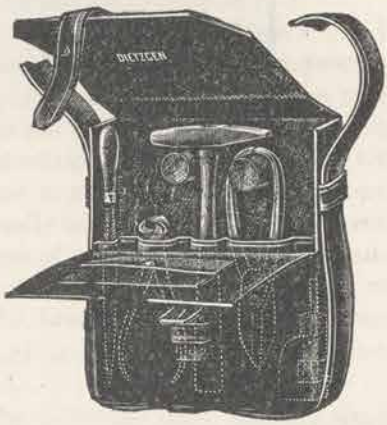


⑤ Hand level.

FIGURE 5.—Accessories.

In this case it is best to do up the tape in a figure-of-eight form as shown in figure 4⑧. To do this take up the tape in 5-foot lengths *without allowing it to turn over.* It will then fall of itself into concentric folds, which after the tape is all in and tied may be easily laid into concentric rings by folding the two loops of the figure-of-eight over each other. The method is as follows:

(*a*) Stretch the tape straight on the ground its entire length. Stand at the zero mark squarely facing the tape and looking along its length. Take the zero end of the tape in the right hand, palm of the hand up, and swing the right hand behind you, pulling the tape through the left hand, palm up. Lay the 5-foot mark with the left hand over the



① Tape repair kit.

② Repaired tape joint.

FIGURE 6.—Tape repair kit and repaired joint.

zero mark (in the palm of the right hand) without permitting the tape to turn over. The right hand closes on it and swings behind for another 5-foot length, and so on, until the tape is gathered in. This will bring the end mark of the tape on top, ready to be taken off by the front chainman when commencing new work.

(*b*) Tapes that are carried this way should have a rawhide strip, about ¼ inch wide, attached to each end for easier handling. These rawhide strips, which must be oiled from time to time, are used to tie the tape together after it is done up in a figure-of-eight. For easier carrying the tape may then be folded into concentric rings by firmly grasping the end of each loop and turning them slightly in opposite

directions. The ends of the rawhide are utilized to bind the rings into one.

(5) Steel tapes must be tested by comparing them with a standardized tape. By knowing the amount a tape varies from standard length, corrections can be applied to the measured distances to make the final results more nearly accurate, provided the measurements were made properly. The Bureau of Standards will standardize tapes free of charge for Government use and funish a standardization certificate with each tape. This certificate should be carefully kept. (See par. 105*a* (2).)

47. Process of chaining.—*a. General.*—(1) In the United States, the foot is used by surveyors and civil engineers as a basis for field measurements. Fractions of a foot are expressed decimally, the nearest 0.1 being taken in ordinary surveys, and the nearest 0.01 foot in more refined work. In railroad and similar line surveys, in which a station stake is set every 100 feet, the unit of measure is really 100 feet instead of the foot. The term "station" was originally applied only to the actual point indicated by the numbered stake, but it is now universal practice in this country to use the word "station" in referring to either the point or the 100-foot unit distance. A fractional station is called a "plus" for the reason that a plus sign is used to mark the decimal point for the 100-foot unit, the common decimal point being reserved for fractions of a foot. The initial or starting stake of such a survey is numbered 0. The 100-foot chain is also called the "engineer's chain" to distinguish it from the 66-foot or 100-link chain, termed the "surveyor's chain" because of its special value in land surveys involving acreage. (10 square chains=1 acre.) The latter is also called the Gunter chain after its inventor. While the metric system is used exclusively or in part in some United States Government surveys, little or no progress has been made toward its introduction in other surveys.

(2) A flat stake is used to mark the stations in a line survey, and a square stake to mark transit stations. The station stake is numbered on the rear face and witnessed by a flat guard stake driven slanting, 10 inches or so to the left. The numerals should be bold and distinct, and made with keel or waterproof crayon pressed into the surface of the wood. Having located a point approximately with the flagpole, the stake should be driven truly plumb in order that the final point may fall near the center of its top. In driving a stake, the axman should watch for signals. It is better to draw the stake with a slant-blow than to hammer the stake over after it has been driven.

(3) In flagging a point, the spike of the pole is placed on the tack and the pole plumbed by holding it vertically between the tips of the

fingers of the two hands, the flagman being squarely behind the pole, with feet apart for steadiness and both arms against the body. When chaining or taping between two points, a range pole is placed on line, about a foot or so beyond the forward (terminal) point to which the measurement is made. With a little practice a range pole may be worked into average ground with its spiked end and remain firmly fixed until the course is taped.

(4) On smooth, level or evenly sloping ground, there are two methods of supporting a tape in the chaining procedures described

PRIMARY TRAVERSE
By direct horizontal measurement

Hd Chain: Cpl M.R. Bear
Rr. Chain: Pvt J.L. McGee

Flagman: Cpl J.C. Lee
Recorder: Pvt A.I. Mair

FROM ⊙3 TO ⊙19

14

Sta	1st Dist	2nd Dist	Mean in yards	Angle	Azimuth
18 — 19	545.22	545.29	181.75		
17 — 18	594.13	594.21	198.06		
16 — 17	476.18	476.32	158.75		
3 — 16	648.94	649.00	216.32		

FIGURE 7.—Record of taped distances.

in *c* and *d* (1) below. During standardization or calibration, the tapes should be supported in the manner which is to be employed in the field.

(*a*) The tape may be supported throughout by the surface of the ground. Where this is possible, a 300-foot tape will give the most rapid and accurate results.

(*b*) The tape may be supported only at the ends. In this method, plumbing to or from the mark or chaining pin at one or both ends of the tape is necessary. The 100-foot tape is usually used in this way as it is adapted to most field conditions.

b. Field notes.—Figure 7 shows a method of recording taped distances. In this case each course was measured twice, the records being entered separately in the second and third columns. In the fourth column are entered the mean (in yards to the nearest hundredth) of the two measurements. The right half of the right-hand page of notes shows a diagram indicating the location of each traverse station and its witness marks with all distances indicated thereon. No effort should be made to draw this diagram to scale. The unused columns, headed "Angle" and "Azimuth," are used later by the party which observes the horizontal angles, or if that is impracticable, the notes are transcribed for the use of that party. In any case, it should be furnished the diagram so that it can find the stations without undue delay.

c. Chaining on level ground.—In general, when measuring distance, horizontal distances are chained or taped. Two persons, called head and rear chainmen, are required. The usual process is as follows: The line to be taped is first marked with range poles. The head chainman with the 100-foot end of tape in his hand, after setting one marking pin at the starting point and checking up the remaining ten pins on his ring, steps to the front. The rear chainman allows the tape to pass through his hands to detect kinks and straighten out the tape. Just before the whole length is drawn out the rear chainman calls "halt," at which the head chainman turns and straightens the tape on the true line under the direction of the rear chainman. In order to allow a clear sight ahead, the front chainman should hold the tape handle with a pin in his right hand well away from his body, supporting the forearm or elbow on the body. The rear chainman holds the handle in his left hand approximately at the starting point and motions with his right to the head chainman, his signals being distinct both as to direction and amount. Finally, when the straight and taut tape has been brought into the true line, the rear chainman, slipping the handle behind the pin at the starting point with his left hand and steadying the top of his pin with his right, calls "Stick." The head chainman at this instant sets his pin to correspond with the end mark of the tape and responds "Stuck," when both chainmen check for accuracy and the rear chainman pulls the pin. Both now proceed, the rear chainman giving the preliminary "halt" signal as he approaches the pin just set by the head chainman. The tape is lined up and stretched, the front pin is set, and the rear pin pulled on signal, as described for the first chain length. This process is repeated until the head chainman has set his tenth pin, when he calls "Out" or "Tally," at which the rear chainman walks ahead, counting his pins as he goes and, if there are 10, transfers

them to the head chainman who also checks them and replaces them on his ring. A similar check in the pins may be made at any time by remembering that the sum, omitting the one in the ground, should be 10. This safeguard should be taken often to detect loss of pins. The count of tallies should be carefully kept. When the end of the line is reached, the head chainman first carries the full tape ahead and then steps back and reads the fraction at the pin, noting the units with respect to the marking on the tape and recording the total distance (number of tape-lengths plus fraction) in the notebook. The number of pins in the hand of the rear chainman indicates the number of applications of the tape since the starting or last tally point. A like method is used in case intermediate points are to be noted along the line. In railroad and similar surveys, chaining pins are usually dispensed with and the ends of the tapes are indicated by numbered stakes. The stake marked 0 corresponds to the pin at the starting point, and the station stakes are marked according to the number of 100-foot units laid off.

d. Chaining on sloping ground.—In measuring on a slope there are two ways of getting the horizontal distance: to measure along the slope and correct this measurement by calculation, and to hold the tape horizontal and get the horizontal distance direct. The latter is the best method and should be used whenever possible.

(1) *First method.*—Measure the angle of slope with either transit or Abney level and tape the slope distance on the ground. Then the tape distance times the cosine of the angle of slope gives the horizontal distance. Slope tables can be prepared very easily, using distances as one argument and slope angles as the other, thus eliminating computation in the field. However vertical angles must be recorded to be available for the computation.

(2) *Second method.*—If the slope is not over 5 percent, the measurement is done as in *c* above, except that the distance at the high point of the tape (always held level or horizontal) is transferred to the ground by a plumb-bob line.

e. Chaining on steep slopes.—If the slope is so great that the entire length cannot be held level, the tape should be "broken" in lengths of multiples of 10 feet. When breaking tape, the head chainman goes ahead the full tape length, dropping the tape approximately on line. He then comes back toward the rear chainman until he reaches a point such that the fractional part of the tape can be held level. In measuring downhill the head chainman marks the point on the ground with the aid of a plumb bob, keeping his finger at the point on the tape until the rear chainman comes up. Every time the rear chainman thumbs a break he hands the head chainman a pin to

replace the one in the ground which represents only a fractional part of the tape length. In measuring uphill the process is reversed, the rear chainman transferring the point on the ground to the tape by means of his plumb bob. The rear chainman always holds the same footmark that the head chainman held until the whole tape length has been measured. This makes it unnecessary to count fractional distances and avoids errors. Measuring down a slope is more accurate than measuring up the slope, since it is difficult for the rear chainman to place his end of the tape accurately over the point by means of a plumb bob on the upslope measurement. If more than two "breaks" are necessary in a tape length, chaining on the slope is more accurate.

f. Avoiding errors.—(1) Chainmen should be thoroughly familiar with—

(*a*) Sources and relative importance of errors.

(*b*) Methods for eliminating and correcting errors.

(*c*) Customary limits of error for different kinds of work.

(*d*) Field work requirements corresponding to different limits of errors.

It is necessary to keep clearly in mind the difference between accidental errors and constant errors, and between cumulative and compensating errors. Distinction between discrepancy and error is also important, since discrepancy between duplicate measurements may or may not indicate precision of work.

(2) Sources of errors in chaining may be due to—

(*a*) Errors in the length and in the graduations of the tape, especially when the tape has been repaired several times.

(*b*) Temperature.

(*c*) Improper manipulation of the tape (tape not horizontal, not stretched properly, careless plumbing, imperfect alinement, sag, pull, etc.).

(*d*) Personal errors, as mistakes in reading the tape and blunders in counting tape lengths.

Most of these errors have an important bearing on the accuracy of the measurement and must be avoided or taken into account in the final calculation. Errors due to minor changes in temperature, and smaller errors due to imperfect alinement, sag, and pull do not influence the accuracy of results when the allowable error is 1 in 5,000 or more.

g. Organization of party.—The chaining party is normally a subparty of the party establishing stations for a traverse. The chaining subparty may work close to the rest of its party (usually between the men establishing stations and clearing lines) and the transit party;

or it may work independently behind the transit party, or in conjunction with the men clearing the lines, etc. In the following exercise the latter is assumed. Subject to modifications due to availability of personnel, etc., a party establishing traverse stations and measuring distances usually consists of—

1 chief of party to select station points and direct the work.
1 flagman to set the range pole on the new station and drive and mark the station and witness stakes.
1 or more axmen to clear lines of sight between stations and remove range pole from rear station.
2 chainmen to measure distances between traverse stations and to witness marks.
1 recorder to record the results of measurements, including diagram of stations, witness marks, etc.

h. Equipment.—A party as constituted above will need the following minimum equipment: 1 calibrated tape with accessories; 2 range poles; axes, brush-hooks, etc., as necessary; stakes and copper tacks; red and white signal cloth (muslin); waterproof crayon; and notebook, pencils, and 6-inch rule for recorder.

i. Exercise I.—Establish stations for a control traverse and obtain two measurements for the distances, the difference between the two measurements not to exceed 1:5,000.

(1) Stations for a control traverse should be not less than 300 or more than 2,000 feet apart, when practicable. Lines between stations must be clear; underbrush and small saplings may be cut if necessary, but stations should be so placed as to make a minimum of cutting necessary. Stations should if possible be so located as to allow a good view in all directions. Each station will be marked by a suitable stake or pin driven flush with the ground, its center plainly marked. Two witness marks will be selected or placed near each station. They will be plainly marked with the station number, the mark itself being a tack, driven into a blaze of a tree, stump, stake, etc., the blaze or mark to face the station directly. The direction, description, and distance of the witness mark to the station to which it refers will be carefully recorded. (See fig. 7.)

(2) The difference between the two measurements for each course should not exceed 1:5,000. Special attention should be given to the sketch which accompanies the notes, as to referencing in stations, descriptions, etc.

(3) Calculate the mean distance in yards, taking into account any corrections that may have to be applied due to errors in the length of

the tape, and record the same in the proper column. If corrections are used, describe them appropriately in the remarks column.

48. Distances by stadia.—These are obtained by using a transit equipped with stadia cross hairs consisting of one vertical and three horizontal cross hairs, the latter so spaced that any intercept between the top and bottom hair when read on a graduated rod, if multiplied by 100, will give the distance between the transit and the point at which the rod was read. For a detailed description of the use of the stadia, see paragraph 70.

49. Distances by other methods.—In instrumental surveying, distances are often obtained by methods other than those by measurement with tape or stadia. These methods require the solution of one or two unknown sides of a triangle, either mathematically or graphically, and belong to one of the various classes of triangulation described in sections XIX to XXIII.

SECTION IX

VERTICAL CONTROL METHODS—ADJUSTMENT OF ENGINEER LEVEL

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Leveling equipment	51
Handling level	52
Adjustments of dumpy level	53
Differential leveling	54
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Cross sections and cross sectioning	56
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50. Description of various methods.—*a. General.*—(1) Vertical control methods are employed to determine more or less precise elevations of points. The method used depends on the type of work for which the elevations must be obtained and is among the following:

(*a*) Differential leveling, or spirit leveling, by which the difference of elevation between points is obtained from direct readings on a graduated rod (*d*, *e*, *f*, and *g* below).

(*b*) Trigonometric leveling, by which the difference of elevation between points is determined mathematically from the measured or computed distance and the vertical angle between them (*c* below).

(*c*) Barometric leveling, by which the elevation of points is estimated from differences in barometer readings caused by changes in atmospheric pressure for different elevations (*i* below).

(2) Level lines that start and close on points which have their elevations definitely fixed, such as bench marks, triangulation stations, etc., are known as level circuits.

(3) Vertical control is classified as first-order leveling (see *d* below); second-order leveling (see *d* below); third-order leveling (see *e* and *g* below), the most commonly used; and leveling of lower order (see *c*, *f*, *h*, and *i* below).

(4) *First-order leveling.*—First-order leveling is used in developing the main level net of the United States. The lines are so placed that eventually no point in the country is more than approximately 50 miles from a bench mark established by leveling of this order. All the lines are divided into sections 1 to 2 miles in length, and each section run forward and backward, the two runnings of a section not to differ more than 0.017 foot $\sqrt{M}$, where M is the length of the section in miles.

(5) *Second-order leveling.*—Second-order leveling is used in subdividing loops of first-order leveling that are too large to be covered by unsupported third-order lines. Second-order leveling includes lines run by first-order methods, but in only one direction, between bench marks previously established by first-order leveling and all double lines of leveling whose sections, run in a backward and forward direction, check within the limits of 0.035 foot $\sqrt{M}$, where M is the length of the section in miles.

(6) *Third-order leveling.*—Third-order leveling is used in subdividing loops of first- or second-order leveling, so that eventually no point on the map is farther from any bench mark (established by leveling of the first, second, or third order) than required by the specification. Third-order lines are not extended more than 30 miles from lines of the first or second order; they may be single-run lines, but must always be loops or circuits closed upon lines of equal or higher order. Closing checks are 0.05 foot $\sqrt{M}$, where M is the length of the circuit in miles.

(7) *Leveling of lower order.*—Leveling that allows closure checks greater than the limit stated for third-order work, such as trigonometric leveling, barometric leveling, etc., is considered as belonging to the lower order of work. No bench marks established by leveling that is less accurate than that of the third order, as above described, are marked by standard bench mark tablets, except that in mountainous regions inaccessible to ordinary spirit-level lines, standard marks may be used on mountain summits to mark elevations determined by trigonometric leveling; such marks should be stamped in a distinguishing manner.

b. Datum and bench marks.—(1) In all surveys the elevations are in direct relation to some common datum. On all standard United States maps, elevations (and contours) refer to mean sea level, which is considered zero. Mean sea level is the mean level of the sea during

its continuous change of level over a certain period of observation and is established as explained in section XXXIII. Sometimes, when an elevation referred to mean sea level (or other some standard datum) is not readily available, a datum is assumed by assigning an arbitrary elevation to a certain point in the area to be surveyed and then determining the elevations of all other points relative to the arbitrary elevation assigned to the first point.

(2) A bench mark is a fixed point of reference, chiefly for elevations, and may or may not be a station of a traverse or other horizontal control system. All first-, second-, and third-order leveling lines should be adequately marked by permanent bench marks (monuments) at average intervals of not more than 3 miles, preferably by metal tablets (fig. 8) set into concrete posts, substantial buildings, out-cropping rock, or large boulders. At places from which it is likely that future leveling lines will be extended, at least three bench marks should be established within a radius of about half a mile but far enough apart to be unaffected by the same disturbing causes. In addition, secondary marks on trees, bridge seats, and similar places should be left for each mile of line, or where required. These secondary marks are often referred to as intermediate, temporary, or supplementary bench marks.

(3) New leveling should be tied to bench marks of previous leveling, by Government and other organizations wherever practicable, to furnish checks and provide means of correlating leveling results. An effort should be made to have level lines run over the routes where traverse stations or other control stations have been established, in order that the elevations of these stations may be determined. All metal bench marks should, at the time they are established, have stamped upon them some mark which will positively identify them. A complete description should be made of each old bench mark visited during the progress of the field work. A detailed description of each new bench mark is made in the notebook used for recording the level work.

c. Trigonometric leveling.—Trigonometric leveling is accomplished by computing differences of elevation from vertical angles and distances; the theory is based on the trigonometric properties of the right triangle. Figure 9 illustrates the two methods, ① using the horizontal distance AB and the vertical angle C and ② using the slope distance AC and the vertical angle C. For an application of the latter method see paragraph 80. Trigonometric leveling is usually a part of traverse work or triangulation, the transit or theodolite being used to read the vertical angle C and the distance measured or computed as explained in the preceding section.

d. First- and second-order leveling.—Military surveys do not require first- or second-order leveling. For a detailed description of the methods used and the special equipment necessary to perform this

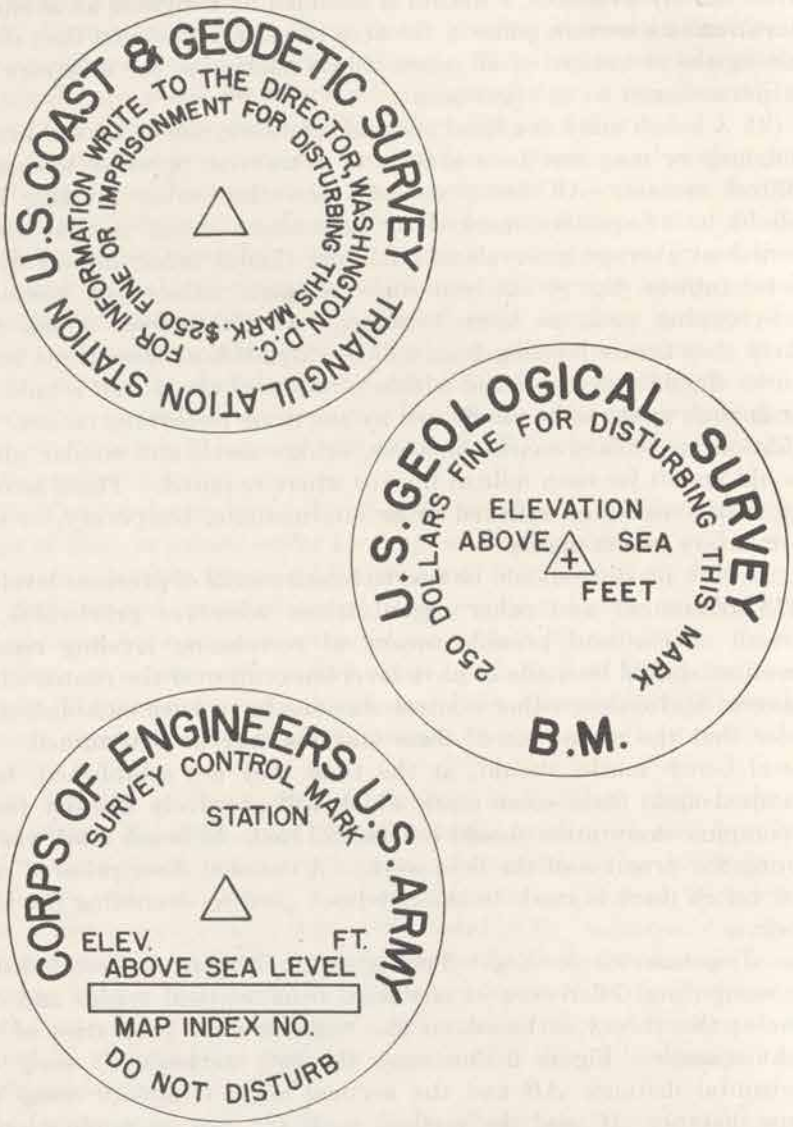


FIGURE 8.—Bench marks and metal tablets.

very precise work, Special Publication No. 140, United States Coast and Geodetic Survey, and Bulletin 788, U. S. Geological Survey may be consulted.

e. Third-order leveling.—Third-order leveling includes normally all leveling where rod readings to the nearest 0.001 foot are required and the line or circuit is tied to bench marks that were established by third-order (or better) leveling. For detailed descriptions of third-order work see paragraph 54.

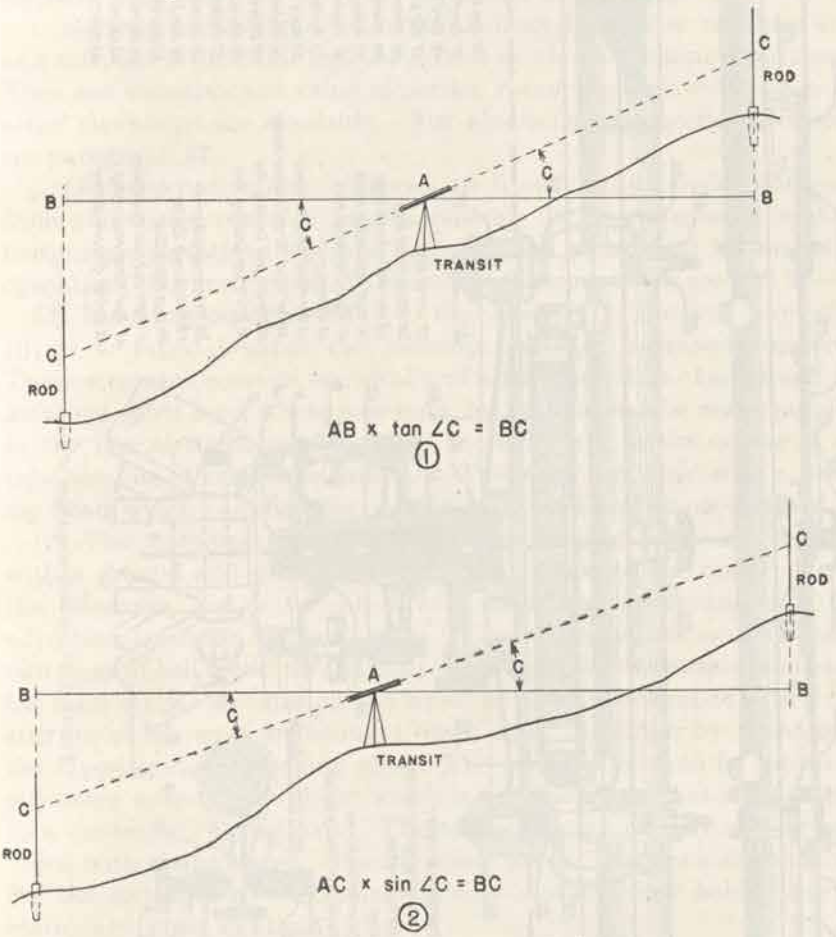
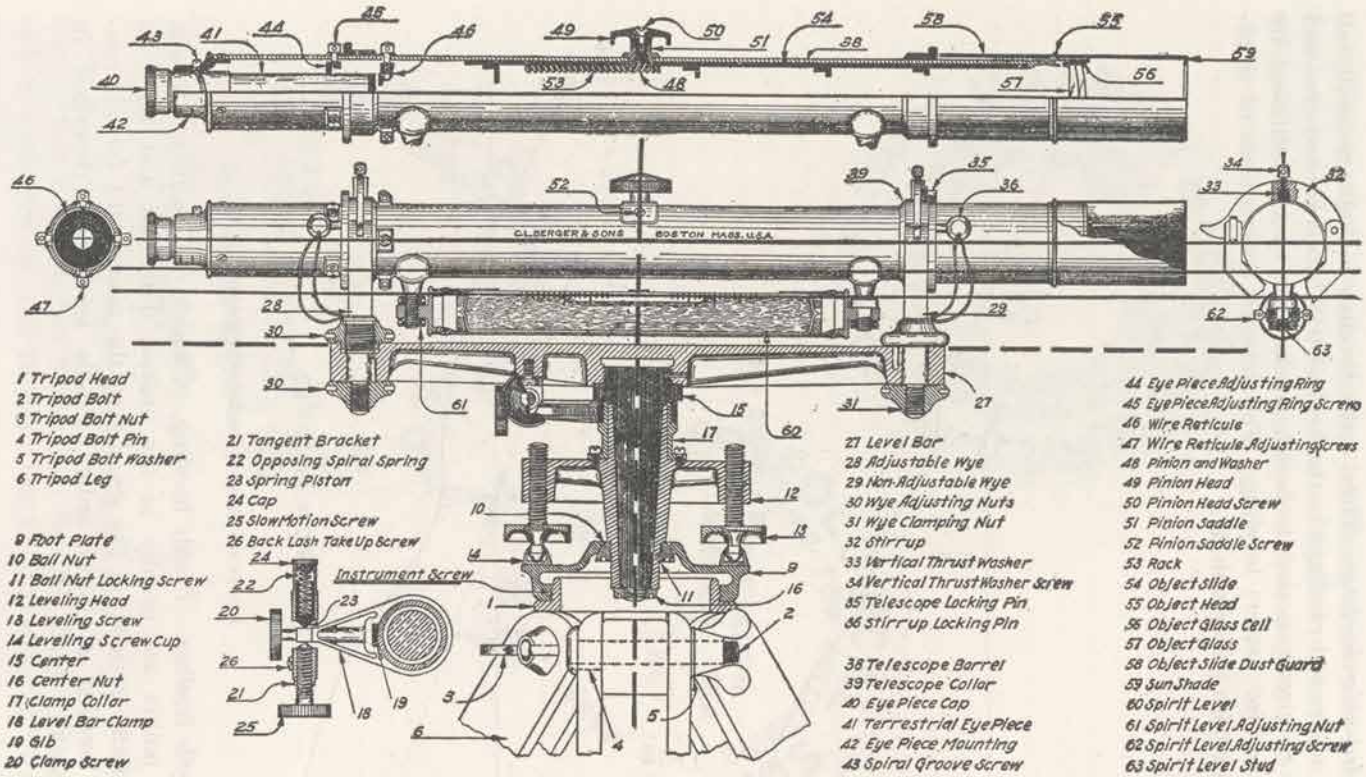


FIGURE 9.—Theory of trigonometric leveling.

f. Profile leveling.—Profile leveling, in which rod readings between turning points are usually to the nearest 0.01 foot and those on intermediate stations on the ground to the nearest 0.1 foot, is considered leveling of lower order. Profile leveling is described in paragraph 55.



1 Tripod Head
2 Tripod Bolt
3 Tripod Bolt Nut
4 Tripod Bolt Pin
5 Tripod Bolt Washer
6 Tripod Leg

9 Foot Plate
10 Ball Nut
11 Ball Nut Locking Screw
12 Leveling Head
13 Leveling Screw
14 Leveling Screw Cup
15 Center
16 Center Nut
17 Clamp Collar
18 Level Bar Clamp
19 Gib
20 Clamp Screw

21 Tangent Bracket
22 Opposing Spiral Spring
23 Spring Piston
24 Cap
25 Slow Motion Screw
26 Back Lash Take Up Screw

27 Level Bar
28 Adjustable Wye
29 Non-Adjustable Wye
30 Wye Adjusting Nuts
31 Wye Clamping Nut
32 Stirrup
33 Vertical Thrust Washer
34 Vertical Thrust Washer Screw
35 Telescope Locking Pin
36 Stirrup Locking Pin

38 Telescope Barrel
39 Telescope Collar
40 Eye Piece Cap
41 Terrestrial Eye Piece
42 Eye Piece Mounting
43 Spiral Groove Screw

44 Eye Piece Adjusting Ring
45 Eye Piece Adjusting Ring Screws
46 Wire Reticule
47 Wire Reticule Adjusting Screws
48 Pinion and Washer
49 Pinion Head
50 Pinion Head Screw
51 Pinion Saddle
52 Pinion Saddle Screw
53 Rack
54 Object Slide
55 Object Head
56 Object Glass Cell
57 Object Glass
58 Object Slide Dust Guard
59 Sun Shade
60 Spirit Level
61 Spirit Level Adjusting Nut
62 Spirit Level Adjusting Screw
63 Spirit Level Stud

FIGURE 10.—Cross section and view of Berger 18-inch wye level.

g. Flying levels.—Flying levels is a line of levels to determine approximate elevations to hundredths or tenths of a foot, run more rapidly and with less precision than usual.

h. Cross sections and cross sectioning.—Level work for cross sections is of the same order as profile leveling. Methods for obtaining cross sections, etc., with the level are explained in paragraph 56.

i. Barometric elevations.—Elevations from barometer readings are, as a rule, not dependable and should not be used for mapping purposes. They are sometimes of value in certain reconnaissance work when no other elevations are available. For obtaining barometric elevations see paragraph 57.

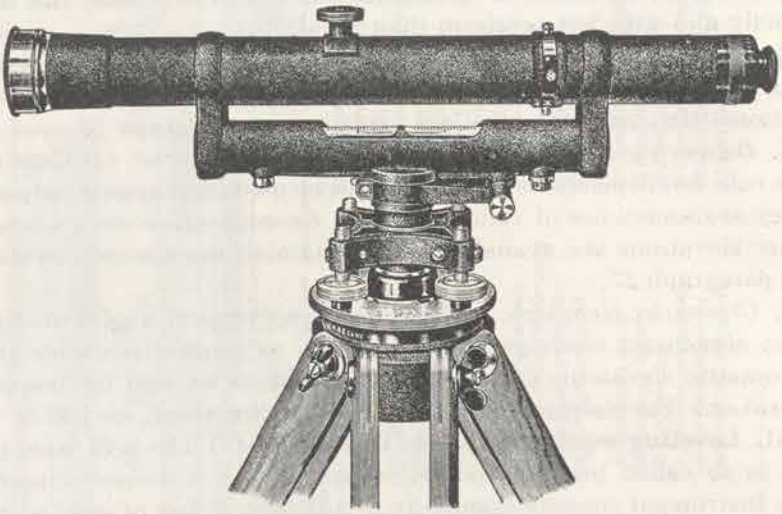
j. Clinometer elevations.—Elevations from vertical angles obtained from clinometer readings are, in general, no more dependable than barometric elevations and are therefore not to be used for mapping operation. For instructions in the use of the clinometer, see FM 21-35.

51. Leveling equipment.—*a. Wye level.*—(1) The wye level (fig. 10) is so called because the telescope rests in Y-shaped supports. The instrument consists essentially of a telescopic line of sight and an attached spirit level whose axis may, by adjustment, be made parallel to the line of sight, so that when the bubble is in the center of its tube the line of sight is horizontal. These are combined with a leveling head which contains the vertical axis, and screws on a tripod.

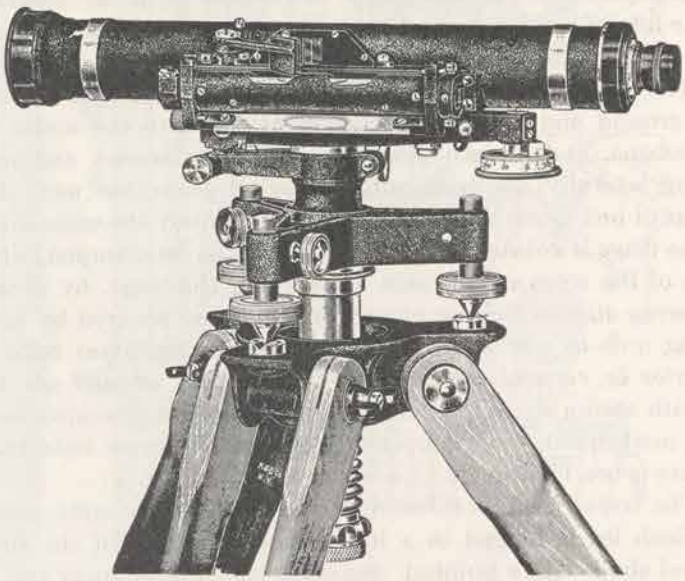
(2) The telescope tube is 18 to 22 inches long. The spirit level, with a ground and graduated scale, is attached to the under side of the telescope, and is furnished with adjusting screws and nuts for adjusting laterally and vertically. The telescope has near its ends two rings of bell metal turned truly and of precisely the same diameter. On these rings it rotates on the wyes; or it can be clamped, when the stirrups of the wyes are brought down upon the rings, by pushing in the tapering stirrup locking pins. The wyes are secured by means of adjusting nuts to a level bar which is attached rigidly at right angles to a center or vertical axis. The telescope may or may not be provided with stadia wires. Four leveling screws are provided for leveling the instrument. A clamp and slow-motion screw hold the level stationary when sighting.

(3) The tripod head is pivoted to three full-length, split, hardwood legs. Each leg is hinged to a lug on the underside of the threaded head, and shod with a pointed, cast-steel shoe on its lower end. The tripod weighs about 10 pounds.

(4) The level is fitted neatly into a hardwood case provided with a lock and two hooks. Fitted into the fixtures attached to the inside of the case are sunshade, wrench, screw driver, adjusting pins, and an



① Older type.



② Improved type.

FIGURE 11.—Dumpy levels.

oilcan for watch oil. A waterproof cover is provided. The weight of the case complete is about 10 pounds.

(5) The same care and precautions should be taken in the use of the wye level as for the transit. The stirrups should be kept closed except when the instrument is being adjusted. Special care should be taken to prevent the wearing of the inside contact surfaces of the wyes by unnecessary turning and twisting of the telescope tube.

b. Dumpy level.—The dumpy level (figs. 11 ① and 12) with inverting eyepiece because of its sturdiness, convenience, and stability of adjustment has superseded the wye level. It has a telescope about 12½ inches long, rigidly attached to a frame consisting of two vertical supports, horizontal bar, and a central vertical spindle. The spirit level is fastened to the horizontal bar just underneath and parallel to the telescope tube, and can be adjusted in a vertical plane. The telescope is provided with stadia wires in addition to the cross wires. The dumpy level is provided with a case and tripod similar to those described for the wye level. The dumpy level fitted with an erecting eyepiece has a telescope about 17½ inches long. The same care and precautions should be taken in the use of the dumpy level as for the transit. Figure 11 ② shows an improved dumpy level or precise level permitting the observer to view the level bubble while sighting. It has several other improvements over the older types, making the instrument more rugged and stable and assuring greater accuracy in less time.

c. Level rods and accessories.—(1) Level rods are of the target type or the self-reading type. Figure 13① and ② shows two rods, the Philadelphia rod with target and a lighter rod without target. The target rod consists of strips of grooved wood sliding one on another and a movable target. For short rod readings, the target is moved up or down the rod; for long rod readings it is set at a stop near the upper end of one strip and clamped, and the strip bearing the target is raised to the required height. The rod reading is found on the side of the raised strip and opposite the vernier which is on the other strip, the scale reading downward. Some target rods have the front face of each strip graduated continuously with respect to the other strip so that the rod can be read from the instrument without setting the target. The self-reading rod is graduated continuously from end to end to permit reading from the instrument. It is usually graduated to 0.01 foot. For a single reading, the error may be larger than with the target rod, which can be read to 0.001 foot; but, since the errors of estimating fractional parts are compensating, the results with the self-reading rod over a long run are very accurate, and it is

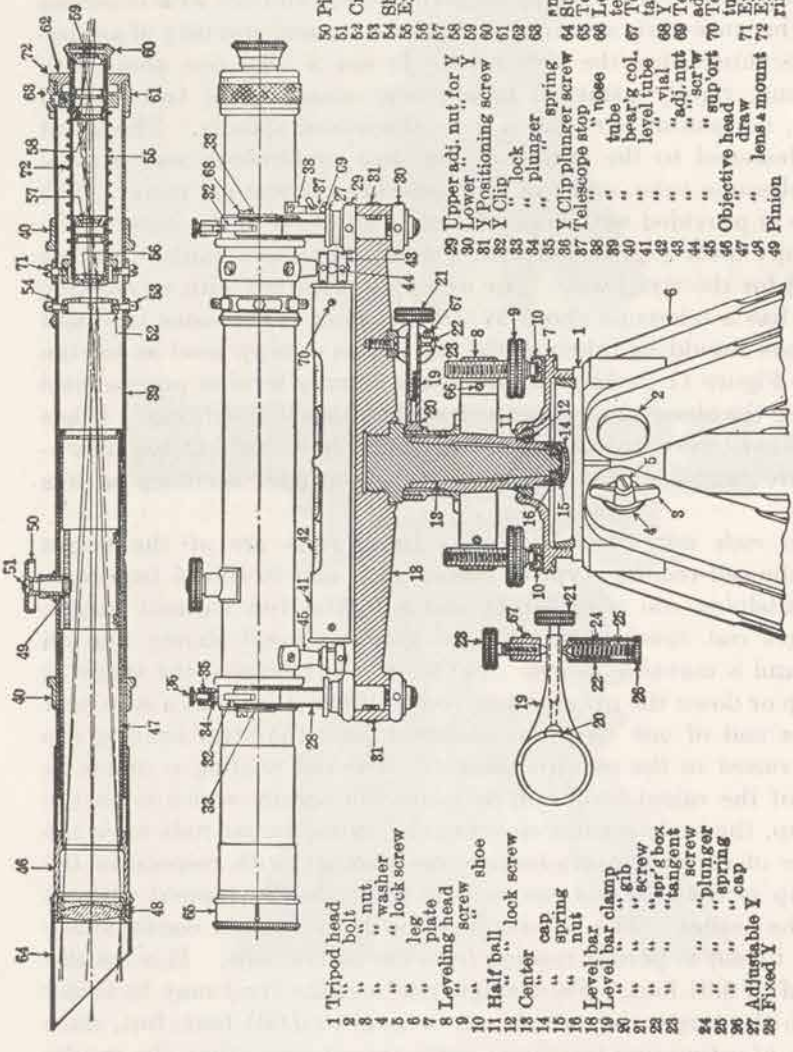


FIGURE 12.—Component parts of K. and E. dumpy level.

used more frequently than the target rod. The target must be used for third-order leveling but need not be for less accurate work.

(2) Other accessories for level work are the Abney level (fig. 13 ③), used for obtaining vertical angles or percentage of slopes for reconnaissance purposes and chaining; the hand level (fig. 13 ④), used for leveling the tape in chaining, etc.; and the steel turning pin (fig. 13 ⑥), used as a temporary turning point on level lines.

(3) Figure 13 ⑤ shows a level rod target with vernier which permits readings on the rod to the nearest 0.001 foot.

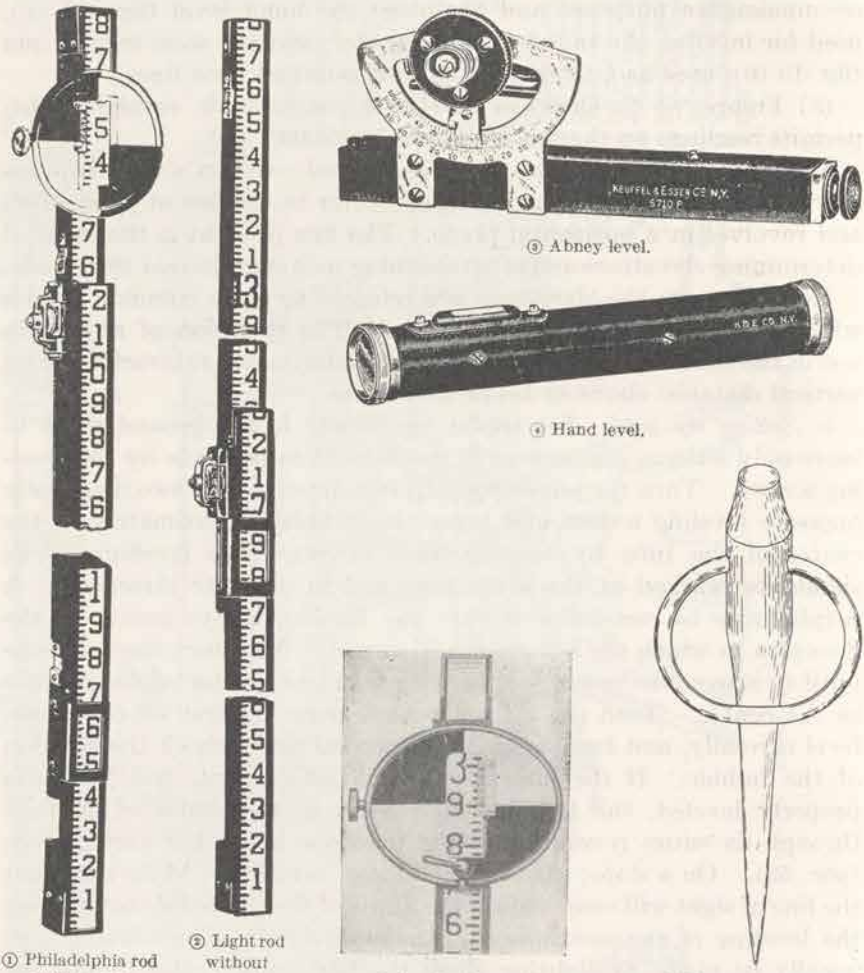
52. Handling level.—*a. Function of level.*—When a level is properly set up the line of sight is perpendicular to the line of gravitation and revolves in a horizontal plane. The line of sight is the basis of determining elevations and of establishing points at desired elevations.

In all surveys the elevations are referred to some common datum which is designated as zero elevation. The elevation of any point not in the datum plane is secured by the addition or subtraction of its vertical distance above or below that plane.

b. Setting up level.—Set tripod legs firmly in the ground so as to leave only a slight adjustment of instrument to be made by the leveling screws. Turn the telescope until it is directly over two diagonally opposite leveling screws and bring the bubble approximately to the center of the tube by turning these screws. The leveling screws should be worked at the same time and in opposite directions. A helpful rule to remember is that the bubble always moves in the direction in which the left thumb is turned. Now turn the telescope until it is over the other pair of screws and bring the bubble exactly to the center. Turn the telescope back over the first set of screws, level carefully, and turn again to the second set to check the position of the bubble. If the instrument is in adjustment, and has been properly leveled, the bubble will remain in the center of the tube through an entire revolution of the telescope about the vertical axis (par. 53). On a slope, set two tripod legs downhill. Make sure that the line of sight will come within the limits of the rod before completing the leveling of the instrument. An approximate determination can usually be made by sighting along the horizontal center line of the telescope.

c. Steps in leveling.—(1) After the level has been set up, the next step is to determine the elevation of its horizontal plane of sight, called the height of instrument (*H.I.*), measuring up to this plane by reading the vertical distance on the rod from some point (a bench mark, *B.M.*), of which the elevation is known. Then the elevation of any point is found by subtracting the rod reading at that point from the *H.I.* (par. 54).

(2) *Example.*—Assume the level to be set up so that a sight can be taken on the rod held on a bench mark with a known elevation (*El*) of 255 feet. It is found that the line of sight of the level cuts the rod at a distance of 4 feet above the bench mark, so that the rod



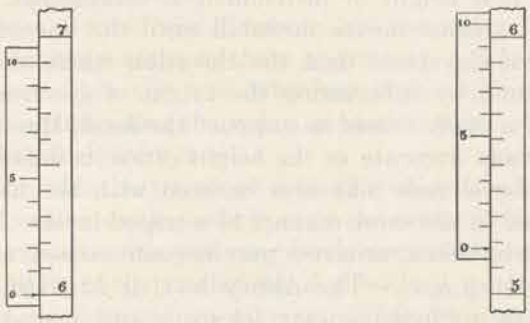
① Philadelphia rod with target. ② Light rod without target. ③ Abney level. ④ Hand level. ⑤ Level rod target. ⑥ Turning pin (steel).

FIGURE 13.—Level rods and accessories.

reading is 4 feet. Then the elevation of the horizontal plane of sight or the height of instrument is equal to 255 plus 4 or 259 feet. This sight taken on a rod held upon a point of unknown elevation is termed a "backsight" (*B.S.*), so the equation can be written: $El+B.S.=H.I.$ The line of sight is now directed at the rod, held on a point of unknown elevation; assuming rod reading 8 feet, the elevation of the unknown

point is equal to 259 minus 8 or 251 feet. This sight taken upon a rod held on a point of unknown elevation is termed a "foresight" (*F.S.*). The second equation thus can be written: $H.I.-F.S.=El.$

d. Reading rod.—The rod is held vertically on the station with its face toward the level, the rodman squarely behind it and facing the levelman so that he can see any signal from the latter, directing the movement and setting of the target. When necessary, the levelman indicates movement of the rod to right or left until it coincides with the vertical wire in his telescope. The rod is vertical when it shows the minimum reading; this is readily noted through the telescope when the rod is swung through a small arc in the line of sight. The levelman should note the accuracy of the telescope bubble before giving the signal for the rod target to be read if the target is being used,



① Construction of vernier scale. ② Rod vernier, reading 5.16 feet.

FIGURE 14.—Vernier scale.

or before and after reading the rod when it is being used as a self-reading rod. The target vernier is read by the rodman, who reports the reading to the note keeper, who checks the setting before making the records. The self-reading rod lends itself to more rapid work than the target rod, and gives nearly as accurate results.

e. Rod vernier.—A rod vernier is an auxiliary scale, used in connection with the target rod, by means of which the rod scale can be read one decimal place beyond its smallest division. The use of the vernier depends on the fact that it is easier to determine coincidence of two lines than to estimate fractions of a scale interval. A vernier is constructed by laying off a distance equal to nine of the main scale divisions and dividing it into ten equal parts, placing it edge to edge along the main scale. (See fig. 14①.) After the vernier starts to move, no lines are coincident until the vernier has moved one-tenth of a main subdivision, at which time the first division line of the ver-

nier coincides with a line of the main scale. The position of the zero on the vernier is the reading desired. In figure 14②, the zero lies between 5.1 and 5.2, and the sixth subdivision of the vernier is coincident with a subdivision of the main scale; hence the reading is 5.16.

f. Use of hand level.—The hand level, having no tripod, is held to the eye by hand and the forward end is raised or lowered until the bubble is in the center of the tube. The levelman may stand erect, in which case he must know the distance that his eye is above ground, or the level may be fastened to the top of a staff of known height, usually 5 feet. In the first case the height of instrument is found by adding the height of eye above ground to the elevation of the ground. In leveling uphill any point where the line of sight cuts the ground is the same elevation as the height of instrument. By moving up to this point, a new height of instrument is established. In leveling downhill the levelman moves downhill until the line of sight cuts a point of known elevation; then the elevation where the levelman is standing is found by subtracting the height of eye from the known elevation. If a staff is used to support the level, the method is the same, but is more accurate as the height of eye is definitely fixed for every shot. Level rods may also be used with the hand level, the level being used in the same manner as a tripod level. No shots over 75 feet should be taken, or errors may become excessive.

g. Use of Abney level.—The Abney level is a modification of the hand level with a supplementary telescope and a graduated arc for reading vertical angles or percentage of slope. A sight is taken through the tube as with the hand level while the bubble of the pivoted level tube, to which is attached the graduated arc, must remain in the center. The vertical angle or percentage of slope is then read from the graduated arc which has a direct vernier enabling the levelman to take readings to 10 minutes, or even less, depending on the markings of the arc and vernier.

53. Adjustments of dumpy level.—*a. Equipment.*—Dumpy level, leveling rod, stakes, hatchet, adjustment pins, note paper, pencil.

b. Adjustment of horizontal wire.—To make the horizontal wire horizontal when the instrument is level and parallax, if any, has been eliminated:

(1) Level the instrument and sight on some sharply defined point which is covered by the horizontal wire.

(2) Turn the telescope with the slow motion screw about its vertical axis so that the point appears to traverse the field of view. If the point remains on the horizontal wire the adjustment is correct. If not, turn the wire reticle slightly after loosening the capstan screws until the condition is satisfied.

(3) Sighting at a plumb line with the vertical hair is an alternative method.

c. Adjustment of the spirit level.—To make the axis of the spirit level perpendicular to the vertical axis—

(1) Set up the instrument and level it over one pair of leveling screws.

(2) Turn the instrument on its vertical axis through 180° or halfway around. The bubble should remain in the center of the tube. If not, move it halfway back to the center by means of the spirit level adjusting nuts.

(3) Repeat until the bubble remains in the center of the tube for any position of the instrument.

d. Direct or "peg" adjustment.—To make the line of sight parallel to the axis of the spirit level—

(1) Drive stakes (at *A* and *B*) about 300 feet apart.

(2) Set up midway between them and take a rod reading at each. Their difference will be the true difference of elevation between them.

(3) Set up so that when the rod is at *A* the eyepiece will be about one-fourth of an inch from it.

(4) Look through the telescope from the objective to the eyepiece and note the reading on the rod opposite the center of the field. (The location of the point on the rod may be determined by setting a pencil point at the center of the small field of view.)

(5) To this reading add or subtract (depending on whether *B* is lower or higher than *A*) the difference obtained in (2) above and set the target at the result.

(6) Sight on the rod at *B* and move the horizontal wire until it cuts the center of the target.

(7) Check the adjustment of the horizontal wire.

e. Exercise II.—Make the adjustments of the dumpy level as described above.

54. Differential leveling.—*a. General description.*—Differential leveling has for its object the determination of the difference of elevation between two stations, or the establishment of the elevation of a new station with reference to a known elevation, irrespective of the distance or the profile of the terrain between them. The level is set up and a backsight taken on the rod at the first station. The rodman paces the distance to the instrument and an equal distance beyond, if possible, to the second rod point, so as to equalize the lengths of backsights and foresights and thus to eliminate errors of instrumental adjustment. The level notes should show these distances, and, if it is not possible to equalize related backsights and foresights, allowance

should be made later so as to have the sum of all backsights and all foresights equal upon completion of the level circuit. At the second rod point the rodman selects or prepares a suitable turning point (*T.P.*). This may be a well defined summit of a solid boulder, a spike driven into the spreading root of a tree, or an iron pin driven firmly into the ground. (see fig. 13 ⑥). After reading this foresight the levelman picks up his instrument, goes by in the direction of the second station, and makes a new set-up. The operation continues in this manner until the final foresight is taken on the second station, the elevation of which, referred to the first, is desired. To check the arithmetical work, compare the difference between the sum of the backsights and foresights with the difference in elevations between the first and second stations. If the results are alike, the computations in the notebook check.

b. Field notes.—Figure 15 ① shows a standard form of differential level notes. The small encircled numbers (in the upper right corners) are not recorded; they are used here only to show the progression of entering the notes as obtained from the example of the field work shown in figure 15 ②.

c. Field method.—Figure 15 ② shows the first part of a differential level line, assumed to be run from *B.M.* 35, the elevation of which is 133.163 (all figures referring to elevations here are in feet). The first set-up shows the *B.S.* to be 6.659, which added to the elevation of *B.M.* 35 gives an *H.I.* of 139.822. The first *F.S.* on ⊙16 was 4.971, which subtracted from the *H.I.* gives an elevation of 134.851 for ⊙16. The lengths of *B.S.* (220) and *F.S.* (220) shown opposite the *H.I.* were determined by pacing, and were recorded together with the other readings as indicated in figure 15 ①. The instrument was then set up halfway between ⊙16 and ⊙17, and the operations (*B.S.*, *H.I.*, *F.S.*, etc) repeated, after which it was moved between succeeding stations until the leveling was completed. (*Caution.*—The rodman must never remove a temporary turning point (*T.P.*) until he gets a definite signal from the observer.) The levelman must never quit work on a temporary turning point but must continue until he comes to the next permanent *B.M.* or station where he will observe and record an *F.S.*

d. Checking methods.—(1) To check the accuracy of the level notes, add all backsights together (see fig. 15 ①); add all turning-point foresights together; the difference between these two sums should equal the difference between the elevations of the first and last stations. The foresight on the last station must be included in the sum of foresights although there is no corresponding backsight. The last station

is a turning point in the sense that, if work should be continued, a backsight would be taken on it. It is good practice to make this check for every page of the field notes before recording or computing subsequent work; this avoids the carrying over of computed errors to the next page. It must be remembered that this check enables one to detect only numerical mistakes made in computing for heights of instrument and elevations of turning points, and does not disclose mistakes in using the level, reading the rod, or recording rod readings; nor is it a check on the computed elevations of intermediate station (side shots) because no backsights are taken on such stations.

(2) Successful leveling requires a thorough knowledge of the sources of errors and of the methods of eliminating them. They are—

(*a*) Instrumental errors, due to imperfect adjustments of the level, sluggish bubble, irregularities in the movement of the object glass slide, errors in rod graduations, and defective rod joints on extension rods.

(*b*) Mistakes in manipulating equipment in setting up the level, including unnecessary clamping of the spindle, unequal backsights and foresights, not centering the bubble, resting the hand on the tripod or telescope, holding the rod improperly, or allowing dirt to accumulate on the base of the rod.

(*c*) Mistakes in reading the rod, as dropping a zero from the reading (like 5.170 for 5.017), etc.

(*d*) Errors in sighting by not having the cross hair coincide with the center of target, or using one of the stadia wires for the (center) horizontal hair.

(*e*) Errors due to changes in position of level or rod, as settling of instrument between backsight and foresight, disturbance of the level by passing traffic, and settling of a turning point between foresighting and backsighting.

(*f*) Errors due to natural sources, such as effects of sun or wind, changes in length of rod from variations in temperature, and, for very precise work, failure to consider corrections for curvature and refraction if backsight and foresight are not equal.

(*g*) Mistakes in recording and computing, by entering the reading with figures interchanged, recording backsights and foresights in the wrong column, neglecting to enter a reading, etc. Errors in computing are usually discovered by the check described in (1) above.

(*h*) Personal errors arising from causes peculiar to the individual.

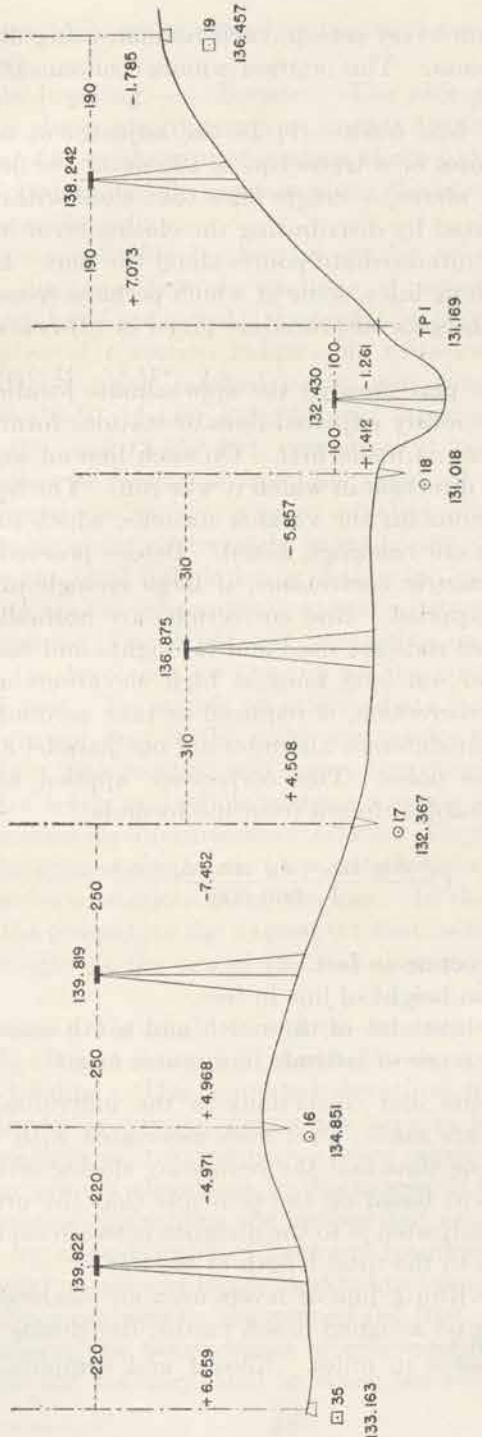
(3) The only method to check the field work is by rerunning the line if it fails to tie within the required allowable error. If the line is of considerable length it is better to use two rodmen and obtain two

LEVEL CIRCUIT — Leveler: Sgt M.T. Todd; Rodman: Sgt C.L. Miller; B & B Dumpy Level 31057 — FROM B.M. 35 TO B.M. 19 — (18)

Sta.	B.S. +	H.I.	F.S. −	Elev.	Sights B.S.	Sights F.S.	Remarks				
							April 21, 1940 (3 hours) Cloudy, moderate.				
B.M. 35	6.659	139.822		133.163	220		Concrete monument				
⊙ 16	4.968	139.819	4.971	134.851	250	220	Peg				
⊙ 17	4.508	136.875	7.452	132.367	310	250	"				
⊙ 18	1.412	132.430	5.857	131.018	100	310	"				
T.P. 1	7.073	138.242	1.261	131.169	190	100	Turning Point				
B.M. 19			1.785	136.457		190	Concrete monument				
	24.620		21.326	133.163	1070	1070	Actual elevation of ⊡ 19 = 136.442 ft.				
			24.620	3.294		1070	Permissible error = $0.05\sqrt{2140} = 0.032$ ft.				
			3.294 ✓	Check		2140	Actual error of closure = $\frac{0.015}{3280}$				

① Notes.

FIGURE 15.—Differential leveling.



Ⓟ Method.

FIGURE 15. — Differential leveling.

sets of readings from every set-up, each rodman using different positions for turning points. This method is more economical of time than to rerun a long line.

e. Adjustment of level lines.—(1) In the adjustment of level lines (or circuits), the notes or a transcript of the notes are first examined for errors (see *d*(1) above). Single lines that close within the allowable error are adjusted by distributing the closing error in proportion to the distance for intermediate points along the line. If a level net embracing several long lines, some of which perhaps cross each other, is to be adjusted, the general procedure given in (2) below should be followed.

(2) A diagram or plat showing the approximate location of all the lines, including previously adjusted lines or stations forming the base of the net or system, is made first. On each line an arrow head is placed to show the direction in which it was run. The field notes are checked and elevations for the various stations, which may be mean results of two or more runnings, listed. Before proceeding further, the rod and orthometric corrections, if large enough to affect final results, should be applied. Rod corrections are normally not to be considered if wooden rods are used and foresights and backsights are balanced. However, on long lines at high elevations a correction, called orthometric correction, is required to take account of the fact that level surfaces at different altitudes are not parallel except at the equator and at the poles. This correction, applied negatively to points going northward, is found from the formula:

$$C=\frac{hm\ (\phi_n-\phi_s)\ \sin\ (\phi_n+\phi_s)}{659{,}000}$$

in which

C=correction in feet.
hm=mean height of line in feet.
$(\phi_n+\phi_s)$=the latitudes of the north and south ends of the line.
$(\phi_n-\phi_s)$=difference of latitude in minutes of arc.

After all adjustments and corrections to the individual values for each separate line are made, level lines associated with one another are considered at one time and the remaining closing errors removed by a final adjustment based on the principle that the probable error of each point to be adjusted is to the distance between each two points as the total error is to the total length of the line.

f. Exercise III.—Run a line of levels over an established traverse starting and closing on assigned bench marks, the closing error not to exceed 0.05 $\sqrt{\text{distance in miles}}$. Record and compute all data as

shown in figure 15①, and make adjustments, if necessary, as explained in *e* (1) above.

55. Profile leveling.—*a. Purpose.*—The object of profile leveling is to find the elevations of points at known distances apart and thus obtain the profile on the ground surface along a given line. A profile shows all elevations and depressions along the line that would be seen in section from the side.

b. Description.— Profile leveling is similar to differential leveling except that distances between stations, if not already established, must be located and measured. In profile leveling it is customary to let the number of a station indicate its distance from the starting point, the latter being marked 0+00 and succeeding stations as 1+00, 1+50, 2+50, 14+00, 123+50, etc., meaning that they are 100, 150, 250, 1,400, and 12,350 feet, respectively, from the first station. Since a number of foresights from the same set-up are often taken on intermediate stations, it is good practice after backsighting on the last turning point to foresight immediately on the next turning point before the level bubble has a chance to "get out," after which the foresights on the intermediate stations between the two turning points are taken. Foresights and backsights on turning points are taken with the level rod on the top of the station, usually to the nearest 0.01 foot. Foresights on intermediate stations with the level rod on the ground next to the stake are usually taken to the nearest 0.1 foot only. This method will insure the same degree of accuracy for the line of levels as a whole without wasting time on readings for ground elevations on intermediate stations. Very often it is desirable to obtain the ground elevations at turning points, which at the same time are numbered stations along the line. In this case an additional reading on the ground, to the nearest 0.1 foot, is taken after the backsight or foresight at the top of the stake serving as turning point is completed.

c. Field notes.—Figure 16 is a form for recording field notes for profile levels along a short line, showing a check on the notes as for differential leveling. The computed elevations for intermediate stations and those for turning points are kept in separate columns as shown. Readings on turning points were taken to the nearest 0.01 foot, and for ground elevations to the nearest 0.1 foot.

d. Field method.—Examine the ground and set intermediate stakes as required for a good profile. Take rod readings to the nearest 0.01 foot on turning points and bench marks; but take ground rod readings on intermediate stations to the nearest 0.1 foot. In calculating elevations, preserve the same degree of exactness in the results as observed in the rod readings; that is, when the rod readings are taken

to the nearest 0.1 foot, use only the nearest 0.1 foot in the height of instrument to calculate the elevations. When a station stake is used as a turning point, the notes should show the ground elevation at this stake to the nearest 0.1 foot on the line preceding the more precise turning point record. Check-levels by the same parties should not differ more than 0.1 foot into the square root of the length of circuit in miles. Backsights and foresights should be balanced, and no sight longer than 350 feet should be taken. In order to secure a

PROFILE LEVELS — Leveler: Cpl L.C. Mahon; Rodman: Cpl M.F. Bear; B. & B. Dumpy No. 33011 — (21)

M.T. VERNON HIGHWAY — FROM ⊙2 TO ⊙7

Sta.	B.S.	H.I.	F.S.	Elev.	Elev. T.P.	Remarks
						April 24, 1940 (2 hours) Clear, moderate.
⊙2	7.29	138.21			130.92	1" iron pipe near road.
0 + 70			7.8	130.4		Road crossing.
1 + 00			8.5	129.7		
2 + 00			5.1	133.1		
3 + 00			6.6	131.6		
T.P.	9.64	147.65	0.20		138.01	Iron pin
4 + 00			8.4	139.2		
5 + 00			7.4	140.2		
6 + 00			3.6	144.0		
7 + 00			4.4	143.2		
8 + 00			2.2	145.4		
8 + 56			2.6	145.0		Road crossing
T.P.	3.65	149.24	2.06		145.59	Iron pin
9 + 00			6.8	142.4		Edge of road.
	20.58	130.92	2.26			
	− 2.26	+ 18.32				
	+ 18.32	Check				

Checked: L.C.M.

FIGURE 16.—Field notes for profile levels.

representative profile, ground rods should be taken not only at every station stake, but also at every important change of slope between station points, and appropriately recorded. The notes should be frequently checked and other safeguards taken to prevent blunders. Keep all records and notes as shown in figure 16.

e. Profile and grade lines.—Profiles and grade lines are plotted by the draftsman from the field notes obtained by the surveyor. Methods of plotting from these notes are explained in section X, TM 5–230. A profile (sec. X, TM 5–230) is a line plotted to a certain scale from known elevations and distances showing the configuration of the

ground surface along the measured line. A grade line is determined from the plotted profile and indicates the finished (graded) line in relation to the profile. A comparison of elevations between the profile and grade line will show how much higher (fill) or lower (cut) the ground must be graded at any point along the line to correspond with the grade line on the plot. For a detailed discussion of setting grade stakes and slope stakes, etc., see paragraph 197.

f. Exercise IV.—Run profile levels over an assigned line, placing stakes 100 feet apart, taking all necessary readings and recording and checking same. Follow methods outlined in this paragraph.

56. Cross sections and cross sectioning.—*a. Cross sections for earthwork.*—(1) For the construction of roads, railroads, or similar projects it is often necessary to estimate or determine the amount of earth to be moved to bring the road, etc., to the predetermined grade throughout its width. Estimates for earthwork are obtained by calculations necessitating the use of cross sections.

(2) Cross sections (see sec. X, TM 5–230) are taken perpendicular to the center lines usually at each 100-foot station. The number of cross sections to be made depends on their interval and on the length of the project. So-called cross levels are taken to determine the ground heights on either side of the main line and at right angles thereto.

(3) Cross levels to obtain the necessary data for cross sections may be run in conjunction with profile levels for the main line, or separately after the main line has been staked and run. The latter method is to be preferred, since it will avoid confusion in the work and in recording the readings. Figure 17 shows a set of cross level notes pertaining to the cross sections plotted in section X, TM 5–230. The second column in figure 17 contains the elevations of the profile transcribed from the profile level notes (sec. X, TM 5–230), the remaining columns the foresights to the points (and their elevations) along the cross levels, 15 feet apart in this case.

(4) The plotting of cross sections from field notes is explained in section X, TM 5–230. Cross sections (and profiles) are sometimes plotted from contoured maps. This too is explained in section X, TM 5–230.

(5) Volume computations for estimating earthwork are made from the cross sections as described in section X, TM 5–230.

b. Cross section levels.—In connection with drainage and irrigation work, the grading of earthwork, location and construction of buildings, etc., it is often desirable to obtain the shape of the surface of a piece of land. This may be done by dividing the area into a system of squares, and then determining the elevations of the corners and other points

where changes of slope occur. The length of the sides of the squares are from 100 feet to 50 or 25 feet. The squares are laid out with the tape or transit, distances measured by the tape or stadia, and elevations determined with the engineer's level. Stakes are set at the corners of the square and marked *A*–1, *A*–2, *B*–1, etc. The data obtained by this method are frequently used in the construction of a contoured map, or to plot cross sections of any portion of the area. Figure 18 shows a page of field notes for cross section levels. The notes

CROSS LEVELS — WATERTOWN HIGHWAY 0+50 To 4+50 (All elevations at top of ground)

Leveler: Cpl. M.F. Bear; Rodman: Pvt. A.I. Muir; B.& B. Dumpy No. 33011; (23)

Sta	Center * Elev.	Left 30'	Left 15'	Right 15'	Right 30'	Remarks
⊡ W17	156.73	B.S. +4.1	H.I. 160.8			
F.S.-		3.7	7.4	9.0	7.2	
0+50	151.5	157.1	153.4	151.8	153.6	
F.S.		5.1	8.2	9.0	6.1	
1+00	151.0	155.7	152.6	151.8	154.7	
F.S.		6.4	9.4	9.7	6.7	
1+50	150.6	154.4	151.4	151.1	154.1	
F.S.		7.6	10.0	10.4	10.8	
2+00	150.1	153.2	150.8	150.4	150.0	
F.S.		8.5	10.6	11.1	11.5	
2+50	149.4	152.3	150.2	149.7	149.3	
⊡ W18	150.19	B.S. +3.3	H.I. 153.5			
F.S.		2.2	1.3	4.5	5.2	
3+00	149.2	151.3	152.2	149.0	148.3	
F.S.		3.6	4.1	5.1	6.3	
3+50	149.0	149.9	149.4	148.4	147.2	
F.S.		6.8	5.2	5.2	7.6	
4+00	148.8	146.7	148.3	148.3	145.9	
F.S.		6.8	6.6	6.5	8.4	
4+50	147.2	146.7	146.9	147.0	145.1	

*Center elevations from profile lines. Checked: J.C.L.

FIGURE 17.—Notes for cross levels.

are arranged as for profile leveling, readings for *H.I.* being taken and recorded to hundredths of feet; for ground elevations at the corners of the squares, etc., to tenths of feet. A descriptive diagram should show the relative location of all points. The notes are usually plotted in the office and on cross section paper to eliminate the necessity of scaling distances.

c. Exercise V.—Run cross section levels and prepare a plot of a given area at the scale of 50 feet equals 1 inch, showing all details and 1-foot contours. Record field notes and complete the plot with cross sections as designated.

57. Barometric elevations.—*a. General.*—Approximate elevations may be obtained with the service aneroid barometer. This records the pressure of the atmosphere in terms of inches of mercury, the reading being taken from a pointer moving on a circular scale (see fig. 20). The corresponding elevation in feet is also shown on the dial. The aneroid barometer contains a vacuum chamber which is mounted within a hollow, cylindrical metal case, about 2¾ inches in diameter and 1¼ inches in depth. The top or front of the metal case is covered

CROSS-SECTION FOR MT. VERNON R.R.

Instrument: Birch
Rod: Oakes, Steele

Level: Berger # 3571
Date: Jan. 4, 1940

(27)

	Surface Elev.	Grade Elev.	Cross-Sections Base 40 – Slope 1½ to 1		
0+50	151.5	150.7	29.0 / -6.0	-0.8	22.4 / -1.6
1+00	151.0	150.1	27.2 / -4.8	-0.9	24.0 / -3.3
1+50	150.6	149.4	26.0 / -4.2	-1.2	25.5 / -3.7
2+00	150.1	148.7	25.1 / -3.6	-1.4	22.5 / -1.6
2+50	149.4	148.0	25.1 / -3.5	-1.4	22.4 / -1.5
3+00	149.2	147.3	24.8 / -3.2	-1.9	22.0 / -1.4
3+50	149.0	146.6	24.5 / -2.9	-2.4	21.8 / -1.3
4+00	148.8	145.9	23.9 / -1.4	-2.9	21.7 / -1.2
4+50	147.2	145.2	23.0 / -1.5	-2.0	21.5 / -1.0

Checked: Sgt. Bedell

FIGURE 18.—Cross section level notes.

with a beveled glass (crystal) fitted into a knurled ring which revolves around the case, and carries the altitude scale. The case is provided with a small ring for hanging the barometer on a nail or hook. The movement of the corrugated back or bottom of the box *B* is connected to the spring *C* and transmitted through levers *a*, *b*, and *d*, thus moving along the line *e*, at the end of which is a chain wound around the shaft to which the pointer is fastened. On the face of the dial are two scales, the inner one corresponding to inches of the mercury column and the outer one to feet of altitude. The altitude scales, reading up to 5,000 feet, are graduated in 20-foot values, and above

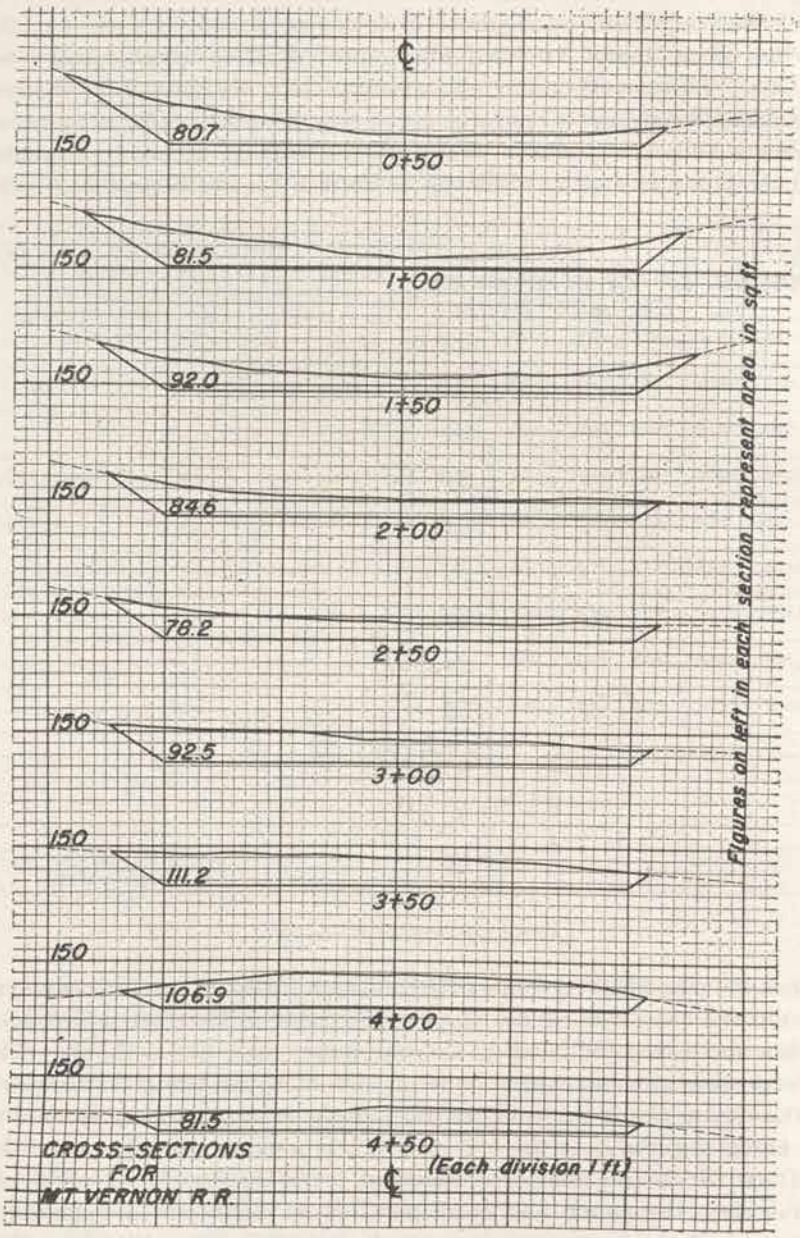
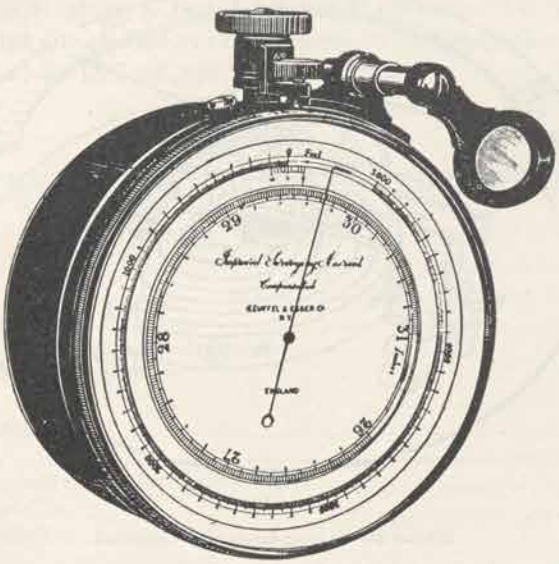


FIGURE 19.—Cross section plot.

5,000 feet in 50-foot values. The service barometer is calibrated for a temperature of 50° F. For temperatures above 50° F. add 2 feet per 100 feet for each 10° F. For temperatures below 50° F. subtract 2 feet per 100 feet for each 10° F. A hand-sewed leather case lined with felt is provided, together with a leather strap ½ inch wide and 5 feet 5 inches long, for carrying. The aneroid barometer must be handled carefully in order to avoid disturbing the delicate mechanism, thus changing the reading of the pointer. It should not be subjected to extremes of temperature, and should be well protected from wind, rain, and dust. Best results are always obtained by reading the



⊙ Front view.

FIGURE 20.—Aneroid barometer.

barometer held in a horizontal position at waist height, after gently tapping the face so as to overcome tendency of mechanism to jam.

b. Use of barometer.—There are three methods employed in leveling with the barometer:

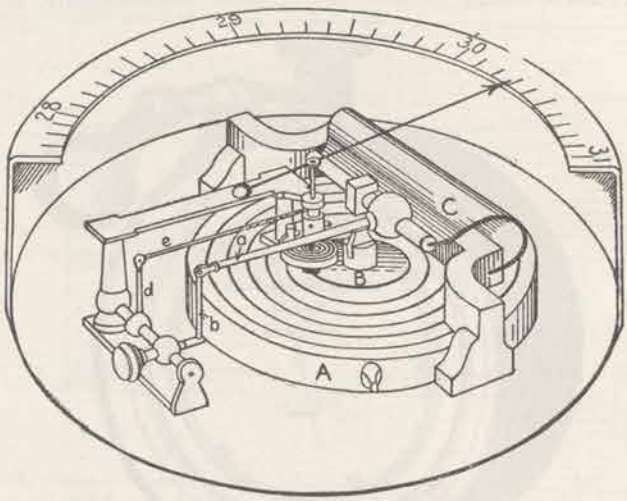
(1) In single observations the barometer is set at the initial elevation and then is carried from place to place and a single reading taken at each station. The elevations secured by this method include all errors due to changes in the atmospheric pressure during the observation period.

(2) Simultaneous observations tend to eliminate the errors introduced by change in the atmosphere. One barometer is kept at the starting point and read at previously determined intervals. Then the change that has occurred in the base barometer at a certain time

is used to correct all readings taken by the other barometers at the same time.

(3) Extended series of observations, usually taken with mercurial barometers, may cover long periods of time. The mean of the readings taken at the various stations is taken as the correct elevation.

c. Care and adjustment.—The aneroid barometer is very sensitive to shocks and must be carefully handled. A screw head, reached through a hole in the back of the outer case, permits the needle to be brought to any desired reading. It is seldom necessary to adjust this



② Interior view.

FIGURE 20.—Aneroid barometer—Continued.

screw, however, as the adjustment is made by the instrument maker. Other types have a movable elevation scale to facilitate adjustment with a standard instrument.

d. Procedure.—With a calibrated aneroid barometer take the aneroid barometer readings at two stations whose heights are to be compared, also the temperature (Fahrenheit) of the two stations with a thermometer. Let us assume that the following data have been obtained:

Station	*Barometer*	*Temperature*
△ Beach (at sea level)	34.046	77.5° F.
△ Top	23.660	70.3° F.

The computation for the difference of elevation between these two stations is then made as follows: Obtain from table VII, TM 5-236,

the difference of elevation between △ Beach and △ Top, interpolating to the nearest 0.1 foot, as shown here:

△ Beach	Barometer 34.046	Elevation	−4.2
△ Top	Barometer 23.660	Elevation	6,462.1
		First difference=	6,466.3

Next obtain coefficient (C) from table VIII, TM 5-236, for ($t+t'$) the sum of the thermometer readings, ($77.5°+70.3°$)=147.8°. Interpolating from the table, C=0.552. This multiplied by the first difference (6,466.3×0.552)=356.9. The latter added to the first difference (6,466.3+356.9) gives a final corrected difference of 6,823.2 feet. Assuming that the elevation of △ Beach is 13.7 feet, the elevation of △ Top would be 13.7+6,823.2=6,836.9 feet.

SECTION X

DESCRIPTION OF HORIZONTAL CONTROL METHODS AND EQUIPMENT

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Equipment	63
Choice of instrument	64
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Direction theodolite	67
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58. Purpose.—The purpose of horizontal control, established with the equipment and by the various methods outlined in this section, is to determine a network of accurately located points, the horizontal positions of which will serve as a skeleton or base for the map to be made. The final accuracy of the map is dependent upon the density of these points, and the closer they are together the less frequently large adjustments need be made. The location of these points may be determined by any of the methods mentioned below.

59. Methods.—*a. Triangulation.*—The main points of the horizontal control net for normal mapping operations are established by triangulation. The farther these points are apart the more precise must be the method by which they are to be located. Very seldom, if ever, will Army mapping require a higher order than third-order triangulation.

b. Intersection, resection, and miscellaneous methods.—Additional control points are often located by methods of minor triangulation included in one of the following:

(1) Intersections, section XX.

(2) Resections, section XXI.

(3) Special triangulation methods, section XXII.

(4) Miscellaneous orientation methods, section XXIII.

c. Traverses.—When it is desired to locate additional points from one or more stations, the locations of which have been previously fixed, and the area is wooded or otherwise unsuitable for triangulation (see *a* above), or the new points are so close together as to make traversing the more economical method, one of the following traverses may be utilized to establish minor or supplementary control points:

(1) *Traverse with transit and tape.*—See paragraph 79.

(2) *Azimuth traverse with transit and stadia.*—See paragraph 80.

(3) *Needle traverse with transit and stadia.*—See paragraph 81.

60. Choice of method.—The method or methods to be employed for the establishment of horizontal control depend upon the following, arranged in order of importance:

a. Desired accuracy of survey or map, time factor considered.

b. Character of terrain—heavily wooded, broken, open country, mountainous, etc.

c. Capabilities of personnel available.

d. Type of equipment available.

Final selection of the methods to be employed calls for considerable experience in all practical and theoretical phases of topographic surveying. Specifications usually state in detail the method by which the work is to be executed, but it is often desirable to leave a certain freedom of action to experienced topographers.

61. Methods of computation and adjustments.—*a. Traverses.*—All traverses coming within the classifications of paragraph 59*c*(1) and (2) are computed and adjusted as described in section XIII. Needle traverses are not computed but should be adjusted by one of the graphical methods explained in paragraph 62. Traverses of the first two classes, if plotted by the azimuth method known as plotting by polar coordinates (sec. XIII, TM 5-230) may also be adjusted by graphical means.

b. Triangulation.—Methods of computing and adjusting horizontal control established by the various forms of triangulation are described in detail in sections XVII to XXIII, inclusive.

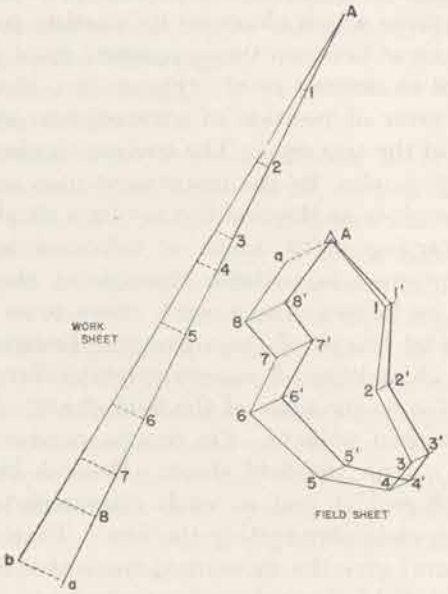
62. Graphical adjustment methods.—There are several methods for graphically adjusting position errors of traverse stations. The

most practical method is illustrated in figures 21① and ②, the first for adjusting a traverse which closes on its starting point, the second for adjusting a traverse between two separately fixed points.

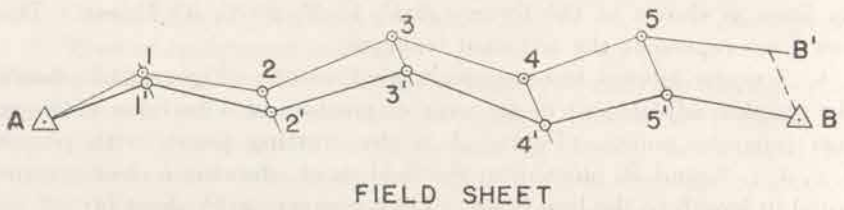
a. Traverse closed on starting point.—Figure 21① shows the graphic adjustment of an error of position of a closed traverse. Point *A* is the starting point of the traverse. The traverse is plotted by stations in the order *1*, *2*, *3*, *4*, etc., by the instrument man on his field sheet and the last course plots as the line *8–a* giving a displacement in the position of the starting point which is indicated by the line *aA*. Assuming that the error is uniform throughout the length of the traverse, proceed as follows: On a work sheet draw a line slightly longer than the total length of the traverse. Designate one end as the starting point *A* and lay off successively the distances *A–1*, *1–2*, *2–3*, *3–4*, etc., to *8–a* to the scale of the field sheet. Draw a perpendicular through the last point *a*. On this perpendicular, lay off the error *ab* equal to *aA* on the field sheet. Draw a line on the work sheet connecting *b* and *A* and at each intermediate point on the line *aA* erect a perpendicular cutting the line. These perpendiculars (broken in the figure) give the amount of correction for each station. On the original plot (field sheet) draw lines parallel to the line *aA* from each station in the direction in which the adjustment is to be made. Lay off the correction for each station and connect the new position by lines as shown in the figure, *A–1′*, *1′–2′*, *2′–3′*, *3′–4′*, etc. The new lines represent the adjusted traverse.

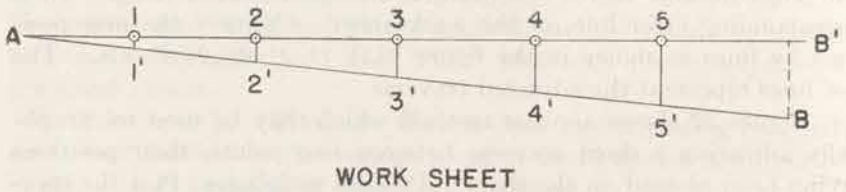
b. Traverse between two separately fixed points.—Figure 21② shows the graphic adjustment of an error of position of a traverse between two separate points. Point△*A* is the starting point, with points *1*, *2*, *3*, *4*, *5*, and *B′* plotted on the field sheet, showing a closing error equal in length to the line *B′–B*. On a separate work sheet lay off, as in the preceding case, a straight line plotting the distances *A–1*, *1–2*, *2–3*, etc., as shown. At *B′* erect a perpendicular the length of *B′–B*. Connect *A* and *B* by a straight line. Draw perpendiculars from all plotted points on line *AB′* to the line *AB*. Now draw lines on the field sheet parallel to *B′–*△*B* from each plotted point as long as their corresponding error line on the work sheet. Connect the new positions by lines as shown in the figure, △*A–1′*, *1′–2′*, *2′–3′*, etc. The new lines represent the adjusted traverse.

c. Figure 22 shows another method which may be used for graphically adjusting a short traverse between two points, their positions having been plotted on the sheet. Proceed as follows: Plot the traverse as actually measured in the field. In sketching or plane table work, this is already done. Draw a line *AB* from the beginning point



① Closed traverse.





② Traverse between two separately fixed points.

FIGURE 21.—Graphic adjustment of traverse.

to the ending point of the plot. Measure the straight line distance between the true positions of *A* and *B*. Measure off this distance, *Ab*, on line *AB*. From any convenient point *O* draw *OA* and *OB*. From *b* draw a line parallel with *OA*, cutting *OB* at *B'*, the corrected position of *B*. From *B'* draw a line parallel with *AB*, cutting *OA* at *A'*, the corrected position of *A*. Draw a line from each station to *O*. Draw *A'–1'* parallel with *A–1*, *1'–2'* with *1–2*, etc.

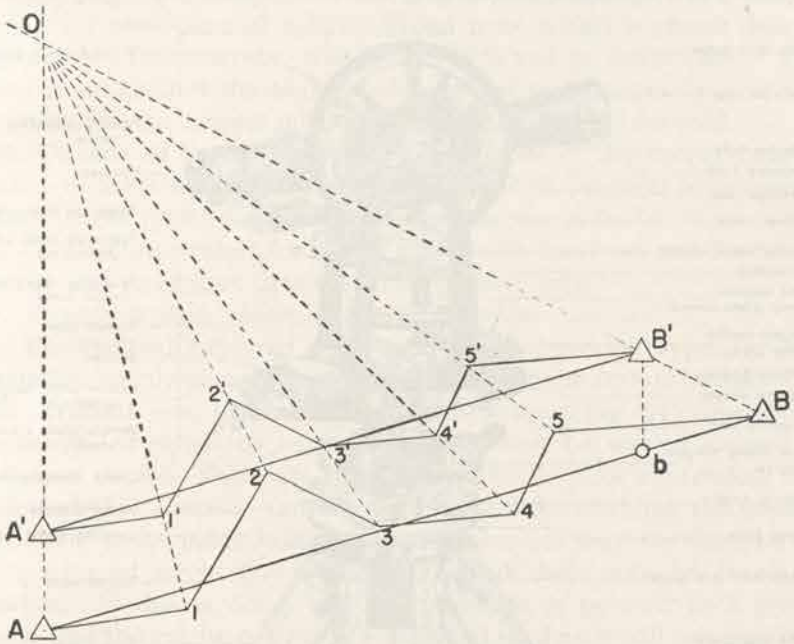


FIGURE 22.—Graphic adjustment of short traverse.

The adjusted traverse *A'*, *1'*, *2'*, . . . *B'* is then traced onto the new field sheet.

63. Equipment.—*a. Transit.*—The transit (fig. 23) is used for measuring horizontal and vertical angles, for measuring distances by the stadia and for leveling. The transit consists of a telescope attached to a vernier which may be moved around a graduated circle. There are suitable attachments for controlling the motion of the telescope and the vernier, and for enabling the graduated circle to be made horizontal. The vernier may be clamped to the graduated circle (zeros together) and they may be revolved together. If this is done with the telescope pointed to a given object and the graduated circle clamped so that it will not revolve, and the vernier then unclamped

from the circle, the vernier and telescope may be turned together until the telescope points to a second object. The angle between the objects may now be read opposite the vernier. Transits are made with horizontal verniers reading to 1 minute, 30 seconds, and 20 seconds and are called 1-minute transits, 30-second transits, or 20-second transits, respectively. Vertical circle verniers usually read to 1 minute, on some transits to 30 seconds. Transits are made in various sizes, the lightest weighing about 5 pounds.

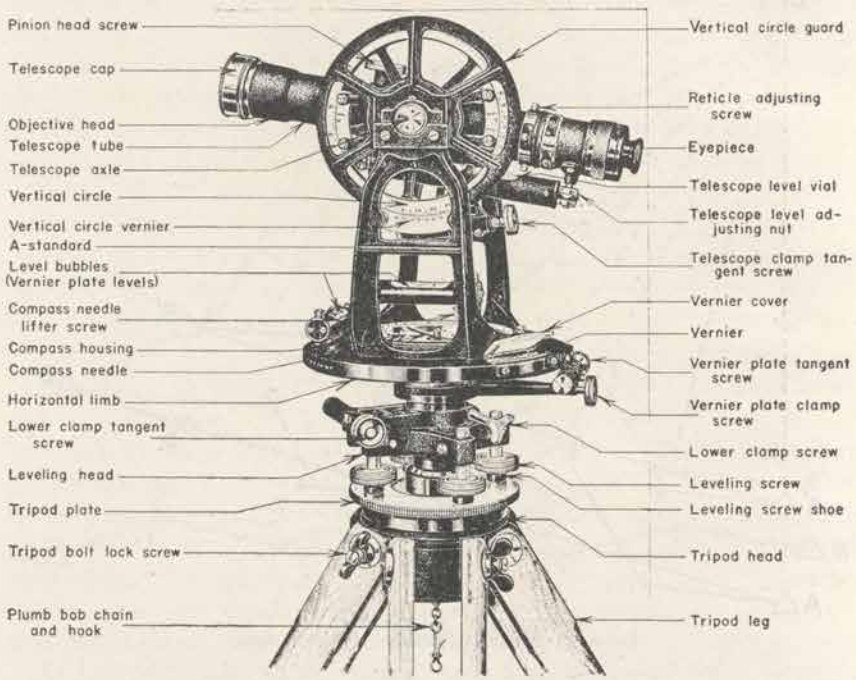


FIGURE 23.—Engineer's transit.

b. Theodolites.—For work requiring a higher degree of accuracy, especially triangulation, instruments of more precise manufacture and refinements are required. Two types of such instruments are in use—the repeating theodolite (fig. 27) and the direction theodolite (figs. 28 and 30). Any large instrument of high accuracy is often called a theodolite, but this term applies more properly to precision instruments provided with a striding level for the horizontal axis. The direction instrument is distinguished by having only one vertical axis so that angles cannot be measured by repetition; also, it has two or more micrometer microscopes for reading the angles. The repeating instrument is constructed with a double vertical axis, as is the case

with the engineer's transit. This permits angles to be measured by repetition.

c. Range pole and plumb bob holders.—Range poles (par. 46*b*) are often used to sight upon in reading horizontal angles for traverses. The sharp point of the pole may be placed on the tack head in a stake and the rod plumbed by balancing it between the finger tips of both hands, the rodman standing squarely behind it, facing the instrument. Another useful accessory when reading horizontal angles is a plumb bob holder consisting of a light tripod from which a plumb bob is suspended. This may be used in lieu of a rod or range pole. The tripod is set so that the plumb bob when at rest is directly over the station mark, the transit man sighting on the plumb bob cord.

64. Choice of instrument.—The selection of the proper instrument to measure horizontal angles depends on the amount of precision desired in the final results. Normally this would be—

a. One-minute transit for needle and stadia traverses and for intersections and resections that are to be plotted only.

b. Twenty-second transit for taped traverses.

c. Theodolites, either of the repeating or direction type, for triangulation requiring an accuracy in closing of 1:5,000 or better.

65. Transit.—*a. Description.*—(1) The transit (fig. 24) consists of a telescope attached to a vernier which may be moved around a graduated circle. There are suitable attachments for controlling the motion of the telescope and the vernier, and for enabling the graduated circle to be made horizontal. The vernier may be clamped to the graduated circle (zeros together), and they may be revolved together. If this is done, and the telescope is pointed to a given object, and the graduated circle is clamped so that it will not revolve, and if the vernier is then unclamped from the circle, the vernier and telescope may be turned together until the telescope points to a second object. The angle between the objects may now be read opposite the vernier. Two spindles, one inside the other, are attached to horizontal circular plates, the outer spindle being attached to the lower plate and the inner spindle to the upper plate. The lower plate carries a graduated circle which in a typical 1-minute transit is subdivided from 0° to 360° into whole and half degrees. Every 10° is distinctly marked or numbered. The upper plate carries two verniers exactly 180° apart, reading to single minutes. On the upper plate are two uprights or standards supporting the horizontal axis. The axis carries a telescope. The motion of the axis is controlled by a clamp and a slow-motion screw, called a telescope tangent screw. At one end of the horizontal axis is a vertical circle with figures from 0° to 90° to 0°, and the graduations in half degrees are read to single

minutes by a double vernier. The telescope is equipped with an eyepiece and with vertical, horizontal, and two fixed stadia wires. Below the telescope is placed a spirit level for use in getting differences of elevation and in adjusting the vernier of the vertical circle. For leveling the instrument there are two spirit levels on the upper plate, one parallel to and one at right angles to the horizontal axis. Leveling is done with four leveling screws. A shifting plate is provided to assist in setting up. On the upper plate is placed a magnetic needle and a graduated circle for reading magnetic bearings. Both the upper and lower plates are provided with clamps for holding them in any desired position and with tangent screws for making exact settings. At the center of the ball-and-socket joint are a chain and hook to which a plumb line may be attached for the setting of the transit over a point in the ground.

(2) More accurate transits, reading to 30 and 20 seconds and having larger horizontal and vertical circles, are made for work requiring a higher degree of accuracy than is obtainable with the 1-minute instrument.

(3) The tripod head is mounted on three wooden legs. On the extension tripod each leg is composed of two parts, an upper part which is hinged to a lug on the under side of the head, and a lower part which slides into the upper. The lower sliding part may be clamped into position by brass screws operating in cast-brass sleeves. The bottom of the sliding part is shod with a pointed, cast-steel shoe. An aluminum or composition cap is provided to protect the threads of the tripod head when not in use.

(4) When stored, the transit is screwed on a threaded metal stud attached to a sliding board that will hold the transit in an upright position and fit into a hardwood instrument case having a hinged door, lock, and two hooks. In the case are fixtures to which the sunshade, wrench, screw driver, plumb bob, magnifying glass, and adjusting pins are attached. A waterproof cover for the transit is furnished.

(5) The transit is a delicate instrument that requires constant care to keep it in good working condition. All adjustments should be made with the greatest care, operating the adjusting screws uniformly and without strain and clamping all motions firmly so as to secure a positive motion of the tangent screws without slipping. When the transit is being carried on the shoulder, the lower clamp should be loose so that the head may revolve freely in case it strikes anything. When moving through brush, doorways, etc., the instrument should be carried under the arm with the telescope in front of the body.

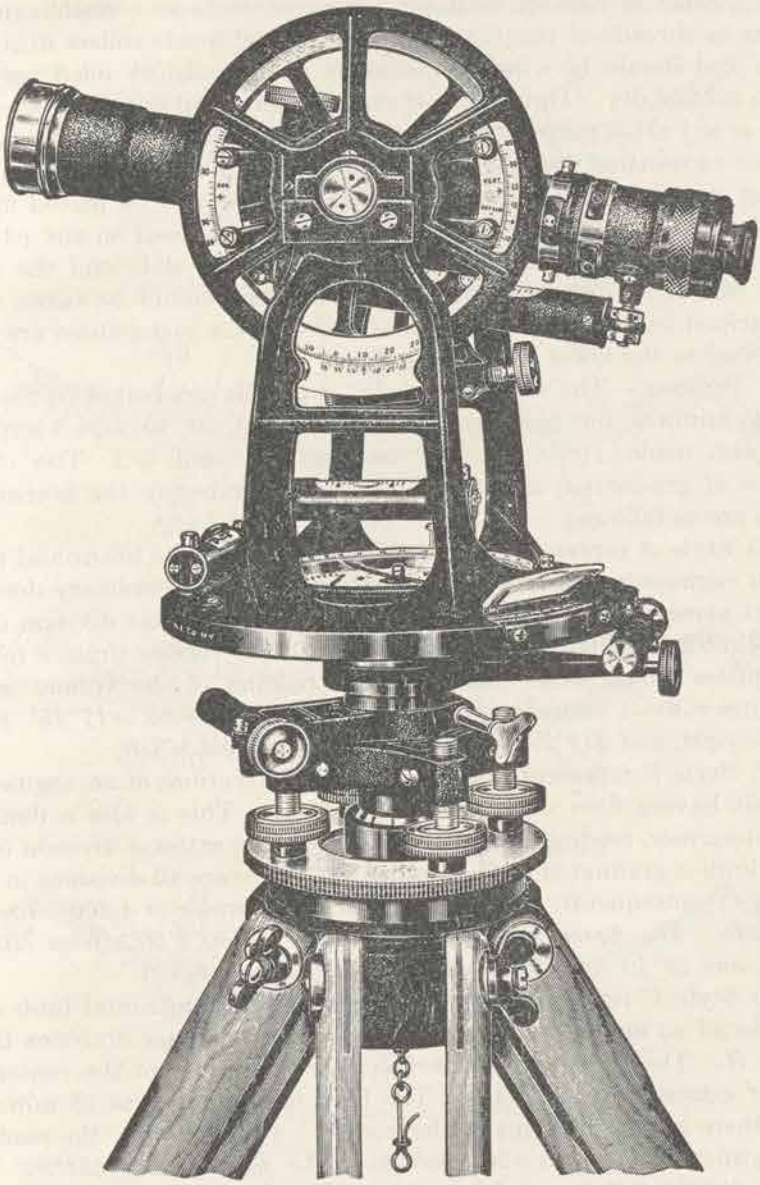


FIGURE 24.—One-minute transit.

The instrument should always be protected as far as practicable from sun, rain, and dust. Care should be taken not to let alcohol or similar liquid come in contact with the lacquered surfaces. Such exposed parts as threads of tangent screws and tripod heads collect dust and grit, and should be wiped occasionally with a slightly oiled rag and then rubbed dry. Only the best grade of watch oil should be used for this or any other purpose in connection with the transit. If the transit is wet on quitting work, it should be rubbed dry with a soft rag before being stored for the night. All instruments should be placed under shelter when not in use. Grease should never be used on any part of the instrument except on the eyepiece focusing slide and the rack and pinion for focusing the objective. Care should be taken that the tripod legs do not become loose and that the metal shoes are well fastened to the lower ends of the legs.

b. Verniers.—The ordinary transit is usually graduated to read to single minutes, but instruments reading to 30, 20, 10, and 5 seconds are also made. (See par. 52*e*, and fig. 14① and ②.) The usual styles of graduation and the methods of numbering the horizontal limb are as follows:

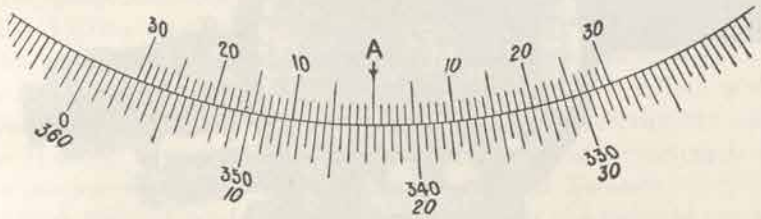
(1) Style *A* represents the usual graduation of the horizontal limb of an engineer's transit with its vernier. This is an ordinary double-direct vernier reading from the center to either extreme division (30). The limb is graduated to half degrees, and the vernier (from 0 to 30) comprises 30 divisions; therefore, the reading of the vernier is 30 minutes÷30=1 minute. *The figure reads 17°+25′=17°25′ from left to right, and 342°30′+05′=342°35′ from right to left.*

(2) Style B represents the graduation and vernier of an engineer's transit having finer divisions than style *A*. This is also a double-direct vernier, reading from the center to either extreme division (40). The limb is graduated to 20 minutes and there are 40 divisions in the vernier; consequently, the reading of the vernier is 1,200÷40=30 seconds. *The figure reads 130°00′+9′30″=130°9′30″ from left to right, and 49°40′+10′30″=49°50′30″ from right to left.*

(3) Style *C* represents the graduation of the horizontal limb and vernier of an engineer's transit having somewhat finer divisions than style *B*. This is a double-direct vernier reading from the center to either extreme division (45). The limb is graduated to 15 minutes and there are 45 divisions in the vernier. Consequently the reading of vernier is 900÷45=20 seconds. *The figure reads 351°30′+5′ 40″=351°35′40″ from left to right and 8°15′+9′20″=8°24′20″ from right to left.*

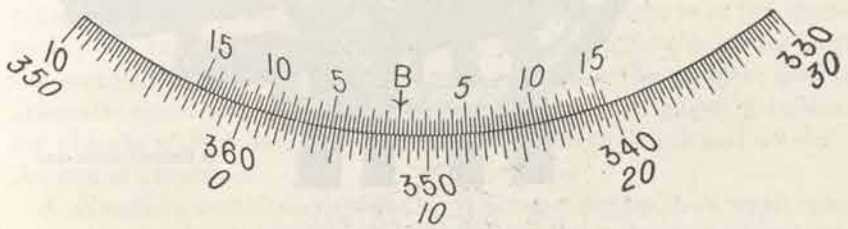
(4) Style *D* represents the graduation of the horizontal limb and vernier of an engineer's transit with more open divisions than style *C*.

This is a single-direct vernier reading from either end to the opposite extreme division (60). The limb is graduated to 20 minutes and there are 60 divisions in the vernier. Consequently the reading of the

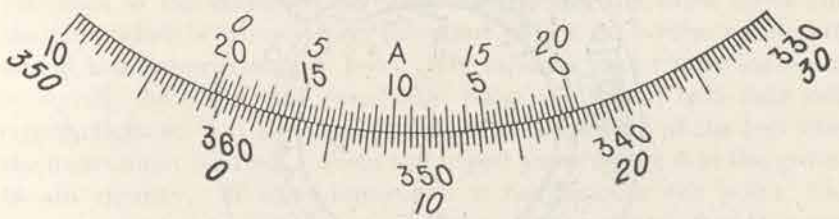


① Style *A*, graduated to 30 minutes, reading to 1 minute.

② Style *B*, graduated to 20 minutes, reading to 30 seconds.



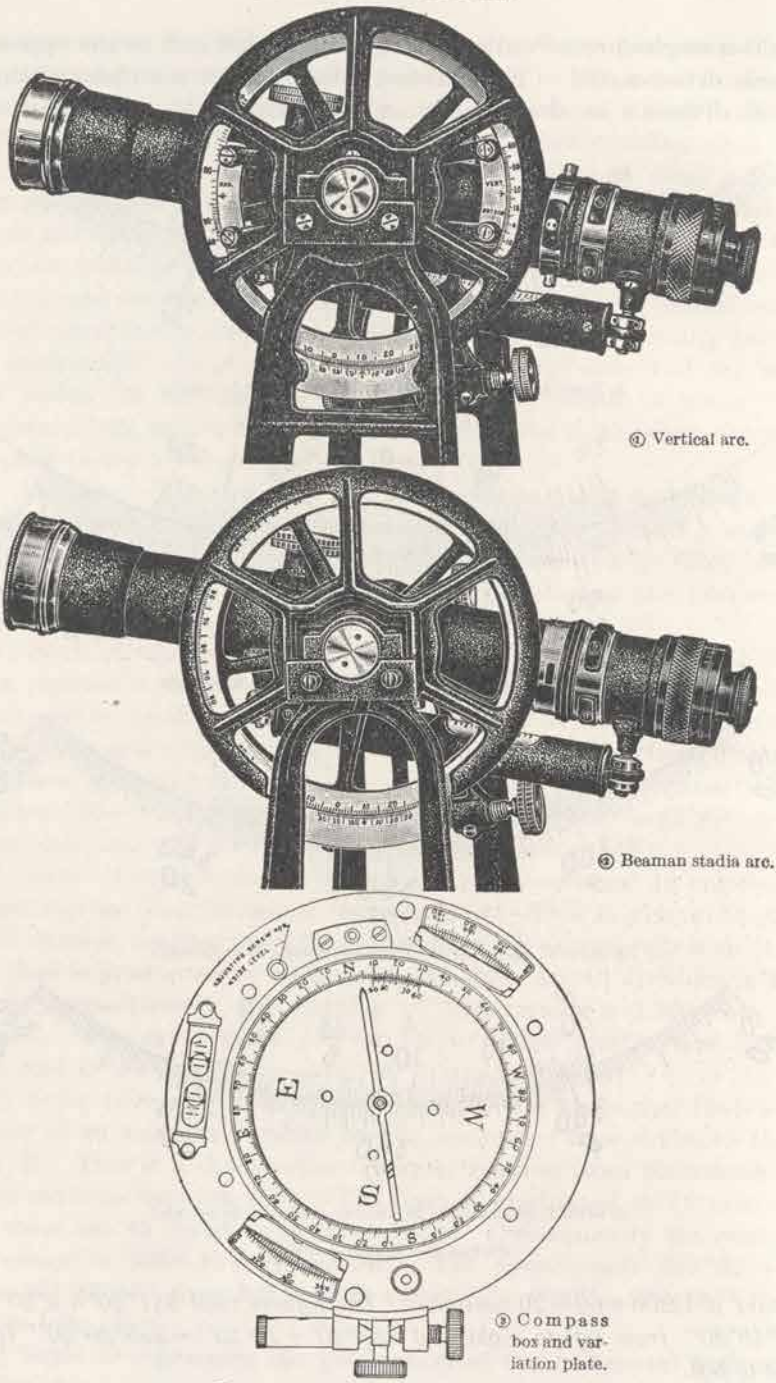
③ Style *C*, graduated to 15 minutes, reading to 20 seconds.



④ Style *D*, graduated to 20 minutes, reading to 20 seconds.

FIGURE 25.—Transit verniers.

vernier is 1,200÷60=20 seconds. *The figures read 341°40′+9′20″=341°49′20″ from left to right and 358°20′+10′40″=358°30′40″ from right to left.*



① Vertical arc.

② Beaman stadia arc.

③ Compass box and variation plate.

FIGURE 26.—Other transit graduations.

(5) Vertical arcs on engineer transit are graduated usually in degrees and half degrees, with verniers reading to minutes or to 30 seconds. Figure 26① shows a vertical arc with vernier reading plus 9°30′.

(6) Some instruments are now equipped with an additional graduation known as the "Beaman Stadia arc" or simply "stadia arc." This (par. 138 *f*) is a graduation based on percentage of slope. Compare in figure 26② the vertical arc reading of +10°34′ with the Beaman arc reading at the index of +18. A scale at the left, opposite the *H* index, gives −3 as the percentage factor to be subtracted from the measured distance to obtain the horizontal distance.

(7) A type of graduation similar to that of the vertical arc will be found on the compass box ring. This enables the observer to read magnetic bearings, usually within 30 minutes of arc and sometimes within 15 minutes. On most transits the box containing the graduated ring can be rotated to set off the magnetic declination for a reading of the true bearing. This is useful when running a needle traverse.

c. Accessory equipment.—As a rule each transit is equipped with a reading glass for verniers. It is best to carry this on a string around the neck. In addition each transit is normally equipped with a plumb bob, sunshade, screw driver, adjusting pins, waterproof cover, and sometimes with a prismatic eyepiece with dark glass slide. The plumb bob is used for centering the transit accurately over the station mark; the sunshade to replace the cap on the objective during work; the screw driver and adjustment pins to adjust the transit; and the prismatic eyepiece which is attached to the regular eyepiece, for observing objects of high altitude, the dark glass being employed whenever the sun is observed.

d. Handling transit in field.—(1) When setting up the transit, place two of the tripod legs in approximately the correct position with reference to the station mark; then manipulate the third leg so that the plumb bob is brought over the point, at the same time keeping the tripod head approximately level. On hillsides, one tripod leg should be uphill, the other two downhill. Keep the tripod bolt nuts sufficiently tight so that they will just sustain the weight of the legs when the instrument is lifted. Press the tripod shoes firmly into the ground to aid rigidity. If the plumb bob is nearly over the point, final centering may be made by moving the shifting plate after loosening the leveling screws.

(2) When leveling the instrument, turn the plates so that each plate level is parallel to a pair of diagonally opposite leveling screws. Great care should be exercised when leveling up; the screws must be snug to avoid tipping of the plates and possibly causing a horizontal shift of

the plumb bob. The screws must not be too tight, on the other hand, as this injures the instrument and causes errors due to strains in the metal. To level, grasp a pair of opposing screws between the thumb and forefingers and turn so that the thumbs move toward or away from each other, thus tightening one screw and loosening the other. After one bubble has been brought nearly to the center of its tube the other bubble is centered in a similar way. Instead of getting one bubble centered exactly, it is better to get both bubbles approximately level, after which one bubble and then the other may be exactly centered. After the instrument has been leveled, check the plumb bob to see that it has not been moved from the point during the leveling process.

66. Repeating theodolite (vernier type).—*a. Comparison with transit.*—The repeating theodolite (fig. 27) is similar in general design to the ordinary engineer's transit but of larger size and superior workmanship. The circle is usually 7 or 8 inches in diameter, and the verniers read commonly to 10 seconds. Figure 27 shows a 10-second repeating theodolite with striding level and individual magnifying (reading) glasses for each vernier. Usually no compass needle is found on these instruments.

b. Handling theodolite in field.—The theodolite should be carried in the box provided for each instrument. In order to obtain the best results the following rules should be carefully observed:

(1) Adjust and level the instrument carefully before proceeding with measurements.

(2) Keep away from tripod when making measurements.

(3) Turn plates gently, taking hold of plates or limbs and not the telescope.

(4) Work steadily and carefully; do not speed.

(5) Do not screw tripod legs too tightly.

(6) Do not turn clamps or leveling screws too tightly.

(7) When setting instrument up over stations, place two of the legs in their approximate positions. Then manipulate the third leg until the plumb bob is approximately centered over the station. Complete centering by loosening leveling screws and moving instrument on tripod head; finally level instrument carefully.

c. Accessory equipment.—The accessory equipment of the repeating theodolite consists of the same items as those listed in paragraph 65*c* except a reading glass.

67. Direction theodolite.—*a. Comparison with repeating (vernier type) theodolite.*—The direction instrument (fig. 28) is distinguished by having only one vertical axis so that angles cannot be measured by repetitions; also, it has two or more micrometer microscopes for read-

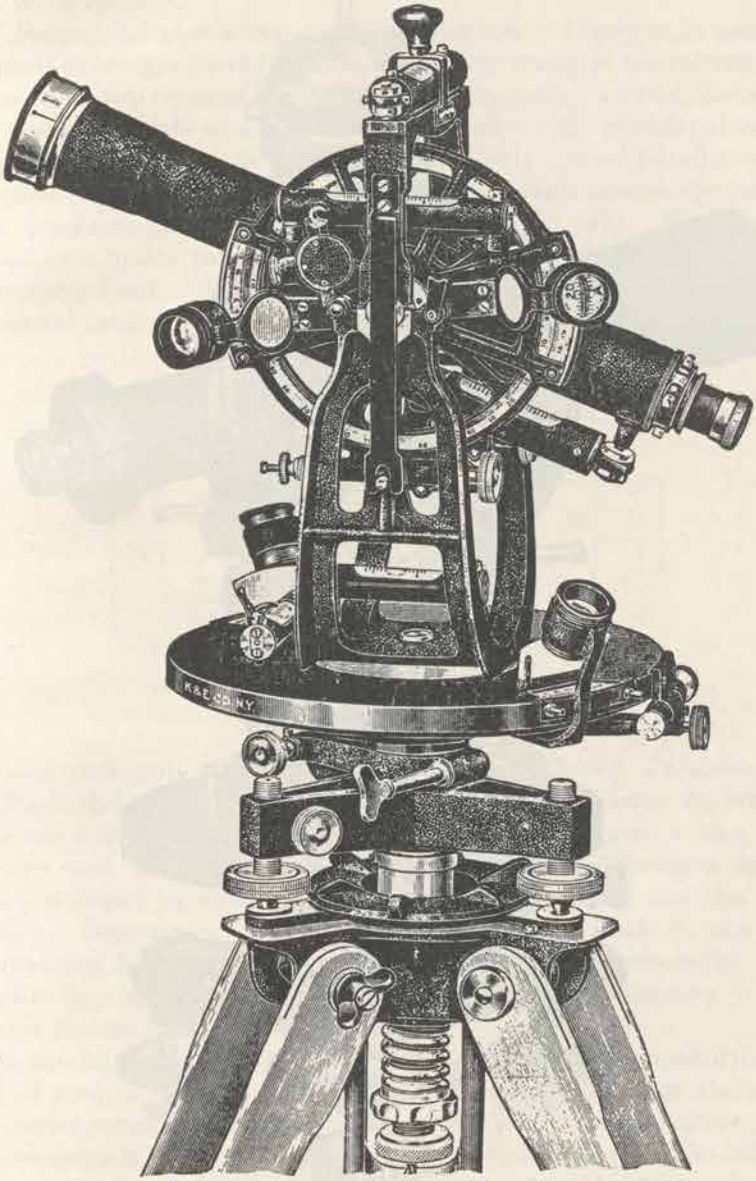


FIGURE 27.—Repeating theodolite.

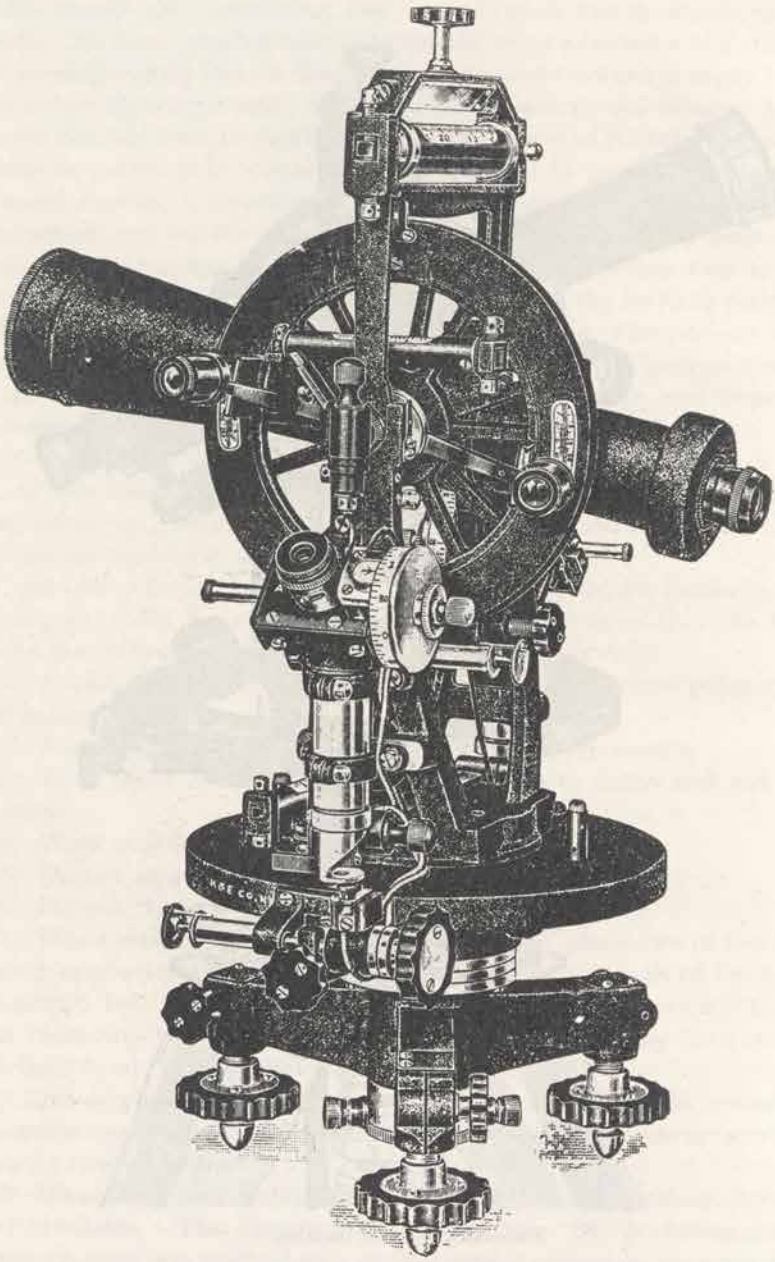


FIGURE 28.—Direction theodolite.

ing the angles. Vertical angles are read as on a transit except that the arc and vernier are graduated on the better class of instruments to read to 10 seconds.

b. Description of micrometers and microscopes.—Precision in measuring parts of the graduated circle is secured by means of the micrometer microscope, two or more of which are equally spaced around the circle. The device consists of a microscope focused on the graduated circle, having in the focal plane a wire, or two closely spaced parallel wires, mounted on a movable slide. In the field of each microscope two or more graduations of the circle can be seen (fig. 29). The slide is moved by a milled thumbscrew carrying a graduated drum called the micrometer head. When the telescope is pointed at an object and the horizontal motion is clamped, the index or fiducial line of the microm-

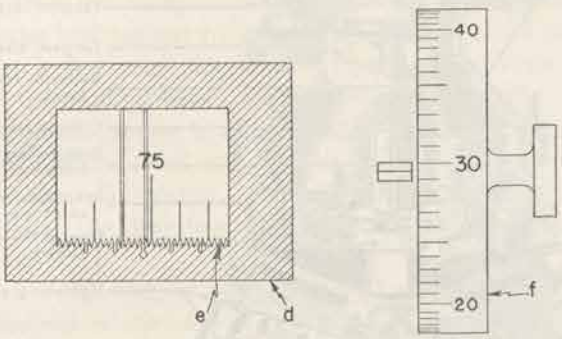


FIGURE 29.—Micrometer microscope.

eter lies ordinarily between two circle graduations. To determine the fractional part of the space between two graduations, the wire or wires are moved with the thumbscrew until they coincide with a scale division and the micrometer head is read. The direction to the object is found by combining the micrometer reading and the scale reading. Sometimes the micrometer is read on each of the two graduations between which the index lies. The micrometer may usually be read to the nearest second, and by estimation to the nearest one-tenth second.

On the left of figure 29 is shown the apparent arrangement in the field of view of the comb, wires, and circle graduations; on the right, the center notch and the micrometer head which furnish direct readings to seconds and by estimation to tenths of seconds. The reading as shown by the movable wires and comb is 74°58′, which, added to the drum reading of 29.5″, gives a reading of 74°58′29.5″.

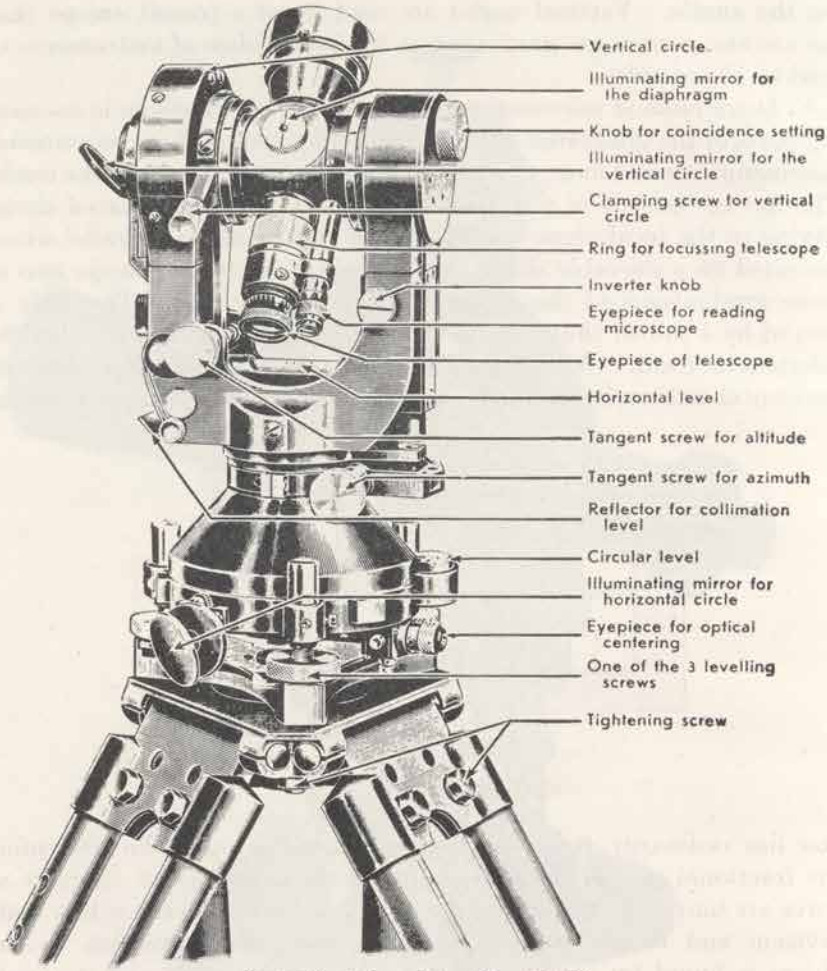


FIGURE 30.—Universal theodolite, Wild T2.

c. Accessory equipment.—The accessory equipment for the direction theodolite consists, with the exception of a reading glass, of the same items as those listed in paragraph 65*c*.

68. Wild type theodolite.—*a. Description.*—The Wild type theodolite is a direction instrument. Because of its ingenious design and ease of handling it may be considered one of the best instruments for general use. It is manufactured in several sizes. The "Wild T2" being perhaps the most satisfactory for military work is here described, together with preliminary instructions for handling it.

(1) *Setting up instrument.*—Set up the tripod over the station point. The central fixing screw, which holds the instrument down to the

tripod, remains always on the tripod, and thus cannot be lost or mislaid. A hole is bored lengthwise through this screw, and serves to receive the plumb line which is carried in a leather pouch on the lower part of the tripod. The tag at the upper end of the plumb line fits accurately into the bore of the central fixing screw; insert it into place from below, giving it a slight turn to secure it (bayonet joint). After setting up the tripod firmly and plumbing it over the mark to within about half an inch, take the plumb line off again. Then loosen the two locking levers of the theodolite casing by firmly pulling outward both ends of the leather strap and lift the sheet steel hood carefully from the base plate. Loosen the three black screws which secure the theodolite to the base plate and withdraw the three slides thus set free; the theodolite is now ready to be lifted from the base plate. Move the telescope from its steeply inclined position to a horizontal one, then take up the instrument by grasping the right hand support (the upper part of which carries the seconds drum), place it on the tripod, and screw up the central fixing screw moderately tight. When handling the instrument, never grasp it by the left hand support (which carries the collimation level and its adjusting arrangement), as this might disturb the adjustment. The theodolite should be so placed on the tripod that the illuminating mirror of the horizontal circle can receive sufficient light.

(2) *Removal of star-shaped base plate.*—If automatically controlled centering is desired in the case of polygon and similar measurements, a small screw is loosened at the bottom of the base plate. Only after loosening this screw can the spring plate, which holds the foot screws, be turned far enough as to permit removal of the theodolite. Owing to the spring system the plate cannot turn by itself even if the safety screw is open.

(3) *Centering and leveling instrument.*—For shifting the instrument on the tripod head in centering over the mark, a play of 2 inches is available. The centering of the instrument is normally done by means of the optical plummet. First bring the bubble of the circular level to center by working the three foot screws. On looking through the eyepiece of the optical plummet, on the lower part of the theodolite (see fig. 30), and focusing, the circular adjusting mark and the ground point will be seen. Move the theodolite about on the tripod head until the ground point appears in the middle of the adjusting mark. Now level the instrument exactly, making use of the horizontal level which is situated between the two supports, and rectify any remaining divergence of the optical plummet by further movement of the instrument on the tripod head. The central fixing

screw should be slightly slackened before every shift of the instrument on the tripod head, and then tightened up again as soon as the centering has been accomplished. In calm weather, the theodolite might be centered over the ground mark by the use of the plumb line alone. The centering can be done before leveling the instrument, because the point of suspension of the plumb line lies in the same plane as the points of the foot screws. For refined measurements, the instrument and the entire tripod should be protected from the direct rays of the sun by the use of a sunshade. Before beginning measurements, the theodolite should be rotated a few times about its vertical axis, and the telescope moved up and down about the horizontal axis. After being centered and carefully leveled, the instrument is ready for taking measurements.

(4) *Focusing.*—Direct the telescope toward the sky and turn the milled black dioptric collar on the eyepiece until the cross lines appear sharp and black. Note the setting of the dioptric collar on the numbered scale because this will be nearly constant for the same observer. Focusing for a sharp image of the object to be sighted is accomplished by turning the nickeled focusing collar (testing for absence of parallax between image and cross lines by movement of the eye). To focus the reading microscope, turn the small milled black collar of the eyepiece, which is alongside the eyepiece of the telescope, until the graduation marks of the circles appear perfectly sharp.

b. Measurement of angles.—On the front and back of the right-hand support will be found the change-over knobs on the same axis for the two circles. To make the horizontal circle visible, the black lines on the change-over knobs are to be set horizontally. For reading the vertical circle the lines should be set nearly vertical.

(1) *Reading of circles with 360° graduation, horizontal circle.*—(*a*) First set the black line of the change-over knob horizontally, so that the two images of the horizontal circle are visible in the reading microscope.

(*b*) To obtain the brightest and most uniform illumination possible of the two images of the circle divisions, turn the illuminating mirror on the tribrach below into the most favorable position. When measuring angles the illumination should not be changed.

(*c*) Then focus the reading microscope by turning its milled black collar until the graduation marks of the circle appear sharply defined in the images.

(*d*) Two separate images are now seen in the reading microscope.

1. Above.—The double image, divided by a fine horizontal line, of the diametrically opposite parts of the horizontal

circle, for reading off the degrees and tens of minutes (fig. 31).

2. *Below.*—The small image of the scale of the seconds drum, for reading off the individual minutes and the seconds.

The reading itself is accomplished in the following manner: After making the pointing by bringing the telescope exactly onto the object, look at the image in the reading microscope before making the coincidence. In the middle of the field of view, on the lower half of the image of the circle, there will be seen a fixed vertical line which does not serve for taking the reading, but only to mark the middle of the field of view, and to form a pointer. The coincidence adjustment of the graduation lines of the two images of the circle should be made in this vicinity by turning the knob of the coincidence adjustment until in the middle of the field of view the graduation lines of the upper half-image appear to be in exact prolongation of those of the lower half-image. The final turning movement of the coincidence adjustment should always be in a clockwise direction. In figure 31 the image of the scale is depicted as it appears after the coincidence adjustment has been made, since the reading of the circle is only taken when the graduation lines exactly coincide: In figure 31① the nearest upright figure to the left of the center line gives the degree division 285°. Now, starting from 285, count the graduations to the division 105 (which lies diametrically opposite to the division 285 on the horizontal circle). In the example, there are 5 intervals, that is, 5 tens of minutes, or 50′; thus from the upper image the reading is 285°50′. In the lower image of the scale of the seconds drum 1 minute is read off on the lower series of numbers and 54.6 seconds on the upper series of numbers and graduations; thus reading 1′54.6″ on the drum. The complete reading is therefore 285°51′54.6″.

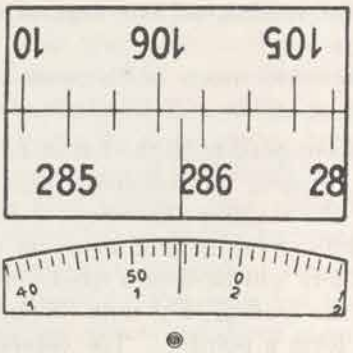
(2) *Vertical circle 360°.*—Graduation and numbering of the vertical circle are the same as for the horizontal circle. With telescope direct, the zenith distances can be read directly. Observing in both telescope positions and according to the readings A_1 and A_2 the zenith distances are given by the formula

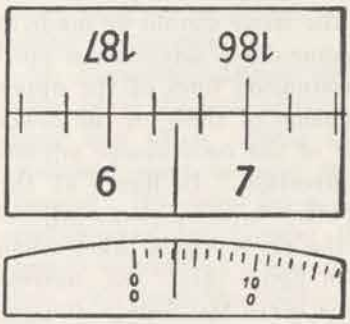
$$Z=180°-\frac{A_2-A_1}{2}\equiv\tfrac{1}{2}(A_1+360°-A_2)$$

The vertical angles are obtained by the formula

$$A=\frac{A_2-A_1}{2}-90°=\tfrac{1}{2}\ (90°-A_1+A_2-270°)$$

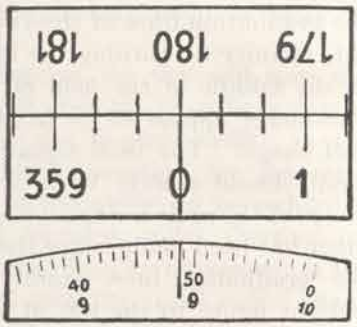
The following procedure is indicated:





Circle . . . . . 6° 30′
Drum . . . . . 0′ 03″,3
Complete reading . 6° 30′ 03″,3

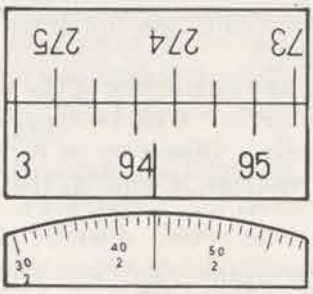
②



Circle . . . . . 0° 0′
Drum . . . . . 9′ 48″,8
Complete reading . 0° 9′ 48″,8

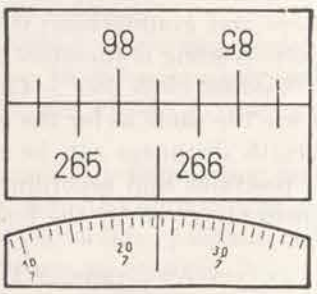
③

Telescope direct



94° 12′ 43″,7

Telescope reversed



265° 47′ 23″,6

Reading with telescope direct — 4° 12′ 43″,7
Reading with telescope reversed — 4° 12′ 36″,4
Mean = correct vertical angle — 4° 12′ 40″,0

④

FIGURE 31.—Reading horizontal and vertical circles on Wild T2 theodolite.

(*a*) The black line of the change-over knob is set vertically, so that the image of the vertical circle becomes visible in the reading microscope.

(*b*) Turn the illumination mirror at the end of the tilting axis toward a clear part of the sky, so that the image of the vertical circle appears bright.

(*c*) Focus the circle image sharply by turning the milled black collar on the reading microscope.

(*d*) Bring the bubble of the collimation level to center by turning the fine adjustment screw of the level until the two ends of the bubble coincide as seen in the level prism. Then check the pointing of the telescope on the object and take the reading in the same manner as for the horizontal circle.

c. Setting horizontal circle.—If angular measurement is to start from 0° (or any other desired graduation point of the circle), the telescope is first directed to the initial object, and then the milled head which serves to rotate the circle (and which is situated on the tribrach and covered by a protecting cap) is turned until the desired initial circle reading appears in the reading microscope. If, for example, the first pointing is to read 0°0′0″, the seconds drum must first be set to zero, then the pointing must be made, and then the circle rotated by the lower milled head until the 0° graduation line coincides exactly with the 180° graduation line in the double image seen in the reading microscope, after which the protecting cap is closed. For other precise angular measurements where it is required to begin with a certain number of degrees, the pointing is first made, then the circle turned by the lower milled head to the desired graduation mark with approximate coincidence; the protecting cap is closed, and exact coincidence of the required graduation lines (0° and 180°, 45° and 225°, 90° and 270°, 180° and 0°, etc.) made by the seconds drum, after which the seconds reading is taken, as for example 0°2′36″.

d. Returning instrument to case.—Withdraw the three slides of the ground plate of the metal hood. Open the clamps of the theodolite and hold the instrument by grasping the right-hand support. Turn the cone with the free hand to set the instrument into the ground plate, taking proper care that the three lugs are placed correctly on the three rests. Now, push in the slides and firmly drive home the three black screws. Move the telescope to a vertical position and tighten moderately the vertical and horizontal clamps. Then put the metal hood on and lock it.

e. Supplementary equipment.—(1) *Electric illumination.*—Take out the two illuminating mirrors and replace by the corresponding lamp

holders. Hang the battery case on the tripod, put the switch in the holder provided for it on one of the tripod legs, and place the plug in its socket on the instrument. For changing the bulbs, the lamp holders can be withdrawn from the illumination supports. The brightness of the illumination can be varied by turning the collar on the switch to change the resistance. The illumination of the tele-

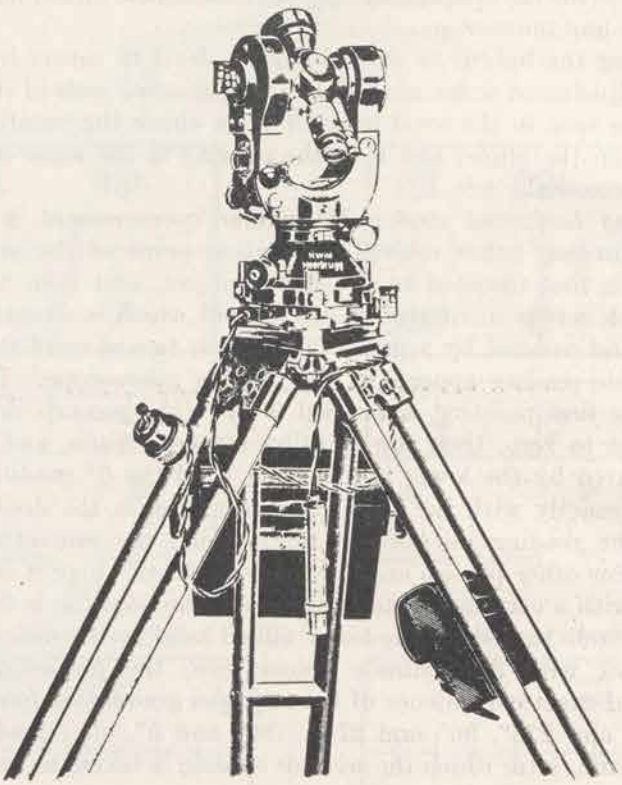


FIGURE 32.—Universal theodolite, Wild T2, with electric illumination and hand lamp.

scope diaphragm can be regulated by turning the milled head on the center of the telescope.

(2) *Diagonal eyepieces.*—Remove the eyepieces from the telescope and reading microscope, replacing same by pushing in the diagonal eyepieces. They make it possible to observe angles of elevation up to the zenith.

(3) *Striding level.*—The striding level is placed on the bright rings of the horizontal axis which are ground exactly concentrical. The striding level is used almost exclusively for astronomical observations.

For the usual terrestrial measurements the alidade level is more convenient and of sufficient accuracy.

(4) *Horrebow level.*—The Horrebow level is used for astronomical observations, especially for refining the methods of corresponding zenith distances. This level is screwed on the vertical clamp.

f. Accessory equipment.—In the ground plate of the metal hood the following accessories are located: adjustment pins, screw driver, brush, oil bottle, and illumination plugs. The parts belonging to the tripod, such as plummet and hexagonal spanner, are kept in the tripod pocket. The central fixing screw is mounted firmly on the crossing of the tripod head.

SECTION XI

ADJUSTMENTS OF TRANSIT AND THEODOLITES—THE STADIA

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Stadia	70
Adjustment of transit	71
Adjustment of repeating theodolite	72
Adjustment of direction theodolite	73
Adjustment of Wild type theodolite	74

69. General.—*a.* A damaged instrument should not be repaired in the field unless required in an emergency. If any parts have been injured, the instrument should be turned in, through proper channels, for repair by expert instrument repairmen. Such repairs can be made with less trouble and less cost if no attempt has been made to turn the various parts to determine the extent of the damage.

b. The adjustment of surveying instruments requires extreme care and good judgment. No man should be allowed to attempt the adjustment of an instrument until he has had several weeks' experience in its use and has become thoroughly familiar with its construction and operation.

c. Before commencing the adjustment of an instrument be sure that it is firmly set up and carefully leveled. An instrument may appear to be out of adjustment when some minor part is loose, therefore it should be looked over carefully before commencing the adjustments. After completing the various adjustments, always make all the tests again before using the instrument. All the adjusting screws should be screwed tight enough to hold, yet not so tight as to injure the threads or put an undue strain on any other part.

d. Adjustments should be made in the order given, as some depend on the accuracy of others previously made, and a change in one may

262341°—40——7

affect the others. If an instrument is badly out of adjustment it is better not to adjust one part completely at once, but to bring the instrument as a whole gradually into adjustment. Nearly all adjustments of instruments depend on the principle of reversion. It should

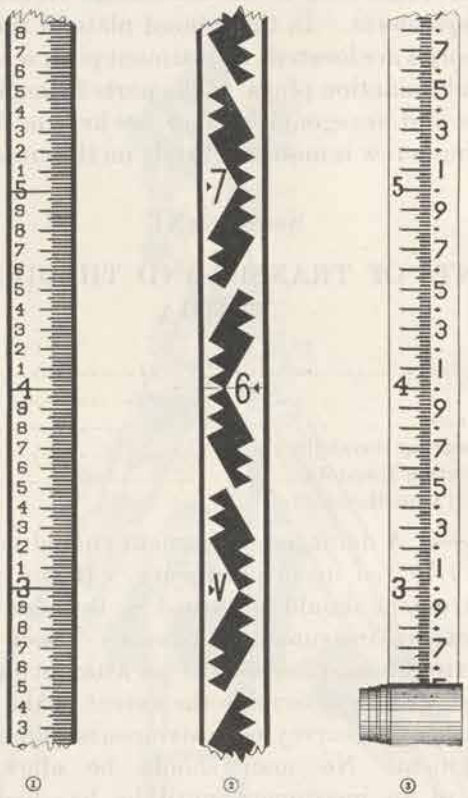


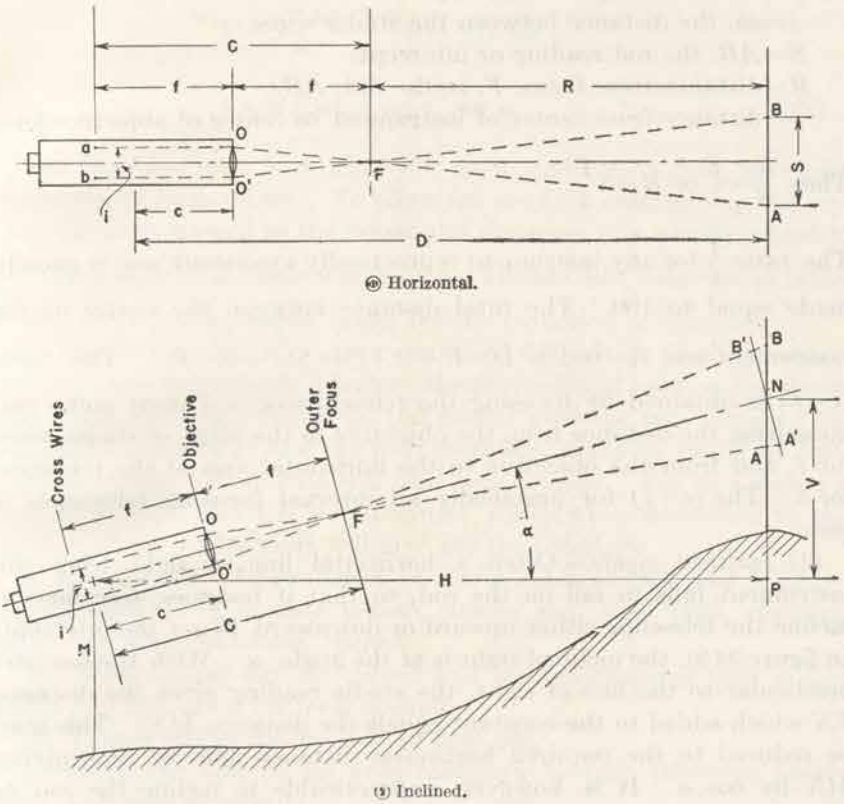
FIGURE 33.—Stadia rods.

be understood that by reversing the position of the instrument the effect of an error is doubled.

70. Stadia.—*a. Description.*—The stadia is a device for measuring distances by reading an intercept on a graduated rod. For this purpose two additional horizontal hairs called stadia hairs are carried in the transit telescope on the same reticle as the cross hairs and are placed equidistant from the horizontal hair. By reading the amount of intercept on a rod between the upper and lower wires, multiplying this by a constant, usually 100, and then adding a second constant, expressed $(c+f)$, the distance from the center of the instrument to the point on which the rod is held is fairly accurately determined.

The theory involves the relation between the sides and altitudes of similar triangles.

b. Stadia rods.—Figure 33 shows parts of three types of stadia rod graduations, ② being the most commonly used type in the Army.



② Horizontal.

③ Inclined.

FIGURE 34.—Sights.

It is a folding rod hinged at the 6-foot mark and when opened for use is 12 feet long. Figure 33① is similar to a standard level rod without target, usually about 3 inches wide and may be of the folding type as the one described above or a single board. Figure 33③ is a flexible pocket rod which can be carried rolled up in the pocket. It is about 3½ inches wide and can be tacked through eyeleted holes to any piece of light board.

c. Theory of the stadia.—(1) *Horizontal sights* (fig. 34①).—The stadia wires, spaced a distance i, are shown at a and b; AB is the space on the rod intercepted by the stadia wires, and $00'$ is the objective

lens. All rays, coming from points on the rod, which enter the telescope come to a focus, F, in front of the lens, at a distance equal to the focal length of the lens, that is, the distance from the lens to the position of the stadia wires, a and b.

Let f=the focal length of the objective lens.

$i=ab$, the distance between the stadia wires.

$S=AB$, the rod reading or intercept.

R=distance from focus, F, to the rod, AB.

c=distance from center of instrument to center of objective lens.

Then $\frac{R}{S}=\frac{f}{i}$ or $R=S\frac{f}{i}$.

The ratio $\frac{f}{i}$ for any instrument is practically a constant, and is usually made equal to 100. The total distance between the center of the instrument and the rod is $D=R+(c+f)=S\frac{f}{i}+(c+f)$. The term $(c+f)$ is obtained by focusing the telescope on a distant point and measuring the distance from the objective to the plane of stadia wires for f, and from the objective to the horizontal axis of the telescope for c. The $(c+f)$ for practically all internal focusing telescopes is zero.

(2) *Inclined sights.*—Often a horizontal line of sight from the instrument fails to fall on the rod, so that it becomes necessary to incline the telescope either upward or downward to get the intercept. In figure 34②, the inclined sight is at the angle α. With the rod perpendicular to the line of sight, the stadia reading gives the distance FN which added to the constant equals the distance MN. This may be reduced to the required horizontal distance MP by multiplying MN by cos α. It is, however, impracticable to incline the rod to make it perpendicular to the line of sight and the practice is to hold the rod plumb, which makes the reading AB longer than the perpendicular reading $A'B'$. The $\angle BB'N$ is nearly a right angle and $BNB'=\alpha$. Also, $A'B'=AB\cos\alpha$.

$$MN=FN+(c+f)=S\frac{f}{i}\cos\alpha+(c+f)$$

$$MP=MN\cos\alpha=S\frac{f}{i}\cos^2\alpha+(c+f)\cos\alpha$$

Use R as in (1) above.

Let H=horizontal distance required.

Put $C=(c+f)$.

Then $H=R\cos^2\alpha+C\cos\alpha$. (1)

The vertical distance $V=PN$, equals $MN \sin \alpha$.
Substituting the value of MN, above

$$V=S\frac{f}{i}(\cos \alpha \sin \alpha)+(c+f) \sin \alpha$$

$$V=S\frac{f}{i}\frac{\sin 2 \alpha}{2}+(c+f) \sin \alpha$$

$$V=R\frac{1}{2} \sin 2 \alpha+C \sin \alpha \qquad (2)$$

These formulas may be worked out on a stadia slide rule, by stadia diagrams, or from tables. To allow for $(c+f)$ in determining the vertical distance, as well as the horizontal distance, it is usually added to the value of $S\frac{f}{i}$, (R), before making use of slide rule, diagram, or table. TM 5–236 contains stadia tables for the reduction of readings.

d. Stadia constant (K).—It is extremely difficult to place the stadia hairs so they will exactly intercept the proportional part on the stadia rod which when multiplied by 100 should give the true distance between the instrument and the rod; therefore, it will be found by testing that many instruments will read *long* or *short*. For this reason a stadia constant, called K is determined, which when multiplied by the reading of the stadia hairs will give the true distance.

e. Exercise VI. Determination of stadia constant (K), *and* $(c+f)$.—(1) *Equipment.*—Complete transit, stadia rod, steel tape, chaining pins, foot rule, notebook, pencil.

(2) *Method.*—(*a*) Set up transit and set ten chaining pins in line about 100 feet apart on level ground.

(*b*) Plumb stadia rod by side of first pin.

(*c*) Set lower hair on an even footmark keeping telescope nearly level, and read intercept.

(*d*) Record same (line 1, column 2, fig. 35).

(*e*) Read intercept on rod at remaining pins and record same (column 2).

(*f*) Measure distance from center of transit to each pin with steel tape and record same (column 3).

(*g*) Focus objective on a distant object, measure f (the distance from the plane of the cross hairs to the center of the objective) and c (the distance from the center of the objective to the center of the instrument). Record same (column 6).

(*h*) Calculate the value of stadia constant K. Enter in column 4 the recorded values of column 3 minus $c+f$ (1.4 in this case). Obtain K, column 5, by dividing each value in column 4 by the corresponding

stadia distance, column 2. Add values in column 5 and obtain mean by dividing the sum by the number of values in that column. The result will be the stadia constant of the instrument used. In recording, etc., follow the form in figure 35.

f. The stadia constant should be frequently checked during the progress of a survey and must always be determined after cross hairs have been replaced in an instrument.

g. Use of stadia.—The transit is set up over a station of known elevation and oriented as outlined in paragraph 80*e*. Measure the

Instrument: Sgt. J.B. Leach Inst: Berger #11848 (25)
Head chain: Cpl. M.F. Bear
Rear chain: Pvt. A.I. Muir

DETERMINATION OF STADIA CONSTANT OF TRANSIT

No.	Stadia	(D) distance	D-(c+f)	K	c+f	Remarks
1	98	98	96.6	.986		April 30, 1940 (2 hours) Fair, warm.
2	195	195	193.6	.993		Measured ten courses with steel tape.
3	304	305	303.6	.999		Read stadia distances and then measured c+f.
4	397	398	396.6	.999		
5	507	509	507.6	1.001		
6	608	610	608.6	1.001		
7	692	695	693.6	1.002		
8	804	803	801.6	.997		
9	895	895	893.6	.998		
10	1005	1010	1008.6	1.004		
			Sum	9.980		
			Mean	0.998		

Checked: J.B.L.

FIGURE 35.—Data for determining stadia constant.

height of instrument above the station point with rod, tape, or plumb bob. Have the rod held on some point, the direction, elevation, and distance of which are desired, and sight on the center of the rod, using the upper motion. Incline the telescope until the center cross hair is roughly on the *H.I.*, then set one stadia wire exactly on the nearest foot mark and read the intercept.

71. Adjustment of transit.—*a. Equipment.*—Berger transit, adjusting pin, leveling rod, stakes, hatchet, paper, and pencil.

b. Adjustment of plate bubbles.—To make the axis of the bubbles perpendicular to the vertical axis—

(1) Set up the transit and bring both bubbles to the center of their tubes by turning the leveling screws.

(2) Rotate the instrument about its vertical axis through 180° or halfway round, and note any movement of the bubbles away from the centers of the tubes.

(3) Bring the bubble of each tube back one-half the distance that it moved by turning the capstan screws at the end of the bubble tube.

(4) Relevel and rotate the instrument again and make a similar correction if the bubbles do not remain in the center of the tube.

(5) Check the final adjustment by noting that the bubbles remain in the centers of the tubes during the entire revolution about the vertical axis.

c. Adjustment of vertical wire.—To make the vertical wire lie in a plane perpendicular to the horizontal axis—

(1) See that parallax is eliminated. Sight the vertical wire on some well defined point and, with the lower motions clamped, rock the telescope slightly up and down on its horizontal axis. If the instrument is in adjustment, the vertical wire will appear to follow the point through its entire length.

(2) If it does not, loosen the screws holding the cross wires and slightly rotate the ring by tapping lightly on one of the screws.

(3) Sight again upon the point and if the vertical wire does not follow the point throughout its length as the telescope is rocked up and down, rotate the ring again.

(4) Repeat this process until the condition is satisfied.

d. Adjustment of cross wires.—To make the line of sight perpendicular to the horizontal axis—

(1) Sight on a point not less than 100 feet distant and clamp both plates.

(2) Transit the telescope and set another point in the ground at a distance from the instrument about equal to the first distance. If the instrument is in adjustment, the two points set and the point over which the instrument is placed will lie in a straight line.

(3) Unclamp lower motion, rotate the instrument about its vertical axis, sight at the first point, and clamp.

(4) Transit the telescope and observe the second point. If the instrument is not in adjustment, the intersection of the cross wires will fall to one side of the second point, and one-fourth of the distance from the newly observed point to the original point will be the error.

(5) Measure this distance and place a point one-fourth of the distance from the last observed point to the original point.

(6) Move the wire reticle horizontally by loosening the screw on one side of the telescope tube and tightening the opposite screw until the vertical wire appears to have moved to the corrected point.

(7) Repeat this operation until no error is observed.

(8) Repeat the test under (2) above, since the vertical wire may have rotated during this adjustment.

e. Adjustment of standards.—To make the horizontal axis of the telescope perpendicular to the vertical axis of the instrument—

(1) Sight with the vertical wire on some high point (at least 25° or 30° above the horizontal), such as the corner of the eaves of a building, and clamp the plates.

(2) Depress the telescope, and mark a second point at about the same level as the telescope.

(3) Transit the telescope, unclamp the lower plate, and rotate the instrument about its vertical axis.

(4) Sight on the first point.

(5) Clamp the vertical axis and depress the telescope; if the vertical wire intersects the second or low point, the horizontal axis is in adjustment.

(6) If not, mark another low point on this line and note the distance between the new point and the original point.

(7) Adjust by turning the small capstan screw in the adjustable Y bearing at one end of the horizontal axis to correct the error which is one-half this distance.

(8) Repeat the test until the vertical wire passes through the high and low points in the direct and inverted positions of the telescope.

f. Adjustment of telescope bubble.—The telescope bubble of the transit is adjusted by the "peg" method which is described in the adjustment of the dumpy level. Set the cross wire on the true reading and move one end of the spirit level vertically by means of the adjusting nuts until the bubble is in the center of the tube. Repeat the test to check the work.

g. Adjustment of vernier of vertical circle.—(1) Bring the telescope bubble to the center of the tube.

(2) Read the vernier of the vertical circle.

(3) If it does not read zero, loosen the small capstan screws holding the vernier and move the vernier until it reads zero on the vertical circle.

(4) Tighten the screws and read the vernier with all bubbles in the center of their tubes to make sure it has not moved during the operation.

h. Exercise VII.—Make the adjustments of a transit as explained in this paragraph.

72. Adjustment of repeating theodolite.—*a. Adjustment of plate levels.*—With the upper motion loose, bring one plate level parallel to the line joining two leveling screws and center the bubble with the leveling screws. Turn the instrument 90° and center the bubble by the third leveling screw. Place the level parallel with the first two screws and center the bubble. Turn the instrument 180° and correct half the bubble error by the leveling screws and half by the adjusting screws on the level. Repeat until this level is in close adjustment. After the instrument is properly leveled, the bubble of the second plate level, if the theodolite is equipped with two, can at once be brought to the center by the adjusting screws.

b. Adjustment of striding level.—(1) Place the striding level in position on the horizontal axis. Level carefully with the horizontal plates clamped, and rock the level slowly back and forth until the foot pieces strike. If the bubble leaves the center, bring it back by means of the side adjusting screws near one end of the tube.

(2) Bring the bubble to the center again by the leveling screws and then reverse the level. If the bubble does not return to the center, adjust half the discrepancy by the foot screws and half by the adjusting screws on the level. Test again, and repeat these operations until the adjustment is perfected.

c. Adjustment of horizontal axis.—Adjust the striding level and place it in position on the pivots of the horizontal axis. Bring the bubble to the center by the leveling screws, then rotate the alidade 180° around its vertical axis. The amount by which the bubble is displaced must be corrected, half by adjustment of a standard and half by the leveling screws of the instrument. This process must be repeated until the striding level bubble shows no displacement when the alidade is rotated through 180°.

d. Adjustment of eyepiece, etc.—Point the telescope toward a light surface, such as the sky. Screw the eyepiece of the telescope in or out until the wires show the sharpest and blackest. Next focus the telescope on a distant object, and then test the adjustment by moving the eye slowly across the front of the eyepiece. If the wires appear to move over the image of the object sighted upon, parallax is present. The focus of the object glass of the telescope should be changed until the objective is at the proper distance to cause the image of the object sighted upon to fall exactly on the plane of the cross wires; in this position no movement of the wires over the field of view will be apparent when the eye is moved across the eyepiece. The adjustment for parallax must be closely watched, for the error due to lack of proper adjustment is not eliminated by the method of observing.

It is especially noticeable if the eye of the observer, because of a strained position, is not in front of the exact center of the eyepiece. The adjustment of the eyepiece must be tested frequently, for as the eyes of the observer tire the focal distance of the lenses of the eye changes, causing a blurring of the wires and an increased eye effort in centering the image of the light between the sighting wires.

e. Adjustment of vertical wire.—Same as for transit.

f. Collimation adjustment.—(1) Point the telescope upon some sharply defined object, and with the alidade clamped, lift the telescope from its wyes, rotate it 180° around its longitudinal axis, and replace it in the wyes, the pivots being in different wyes than in the original position. If the object is not bisected after reversal, correct half the discrepancy by shifting the reticle by means of the screws provided and again bisect the object by using the tangent screw. Repeat the test to check the adjustment.

(2) If the instrument cannot be reversed in the wyes, sight on a point several hundred feet distant, clamp the instrument, and bisect the point with the wires. Plunge the telescope and mark a second point in the opposite direction and at about the same distance to avoid changing the focus. Rotate the instrument about its vertical axis and bisect the first point. Plunge the telescope and mark a point beside the second point, unless they are found to coincide. Adjust by bringing the vertical hair back one-quarter of the disagreement. Repeat the process until no discrepancy can be detected.

g. Adjustment of telescope level and vertical circle vernier.—These adjustments are described in paragraph 73*g*.

73. Adjustment of direction theodolite.—*a. Adjustment of plate bubbles.*—Same as in paragraph 72*a*.

b. Adjustment of striding level.—Same as in paragraph 72*b*.

c. Adjustment of horizontal axis.—Same as in paragraph 72*c*.

d. Adjustment of eyepiece.—Same as in paragraph 72*d*.

e. Adjustment of verticality of sighting wires.—To test if this adjustment is necessary, point upon a well defined object with the instrument leveled. Swing the telescope slowly in elevation while watching the position of the object in the field of view. If the object changes its position with relation to the vertical wires as the telescope changes in elevation, the diaphragm must be rotated around the longitudinal axis of the telescope. An examination of the telescope will usually quickly disclose the mechanical means for accomplishing this purpose.

f. Collimation adjustment.—This adjustment is the same as described in paragraph 72*f*.

g. Adjustment of vertical circle level.—(1) Such theodolites as are equipped with vertical circles have them rigidly attached to the

horizontal axis of the telescope, with the level bubble mounted on the vernier frame, the verniers not being adjustable. Since the method of observing entails the reversal of the telescope during each measure of a vertical angle, thus eliminating the error of the level, it is customary to leave the level bubble on the vertical circle unadjusted.

(2) If it is desired to adjust the bubble, proceed as follows: For convenience this method, called the "peg" method is repeated here. Set up the theodolite and level it. Two pegs, A and B, are driven into the ground for rod supports at equal distances from the theodolite and about 180° apart in azimuth. A level rod is read successively on the two pegs, with the vertical movement of the telescope clamped, the bubble of the level on the vernier frame in the center of the tube, and the vertical circle set at the reading corresponding to the horizontal position of the telescope. The difference in the rod readings is the true difference of elevation of the two pegs. Next set up the theodolite very near peg A. With the bubble centered on the vertical circle level and the vertical circle reading the same as before, read the rod on that peg by looking through the objective end of the telescope. If this reading is greater than the first reading on peg A, add the difference of the two readings to the first reading on peg B and, if it is smaller, subtract the difference from the first reading on peg B. Set the horizontal wire of the telescope on the rod at peg B at the reading so obtained. The telescope is now horizontal. Adjust the vernier to read zero or whatever corresponds to the horizontal position of the telescope and then adjust the bubble to the middle of the tube.

(3) The error due to an inclined horizontal axis of the vertical circle is usually of no consequence if ordinary care is used in leveling the instrument. The formula for obtaining the true altitude is

$$\sin h = \sin h' \cos i$$

where h is the true altitude, h' the observed altitude, and i the inclination of the horizontal axis.

h. Adjustment of centers.—In precision theodolites so little tolerance is permitted in the fit of the centers that an adjustment is often necessary to regulate the variations of friction caused by wear and by changes of temperature. This usually is done by the use of a nut or screw at the lower end of the vertical axis which, by pressing upward upon the lower end of the alidade axis, lessens the weight of the alidade and telescope upon the conical bearing surfaces. This adjustment must be made with caution, for if too much of the weight is removed from the bearing surfaces, there is play in the centers, with a resultant loss of accuracy. To make the adjustment, raise the vertical axis

with the adjusting nut or screw until the alidade appears to move just freely enough on its vertical axis. Test for play in the conical bearings by pointing on an object and noticing if a slight pressure on the alidade will move the telescope off its pointing. A better test is to point the telescope on some well defined object, read the circle, swing the telescope around clockwise, and again point and read. Repeat the process, except that the alidade is swung in the opposite direction. If a series of three or four sets of such alternating pointings shows the effect of drag, the pressure of the screw upward on the vertical axis should be increased; if the readings are erratic and cover a considerable range, the pressure should be lessened.

i. Adjustment of micrometers and microscopes.—(1) A single micrometer microscope is in correct adjustment when the following conditions are fulfilled:

(*a*) When the micrometer lines and image of the graduated circle are so closely in the same plane that no parallactic movement can be detected by shifting the eye laterally.

(*b*) When the corresponding amount of revolutions of the screw will exactly traverse the interval between adjacent graduations on the circle.

(*c*) When the micrometer reads zero seconds with the pair of parallel wires coinciding with the zero point on the comb.

(2) After these conditions are fulfilled the micrometers must be spaced around the circle at equal intervals.

(3) First, adjust the eyepiece by sliding or screwing it in or out until the parallel wires are in the most distinct focus. The comb then will be sufficiently visible.

(4) Next, loosen the screws which hold the microscope tube in the bracket clamps and move the tube up or down until the graduation marks are as distinct as possible, then tighten the screws.

(5) It may be necessary to adjust the objective of the microscope radially. Usually some sort of hinged joint is provided for that purpose. The adjustment should be made to bring the outer edge of the graduations near the center of the field of view, yet still leave the degree numerals visible.

(6) If the parallel wires of the diaphragm are not parallel to the graduation marks on the circle, either turn the micrometer box slightly on the tube or loosen the binding screws and turn the tube slightly in its supports. If the zero notch of the comb, usually marked by a deeper cut than the other notches or by a hole beneath it, is not in the center of the field of view, adjust it by the screw provided for that purpose. Center the pair of parallel wires on the zero notch of the

comb, then hold the spindle of the micrometer firmly and turn the graduated drum on its friction mounting until the zero of the drum coincides with the index line mounted on the micrometer case adjacent to the drum. Check to see that the parallel wires have not moved off the zero of the comb. If the drum is held in position by a screw as well as by a friction mounting, the screw must be loosened before making the adjustment.

(7) If the micrometer screw is threaded to have one revolution equal to 2 minutes of arc, then the graduated drum will have two graduations on it marked 0 seconds, the numerals of the second minute usually having some distinguishing characteristic. Care must be taken always to adjust and read to the proper zero mark.

(8) By "run" of the micrometer is meant the difference in seconds of arc between the intended value of one turn of the micrometer the space between two adjacent graduation marks of the circle. This quantity has sometimes been called the "error of run," but the former term seems preferable and is more generally used.

(9) Assuming that adjacent graduations of the circle are 5 minutes apart and that one revolution of the micrometer screw will cause the parallel wires to traverse approximately one-fifth of the interval between adjacent graduations, the purpose of the adjustment for run is to bring about, as closely as may be, the condition that exactly five revolutions of the screw will move the parallel wires from one graduation to the next. If fewer than five complete turns of the micrometer screw are necessary to move the parallel wires from one graduation to the next, the image needs to have its magnification increased, and the objective should be moved nearer the circle by protruding it from the tube by its screw adjustment.

(10) When the objective is protruded from the tube to correct for run the image is thrown farther up in the tube. Without changing the relation of the eyepiece to the micrometer wires, move the whole micrometer tube upward to bring the image of the circle into the plane of the micrometer wires. This process may have to be repeated several times to get the run down to the required limit.

(11) If more than five complete turns of the screw are required to make the parallel wires traverse the space between adjacent graduations, the adjustment should be made in the direction opposite to that described above.

(12) In actual practice the preliminary tests for run should be made on five or six equidistant parts of the circle, since the error of run will vary somewhat due to eccentricity of the circle, and the adjustment should be made on a portion of the circle where the error is near the mean. Also, the final adjustment should be tested by

taking the mean of at least 10 readings of the screw value of the space between graduations, and these readings should be made a part of the record of horizontal directions.

(13) Where two pairs of wires are used mounted at an arbitrary distance apart (less than the interval between adjacent graduation marks), the correction for run can be applied to the reading of each pair of wires separately, which is better than when a single pair of wires is read on one graduation only at a setting. However, the readings of the two pairs do not tend to eliminate the run. The principal benefit derived from using two pairs of wires lies in the comparison obtained by sighting on two graduation marks instead of one. After the individual adjustments have been made, set one microscope exactly on an even degree graduation mark with micrometer comb and drum reading zero. Set the other microscope, or microscopes, at 120° or 180° distance around the circle, depending on whether it is a three-micrometer or a two-micrometer theodolite; then proceed as follows: Note if the zero of the comb when near the center of the field of view is very distant from the proper degree mark. If it is, move the microscope in the proper direction by whatever mechanical means for such adjustment is provided until the proper degree mark is near the zero of the comb. (One of the microscopes, which is the one set first, is not usually adjustable circumferentially.) Then move the comb until the zero mark coincides with the proper degree mark and adjust the graduated drum to read zero when the parallel wires read zero on the comb, as already described. If the theodolite has a third micrometer microscope, it should be adjusted to the first set microscope in the same way.

(14) Because of eccentricity of the circle the micrometers will not maintain a constant difference when read on different portions of the circle. A sufficient number of readings should be taken around the circle to determine the approximate amount of this eccentricity and the adjustment for equidistance should be made at a point where the eccentricity can be closely estimated. It facilitates the taking of means of micrometer readings if the second microscope is set a small amount ahead of the first to read somewhat more than the latter.

(15) On three-micrometer theodolites it is usual to have a small reading microscope with a single wire on its diaphragm by which the degrees and the next preceding 5-minute graduation of the circle is read. Such a microscope should be adjusted to read in agreement with the first set micrometer microscope by whatever means are provided for such adjustment.

(16) For making micrometer readings it is important that the circle be evenly illuminated from above, or else that the light be admitted

normal to the circle and directly opposite the graduation to be read, as otherwise there will be an appreciable error in the readings due to phase.

74. Adjustment of Wild type theodolite.—The following tests and adjustments should be carried out in the order given:

a. Adjustment of foot screws and micrometer screws.—The three foot screws must work with moderate ease, but should be without any play; if one of them turns too easily, the adjusting screw should be tightened until the movement is free from backlash. The adjusting screws of the micrometer screws are situated behind the milled head; they should very seldom require any attention.

b. Adjustment of horizontal level.—(1) Bring the bubble of the circular level to center by working the three foot screws, thus bringing the vertical axis into an approximately vertical position. Turn the instrument until the centrally situated horizontal level lies parallel to a line joining two of the foot screws, and bring its bubble to center. Then turn the theodolite 180°. If the bubble is no longer central, correct half the difference by the foot screws and the other half by the adjusting screws of the level.

(2) Then turn the theodolite 90°, so that the level lies perpendicularly to the line joining the same two foot screws, and bring the bubble to center by working the third foot screw; this brings the vertical axis to verticality in space.

(3) This procedure is called leveling the theodolite. If the horizontal level shows only a small discrepancy (1 or 2 divisions), it is more convenient to note the position of the bubble when the axis is truly vertical, without correcting it.

(4) The leveling of the instrument is then done with reference to this position of the bubble.

c. Adjustment of optical plummet (centering arrangement).—After the theodolite has been accurately centered over the ground mark by means of the plumb line inserted in the central fixing screw, and the vertical axis made truly vertical by the horizontal level, carefully remove the plumb line and determine whether the ground mark appears exactly in the center of the small circle. If correction is necessary, it can be made by means of the adjusting screws on the eyepiece of the optical plummet.

d. Collimation adjustment.—The horizontal collimation of the telescope can be adjusted by sliding the diaphragm. A screw driver serves this purpose. Swing the telescope on to a clearly marked point and take the reading of the horizontal circle, repeat the measurement in the other telescope position. The difference of the two circle readings reduced by 180° is equal to twice the collimation error. Set

the micrometer drum by the average value of the two drum readings and turn the tangent screw for azimuth up to the point of coincidence of the graduation lines. Now, slide the diaphragm laterally to obtain coincidence again between the vertical line and the object sighted. There are three adjusting screws, one horizontal and two slanting. If, for instance, telescope direct requires moving the diaphragm to the left, loosen to exactly the same extent the two slanted screws and thereafter tighten moderately the horizontal screw to the left. Repeat these measurements and corrections until the simple collimation error is under 10 inches and examine by tilting the telescope whether the vertical line is taking a vertical course. In case of need adjust the diaphragm by turning the slanting screws in opposite directions to each other. A correction of the horizontal collimation is very seldom necessary.

e. Adjustment of collimation level.—(1) If there is an inadmissible horizontal collimation error, it is necessary to remove it first.

(2) After bringing the image of the vertical circle into view by moving the change-over knob, bring the horizontal cross line onto an object, and, after bringing the collimation level bubble to center, read the circle. Repeat the operation with the telescope reversed. The sum of the two readings should be 360°. If it is not, a so-called index error of the vertical circle, or collimation error, exists. To eliminate this, calculate the zenith distance as indicated previously, set the micrometer drum to the corresponding reading, and bring the telescope in direct position, exactly onto the object. With the tangent screw for the collimation level make coincidence of the graduation lines and bring the level to center by means of its adjusting screws.

(3) If the collimation error has been sufficiently eliminated, we may have, for example, the following readings:

Telescope direct	86°	43′	24″
Telescope reversed	93°	16′	43″
Sum	180°	00′	07″

There is thus a collimation error of 7 seconds still present, which of course needs no further elimination.

f. Testing tripod.—It should be verified that no play exists at the junction of metal and wood, either at the feet or at the head of the tripod. The hexagonal spanner supplied serves to tighten up the three clamping screws under the head of the tripod. This tightening up will cause the legs of the tripod to work more stiffly. The screws should be sufficiently tightened for the legs to retain their position when free with a spread of 24 inches.

g. Exercise VIII.—Test the adjustments of an assigned theodolite in accordance with the instructions given for the particular type of instrument assigned.

SECTION XII

TRAVERSES

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Traverse with transit and tape	79
Azimuth traverse with transit and stadia	80
Compass traverse with transit and stadia	81

75. General.—*a.* A traverse is a series of consecutive courses of which the lengths and relative angles have been determined, thereby locating the stations with respect to each other. A traverse which comes back to the starting point is called a closed traverse. Traversing is one of the most important and frequent operations in surveying.

b. The methods of running traverses may be grouped in accordance with the methods of measuring and recording transit angles; the methods for linear measurements are chaining or measuring with the stadia. The methods may be grouped as follows:

(1) By direct or deflection angles.

(2) By azimuths or by bearings.

c. Running traverses by direct angles consists simply in measuring each angle directly from a back sight on the preceding station. If desired, the accuracy of measurement may be increased by repetition (par. 79).

d. Running traverses by deflection angles is a modification of the method last described, especially applicable to railroad work. Instead of measuring the angle direct from the backsight, the telescope is plunged and the deflection angle is measured right or left.

e. Traverses are often run by azimuths (par. 77*e*). When using this method, remember that when the telescope is pointed in zero direction, usually true north, the controlling vernier must read zero. Transits are generally equipped with two verniers, designated *A* and *B*, and, in that case, the other vernier must read 180°. The azimuth of the initial course is preferably obtained by astronomical observation, from a triangulation system, or from a previous traverse.

f. Running traverses by bearings is essentially the same as the azimuth method, except that bearings instead of azimuths are read direct from the transit. As the transit compass is used for orientation by bearings it is usually more practical to obtain and record their

corresponding azimuths from the transit verniers. (See par. 81.) In connection with reconnaissance work, or when a moderate degree of accuracy is acceptable, only alternate traverse stations need be occupied by the transit when running a needle traverse or compass traverse.

76. Azimuths and bearings.—*a.* Azimuth is the most convenient and commonly used way of expressing direction. For most military

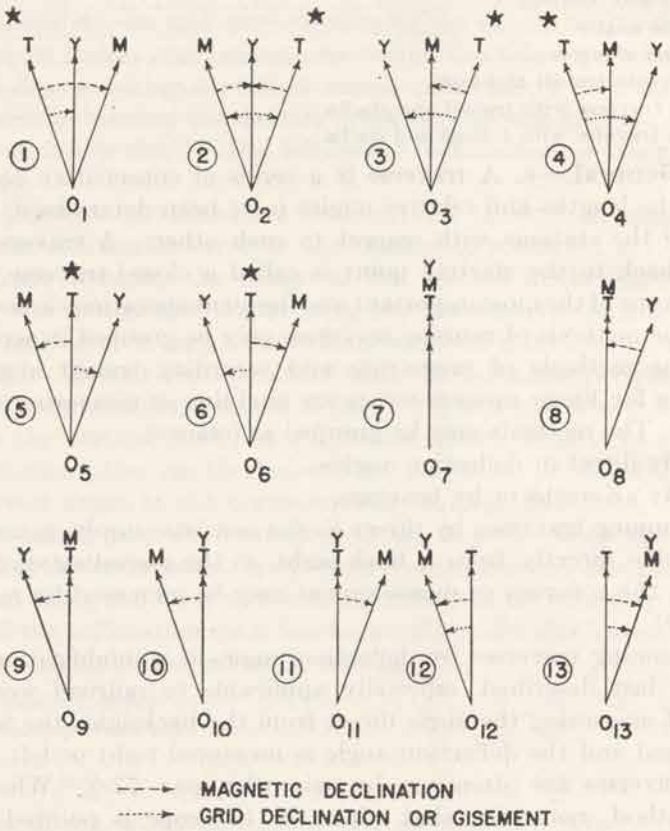

FIGURE 36.—Possible relations between grid north, true north, and magnetic north.

purposes, azimuths are angles reckoned clockwise from 0° at magnetic, true, or grid north through 360°. Thus, there are three different azimuths for a given line—magnetic azimuth, true azimuth, and grid azimuth. Consequently, the necessity for specifying the kind of azimuth expressed is obvious.

(1) The *magnetic azimuth* of a given line is the angle measured clockwise from magnetic north to the line.

(2) The *true azimuth* of a given line is the angle measured clockwise from true north to the given line.

(3) The *grid azimuth* of a given line is the angle measured clockwise from grid north to the given line.

(4) *Grid, true, and magnetic azimuths.*—The relation between grid, true, and magnetic azimuths is shown in figure 37. From the informa-

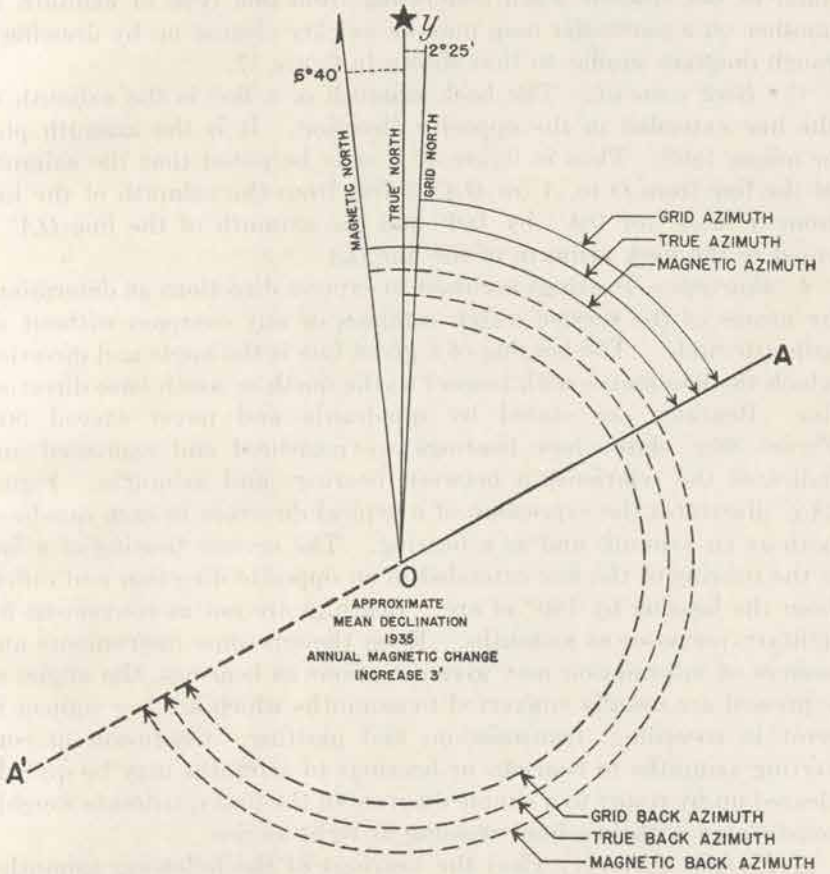

FIGURE 37.—Example of relationship between three base directions on a map, showing corresponding azimuths and back azimuths of line *OA*.

tion contained in the margin of the map (fig. 38 ①), in the case illustrated in the year 1935, the mean magnetic declination at Washington, D. C., was 6°40′ west with an annual change of 0′. The average grid declination was 2°25′ east and does not change. The grid azimuth is in consequence less than the true azimuth by 2°25′ and in 1935 was less than the magnetic azimuth by 9°05′ (i. e., 6°40′ plus

2°25′). The true azimuth was, in 1935, 6°40′ less than the magnetic azimuth and 2°25′ greater than the grid azimuth. In order to determine the interrelationship of the azimuths for the year 1937, account must be taken of the annual change, if any, of magnetic declination. For instance, the increase in 2 years, if the annual change is +3′, would be 6′ (i. e., 2 times 3′). Any confusion or uncertainty in the mind of the student when converting from one type of azimuth to another on a particular map may be quickly cleared up by drawing a rough diagram similar to that shown in figure 37.

(5) *Back azimuth.*—The back azimuth of a line is the azimuth of the line extended in the opposite direction. It is the azimuth plus or minus 180°. Thus in figure 37 it may be noted that the azimuth of the line from *O* to *A* (or *OA*) differs from the azimuth of the line from *O* to *A′* (or *OA′*) by 180° and the azimuth of the line *OA′* is equal to the back azimuth of the line *OA*.

b. Bearings.—Bearings are used to express directions as determined by means of the service watch compass or any compass without an azimuth circle. The bearing of a given line is the angle and direction which the line makes with respect to the north or south base direction line. Bearings are stated by quadrants and never exceed 90°. Figure 38② shows how bearings are measured and expressed and indicates the relationship between bearings and azimuths. Figure 38③ illustrates the expression of a typical direction in each quadrant both as an azimuth and as a bearing. The reverse bearing of a line is the bearing of the line extended in an opposite direction and differs from the bearing by 180° of arc. Bearings are not as convenient for military purposes as azimuths. Even though some instruments and sources of information may give directions as bearings, the angles so expressed are usually converted to azimuths which are less subject to error in recording, transmission, and plotting. Confusion in converting azimuths to bearings or bearings to azimuths may be quickly cleared up by resort to a simple diagram of the four quadrants roughly constructed with two lines crossing at right angles.

c. Exercise IX.—(1) Find the bearings of the following azimuths:

1. 132°
2. 217°40′
3. 101°35′
4. 343°01′
5. 67°14′
6. 271°57′
7. 91°57′
8. 219°51′
9. 269°59′30″
10. 179°59′30″
11. 271°00′15″
12. 300°00′15″
13. 17°01′01″
14. 349°59′59″
15. 73°14′15″.5
16. 143°17′59″.3

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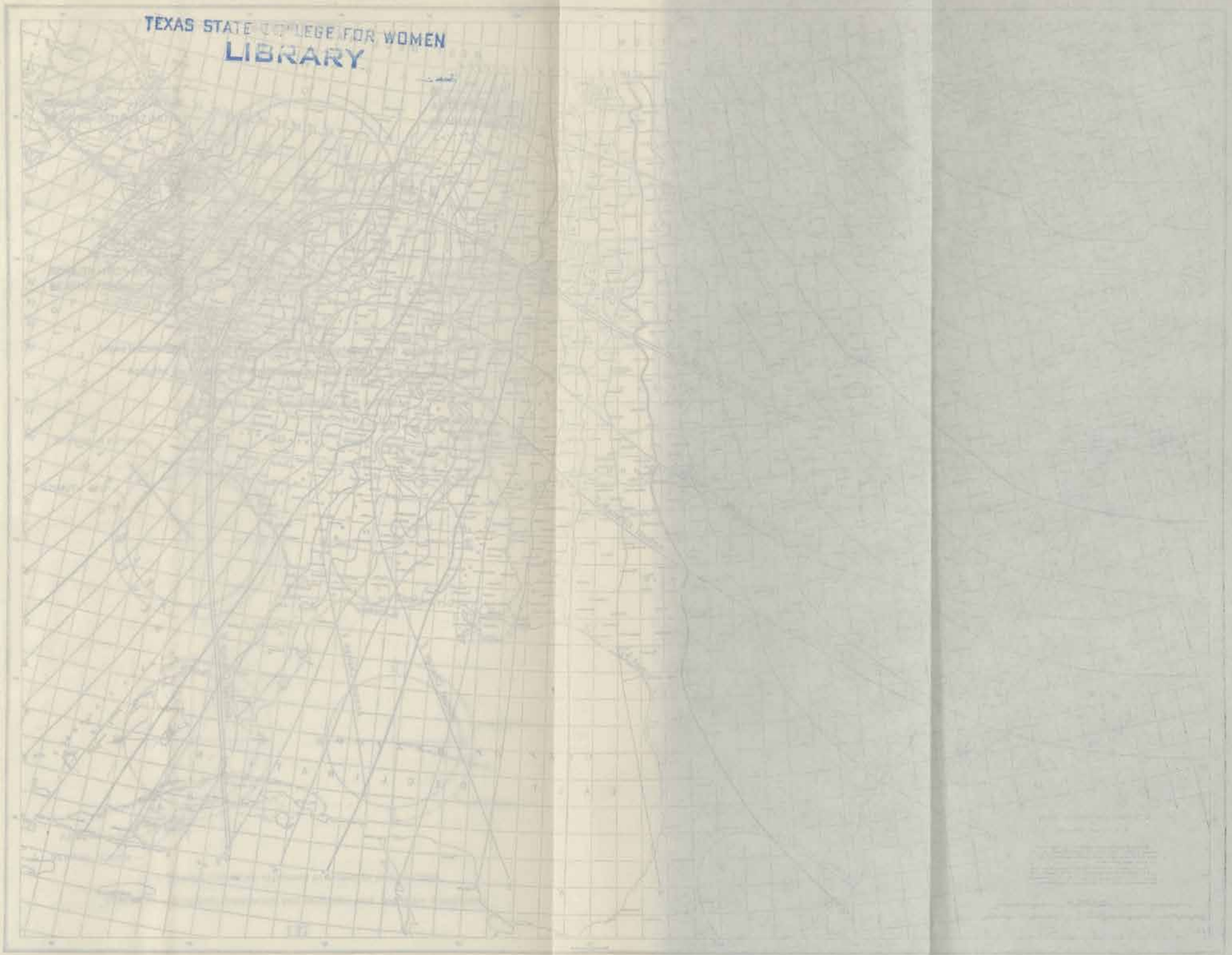

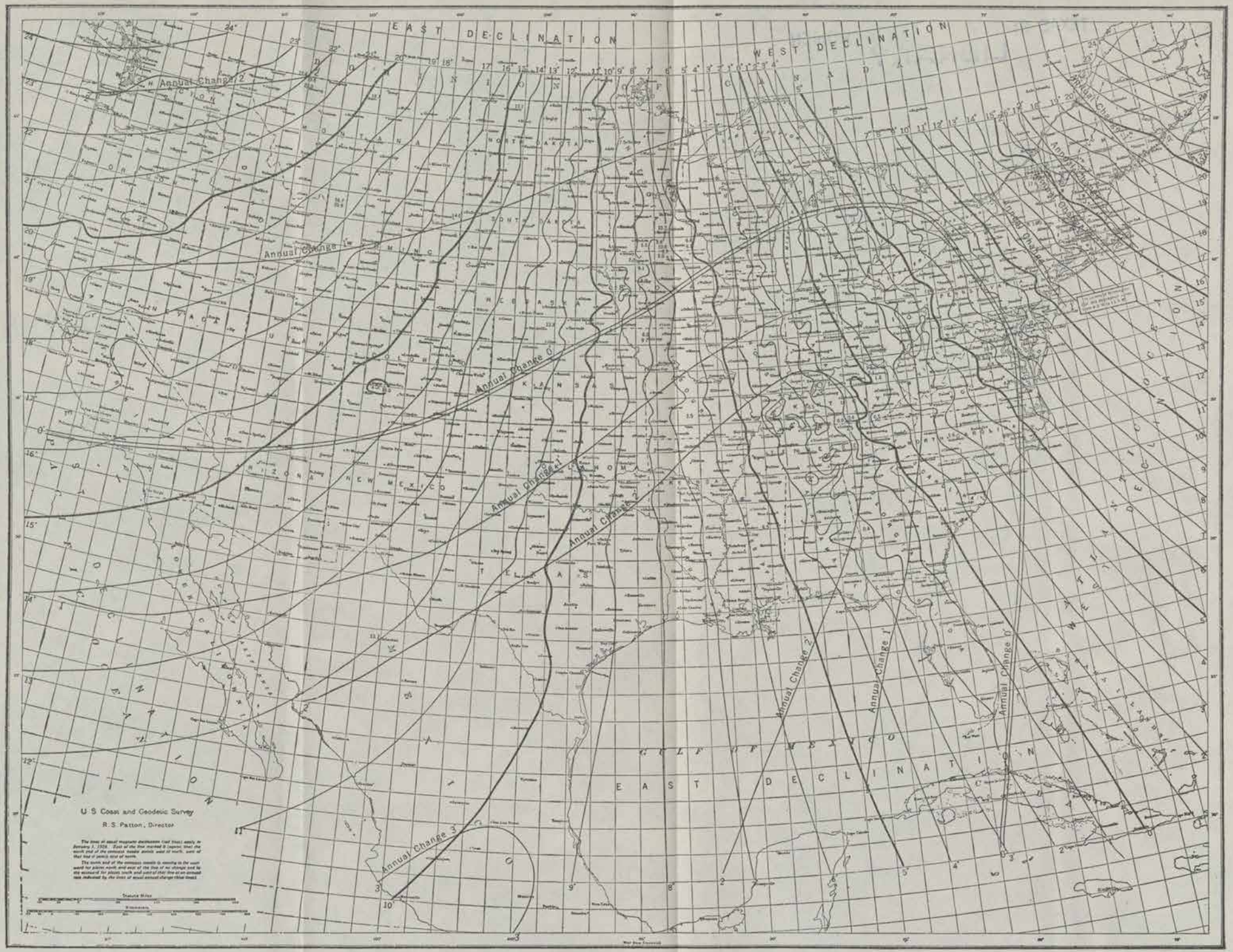

◎ Lines of equal magnetic declination and of equal annual change in the United States for 1935.

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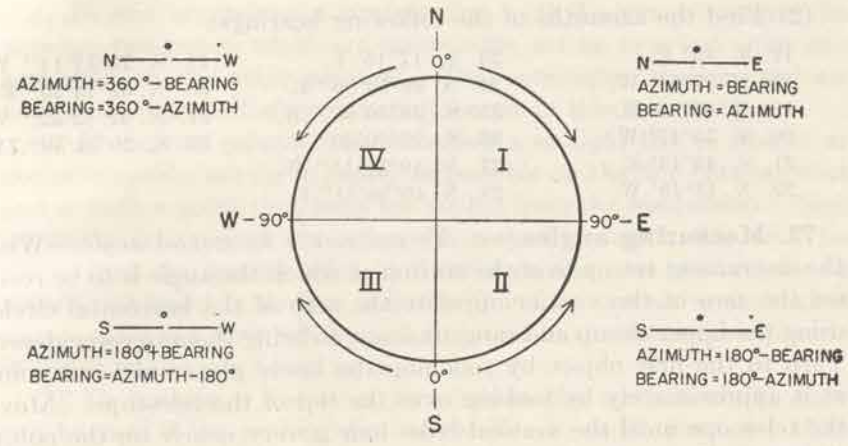
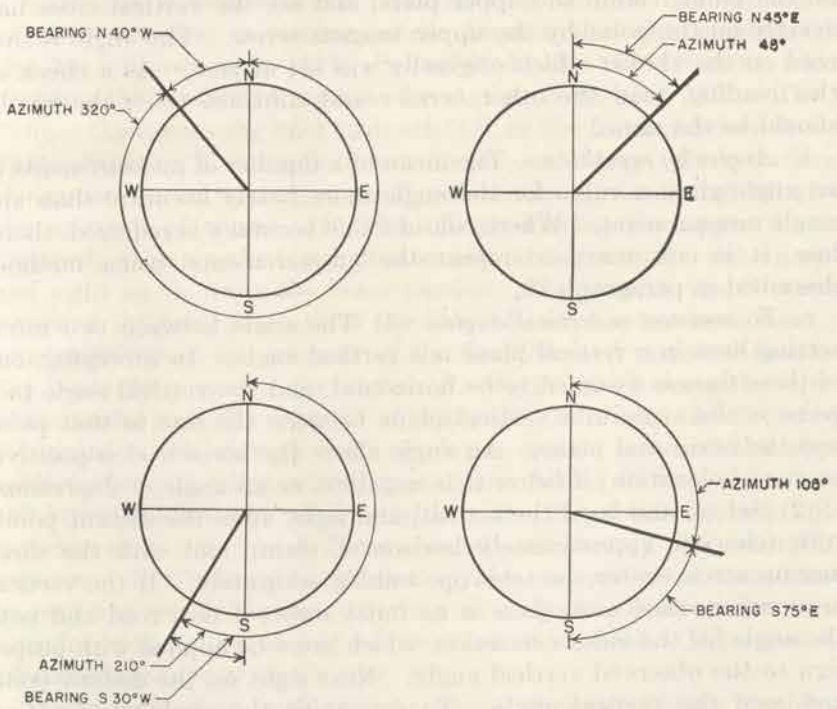
FIGURE 38.—Magnetic declination and azimuths and bearings.



② Relations between bearings and azimuths.



③ Typical directions expressed as azimuths and bearings.

FIGURE 38.—Magnetic declination and azimuths and bearings—Continued.

(2) Find the azimuths of the following bearings:

17. S. 43° E.	23. S. 17°15′ E.	29. S. 73°17′14″ W.
18. N. 17°50′ W.	24. N. 89°59′59″ E.	30. N. 62°13′20″ E.
19. N. 43°54′ E.	25. S. 29°30′30″ W.	31. N. 52°52′52″ W.
20. S. 55°17′ W.	26. S. 29°30′30″ E.	32. S. 29°31′59″.7W
21. S. 43°43′ E.	27. N. 49°59′15″ W.	
22. N. 43°43′ W.	28. N. 49°59′51″ E.	

77. Measuring angles.—*a. To measure a horizontal angle.*—With the instrument set up over the station at which the angle is to be read, set the zero of the vernier opposite the zero of the horizontal circle, using the upper clamp and tangent screw to bring them to coincidence. Turn to the first object by touching the lower plate only and point at it approximately by looking over the top of the telescope. Move the telescope until the vertical cross hair is very nearly on the point, clamp the lower plate by means of the lower clamp thumbscrew, and set exactly on the point by the lower motion tangent screw. The line of sight is now on the first object. To measure the angle, loosen the upper clamp, turn the telescope to the second point, set nearly on the point, clamp the upper plate, and set the vertical cross hair exactly on the point by the upper tangent screw. The angle is then read on the vernier which originally was set at zero. As a check on this reading, read the other vernier and subtract 180°; the results should be the same.

b. Angles by repetition.—The mean of a number of measurements of an angle gives a value for the angle more nearly accurate than any single measurement. Where considerable accuracy is required, therefore, it is customary to repeat the measurements, using methods described in paragraph 79.

c. To measure a vertical angle.—(1) The angle between two intersecting lines in a vertical plane is a vertical angle. In surveying, one of these lines is assumed to be horizontal, and the vertical angle to a point is the angle in a vertical plane between the line to that point and the horizontal plane. An angle above the horizontal is positive, or one of elevation; if below it is negative, or an angle of depression.

(2) Set up and level the transit, and sight upon the distant point; turn telescope approximately horizontal, clamp and with the slow-motion screw center the telescope bubble accurately. If the vertical arc vernier reads zero, there is no index error; if not, read and note the angle for the index correction, which must be applied with proper sign to the observed vertical angle. Next sight on the distant point and read the vertical angle. To determine the angle of elevation (or depression) between two points, it is necessary to take into account the height of instrument and the height of target at the distant point.

d. To run or prolong a straight line.—(1) To run a straight line between two points which are intervisible, set up over one point as *A* and sight on the other point *B*. This establishes the line and any number of intermediate points may be set in this line of sight.

(2) If the two points between which a straight line is desired are not intervisible, set up as nearly as possible on the line between them and at such a point that both are visible from the instrument. Sight on one point, plunge the telescope, note how much this trial line varies from the second point, and estimate the next position for the transit. A point in the line is finally found by successive approximations. This is a slow process and one must have considerable practice in order to become adept in it.

(3) To prolong a line from two points, the method of (1) above can be used if the prolongation of the line is visible from *A*. If the prolongation of the line is not visible, from *A*, set up over *B*, sight at *A*, and plunge the telescope. The vertical wire will now produce the straight line if the instrument is in adjustment, and additional points may be set in on the continuation of the line.

e. To run a traverse.—(1) The following method for running an azimuth traverse, is recommended: Assume that the aximuth *AE* (fig. 39) is known to be 112°. Set up at *A*. Set the *A* vernier, which becomes the controlling vernier for station *A*, to read 112°. The *B* vernier then reads the back azimuth *AE*, or the forward azimuth *EA*, 292°. Sight on *E*, using the lower motion, and clamp. Now unclamp the upper motion and sight on *B*. Read the *A* vernier, which gives the azimuth of course *AB*, 40°. Next, leaving the upper motion clamped, unclamp the lower motion and advance to *B*. Set up at *B* and sight on *A*, using the lower motion. Since, at *A*, the *B* vernier recorded the back azimuth of the line *AB* and has not been reset, it must now record the azimuth of the line *BA*, and therefore becomes the controlling vernier for station *B*. Sight on *C*, using the upper motion, and read the azimuth of course *BC*, 82°, from the *B* vernier. Continue in this manner with the lower motion free and the upper motion clamped when changing station. Make all backsights with the lower motion and all foresights with the upper motion. Remember that the *A* and *B* verniers are alternately the controlling verniers. It is advisable, particularly for inexperienced transitmen, to read and record compass bearings as a check. Both vernier readings for each course should be recorded as a check. Distances are measured by tape or stadia (par. 80).

(2) In connection with reconnaissance work, or when a moderate degree of accuracy is acceptable, only alternate traverse stations need be occupied by the transit in running a *compass or needle traverse.* At

each station occupied, the transit is oriented by the compass needle. Referring to figure 39, set up at *A* and set verniers to zero and 180°, leaving lower motion free. Orient limb of transit by compass needle so that transit points to magnetic north and clamp lower motion. Sight on *E* and *B* successively, reading the respective back and forward azimuths or bearings. The next set-up is made at *C*, following the same procedure. Distances are usually determined by stadia

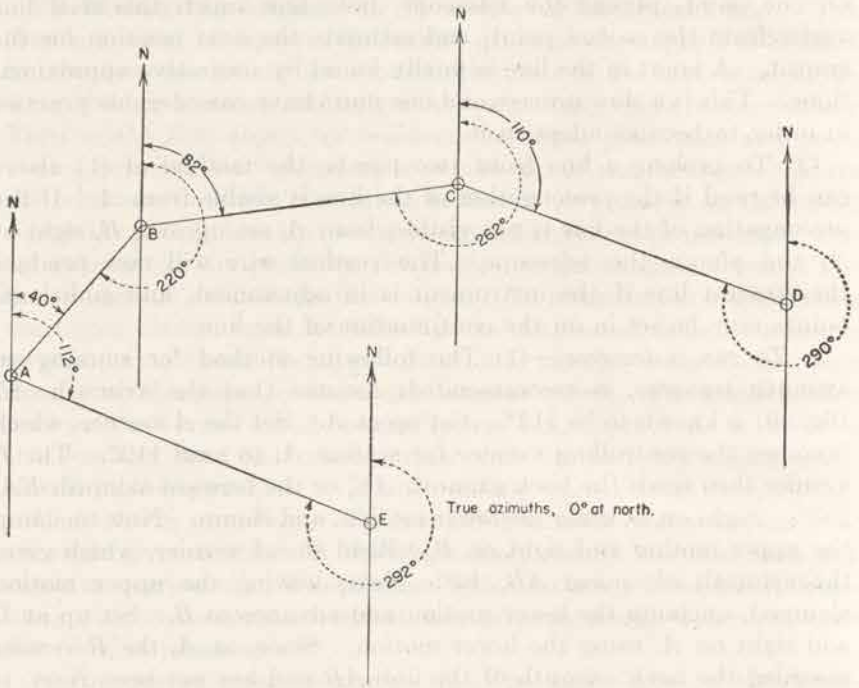


FIGURE 39.—Running a traverse.

making back and forward measurements from each transit station. See paragraph 81.

78. Prevention of errors.—*a. General.*—To come within a given limit of error in transit surveying both linear and angular errors must be within required limits. The most important points may be summarized as follows:

(1) The true error of any measurement is never known but can be estimated from duplicate measurements.

(2) Accidental or compensating errors, as a rule, are less important than constant or cumulative errors.

In transit surveying two classes of errors, both of which can be avoided or minimized, are the usual cause for unsatisfactory work, namely—

personal errors and instrumental errors. The first named, caused by haste or carelessness, may be called mistakes and should be guarded against at every step of the work. The second named can be avoided with an otherwise undamaged instrument by a proper program of observation (see par. 79*e* (12)).

b. Instrumental errors.—The instrumental errors which most adversely affect the accuracy of final results are that the plates are out of level and that the line of collimation is out of adjustment. The plates and line of collimation should be frequently and carefully checked.

c. Avoidable errors.—All errors, with the exception of those caused by faulty construction or condition of an instrument, are avoidable and can be eliminated if the work is performed as described in the paragraphs following.

79. Traverse with transit and tape.—*a. Organization of party.*—Assuming that distances are to be measured (with an accuracy of at least 1:5,000) using a calibrated steel tape and that angles are to be obtained by repetition, the party should include:

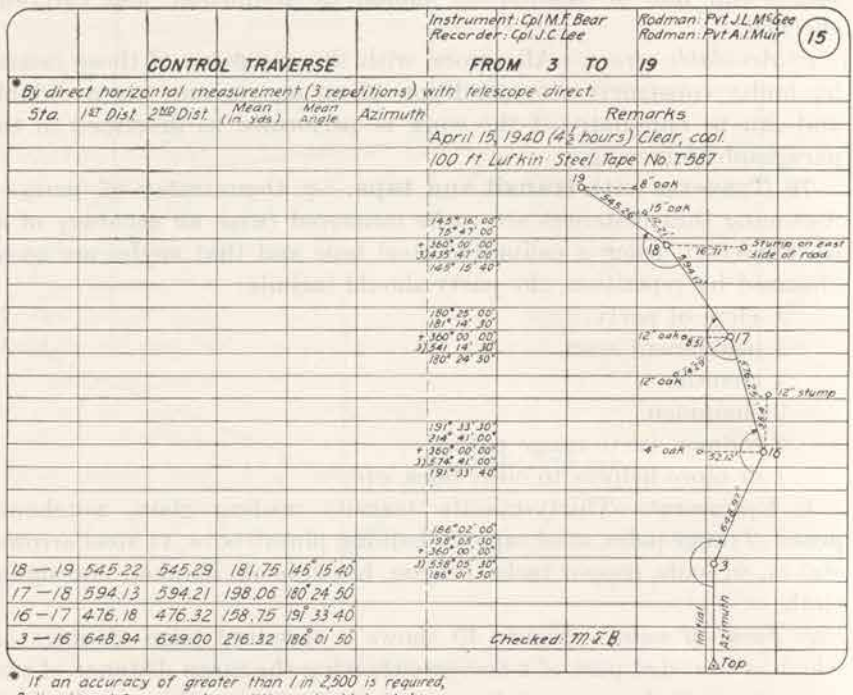
1 chief of party.
1 instrument man.
1 recorder.
2 chainmen.
2 rodmen (with range poles).
1 or more helpers to clear lines, etc.

b. Equipment.—Thirty-minute transit, reading glass, notebook, pencil, 2 range poles, steel tape, 2 chaining plumb bobs, 11 steel arrows, stakes, 8d nails, copper tacks, crayon, brush hook, hatchet, and signal cloth.

c. Form of notes.—Figure 40 shows a standard page of notes on which is recorded part of a traverse showing the mean distance of two measurements between stations and the mean angle as calculated from three readings, for instance: The mean distance from 3 to 16 (216.32) is the mean of the two measurements or ½ (648.94+649.00) reduced to yards, and the mean angle (186° 01′ 50″) is the mean of the angle at ⊙3 between △Top and ⊙16, the latter being the first station of the new traverse. Assuming that the azimuth from ⊙3 to △Top is given, the first angle at ⊙3 is read (186° 02′ 00″) and recorded as shown in the notes opposite ⊙3 of the diagram. The measurement of the angle then is repeated until three readings are obtained on the horizontal circle, in this case the final reading being 198° 05′ 30″. Since the first angle reading was over 180° (but less than 240°), 360° must be added to the final reading, which gives a total of 558° 05′ 30″. This divided by three gives a mean angle of

186° 01′ 50″, as shown in figure 40. The notes must show a diagram including witness marks for all new stations and their distances from the traverse station. The recording is usually done from the bottom of the page toward the top. For the method of reading angles by repetition, see *e* below.

d. Control (starting and closing) data.—All traverses of the types described in this and paragraphs 80 and 81 must start at some fixed point (triangulation or traverse station) previously located and ter-



* If an accuracy of greater than 1 in 2,500 is required, 2 direct and 2 reversed repetitions should be taken.

FIGURE 40.—Control traverse notes.

minate at another fixed point, or in the case of a closed traverse, at its starting point. See paragraph 83.

e. Field work.—The measurement of distances is identical with the method described in paragraph 47, except that the recorder here records the horizontal angles and transcribes distances from notes made by the head chainman. To measure angles by repetition, proceed as follows:

(1) Send rear rodman (with range pole) to rear station of azimuth line, unless said station is a triangulation station or other point marked with a signal. Send front rodman to first new station of traverse.

(2) Set the transit over the starting point, center, and level. This is called "setting up."

(3) Set *A* vernier to read zero, sight at the left hand (rear) station approximately, clamp the lower motion, and make an exact bisection with the lower tangent movement.

(4) Unclamp the upper motion, sight at the right hand (front) station approximately, clamp, and make an exact bisection with the upper tangent movement.

(5) Read *A* vernier to the nearest 30 seconds and record. This is the trial measurement of the angle sought.

(6) Unclamp the lower motion and sight again at the rear station approximately, clamp the lower motion and make an exact bisection with the lower tangent movement.

(7) Unclamp the upper motion and sight at forward station approximately and make an exact bisection with the upper tangent movement. (This is the second measurement of the angle.)

(8) Without reading any vernier, unclamp lower motion, sight at rear station, clamp lower motion, and make an exact bisection with the lower tangent movement.

(9) Unclamp upper motion and sight at forward point approximately, making an exact bisection with the upper tangent movement. (This is the third measurement of the angle.)

(10) Read *A* vernier to the nearest 30 seconds and record. One-third of this reading will be the angle.

(11) Measure and record the other angles of the traverse in like manner.

(12) Three direct repetitions are described above. Reversing the telescope for half the observations of an angle eliminates many errors due to imperfections of the instrument and its adjustment. Observing two direct and two reversed clockwise repetitions with a 20-second transit should yield third-order traverse angles. The *A*, *B*, and mean of the verniers are recorded on the initial setting and the fourth (second reversed) repetition, and only the *A* vernier need be read on the first angle, thus:

00°	00′	00″	20″	10″	(initial setting)
186°	02′	00″			(1st angle)
24°	07′	20″	20″	20″	(4th repetition)
744°	07′			10″	(4th—initial 360×(1st angle÷90°))
186°	01′			48″	(above÷4)

f. Special notes on boundary traverse.—Distances and angles for a boundary traverse are usually obtained by the same methods as described above, distances taped and angles by repetition. Due to the

peculiar nature of a boundary survey, usually involving properties of several parties, complicated by possible boundary disputes, the surveyor should be familiar with interpreting the descriptive part of deeds conveying property from one owner to the next. Descriptions in some deeds are often vague and ambiguous and the surveyor must frequently reconcile smaller differences between contending parties, based on honest and conscientious interpretations of his findings. Copies of deeds, or that part of deeds containing description (metes and bounds) of property, can be obtained from the land records of the county court of the county in which the property is situated. These land records are usually well indexed and cross indexed and the officials supervising the records will be found most willing to cooperate.

g. Exercise X.—Measure the angles of an assigned traverse by the repetition method described in *e* above and record and calculate the mean angle for each set-up as shown on the form in figure 40.

80. Azimuth traverse with transit and stadia.—*a. Organization of party.*—A transit party which is given the task of establishing stations by an azimuth traverse with transit or stadia, such traverse also being termed stadia control traverse, should consist of—

1 instrument man and chief of party.
1 recorder.
2 rodmen.
1 or more helpers to clear lines, etc.

b. Equipment.—A 1-minute transit, 2 stadia rods, stadia computer or stadia reduction table (see table VI, TM 5–236), *H.I.* stick (about 6 feet long and graduated to tenths of feet), stakes, hatchet, brush hook, notebook, and pencil. Figure 41 shows a Cox stadia computer, with an example.

c. Form of notes.—Figure 42 shows some notes with all measurements reduced to their proper values. The methods of observing and recording them are explained in *e* below.

d. Control (starting and closing) data.—In addition to the azimuth from the starting point of the traverse to some other established point the elevation of the starting point should be known. In this case (fig. 42) the known azimuth ⊙16 to ⊙3 was given as 330°15′ and the known elevation of ⊙16 as 135.0 feet. Also, an azimuth from the terminal station of the traverse to some other established point must be known as well as the elevation of the last point of the traverse occupied by instrument, so that the degree of accuracy of the work can be calculated and a reasonable amount of closing error adjusted.

e. Field work.—To run an azimuth traverse by the method outlined in paragraph 77*e* (1), proceed as follows:

(1) Set up on the starting point (⊙16, fig. 42) and measure the *H.I.* with the *H.I.* stick or stadia board. Unclamp needle.

(2) Set *A* vernier at the azimuth (330°15′) to the known point (⊙3), and signal rear rod "Edge."

(3) With lower motion and telescope direct, sight on rear rod and clamp. Read needle "North (or south) so many degrees and minutes east (or west)," which is recorded in the notebook (N. 30°00′ W.). Signal rear rod "Down" and front rod "Face."

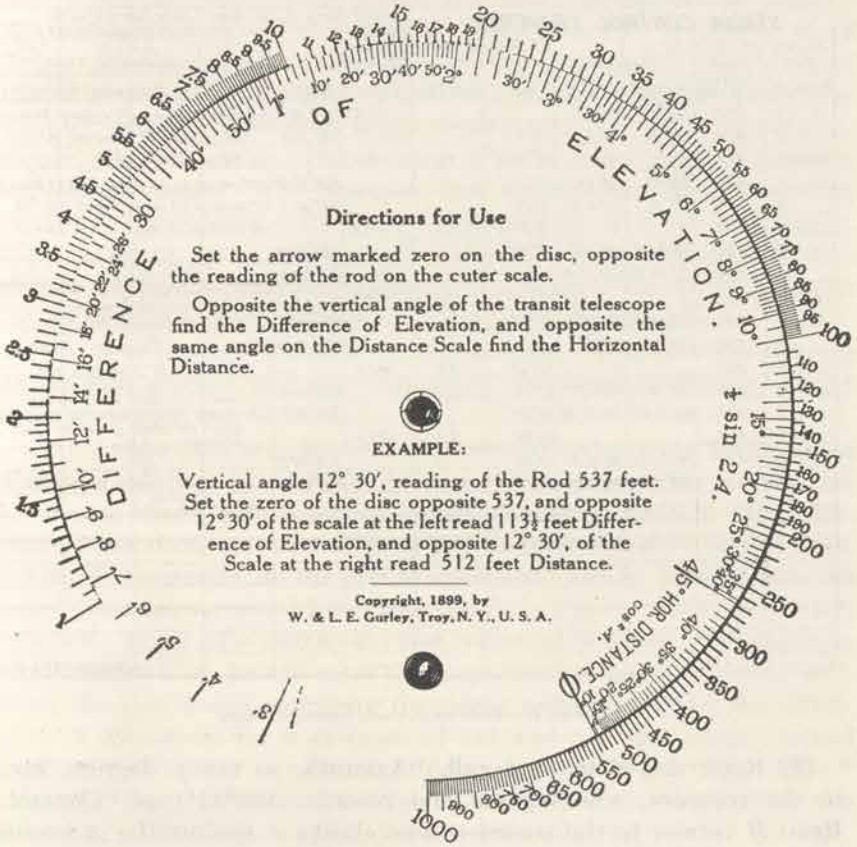


FIGURE 41.—Cox stadia computer.

(4) Unclamp upper motion, reverse telescope, and set center horizontal hair on front rod (⊙17) at the *H.I.* with upper motion.

(5) Read vertical vernier, calling, "Reversed, plus (or minus) so many degrees and minutes," to the recorder, who repeats and records on scratch paper (—0°11′).

(6) Plunge telescope and with *upper* and vertical motions set the lower stadia wire on one of the hundred-foot divisions, if possible, so that the center wire will be at about the *H.I.* Count the graduations from the lower to the upper wire and call, "Distance, so many feet," to the recorder, who repeats and records (475) in the notebook.

(7) Set middle wire at the *H.I.* Signal "Edge."

(8) With the *upper* motion, set on the edge of the rod and signal front rod "Down" and rear rod "Come in."

STADIA CONTROL TRAVERSE — Ft. Belvoir, Va.
From Sta. 16 To ⊙ 22 — Azimuth
Chief of Party: Sgt. M. I. Blair; Observer: "; Recorder: L. M. Toe; Rodman: Lee & Roe
Date: April 17, 1940; Inst: Berger 13281; K = 1.01; C + f = 1.05
(12)

From - To	Stadia Dist. Corr. Dist.	Control	Check	V.A.	Diff. Elev.	Elev.	Needle	Remarks
16 - 3	648	A 330°15'	150°15'			135.0	N30°00'W	⊙3-wooden peg with copper tack on
17	475	138°41'	318°41'	-0°11'			S41°30'E	S.E. corner of concrete, Belvoir
Mean	481			-0°11'30"	1.6	133.4		Rd. and 23rd street
17 - 16	475	B 318°41'	138°41'	+0°12'			N41°00'W	⊙16-same as ⊙3, about 15' W of Belvoir Rd
18	595	138°16'	318°16'30"	-0°31'			S41°30'E	⊙17-same as ⊙3, about 15' W of B.M. "35"
Mean	602			-0°31'30"	5.5	127.9		⊙18-same as ⊙3, about 30' E. of bend in road
18 - 17	595	A 318°16'30"	138°16'	+0°32'			N41°30'W	
19	545	172°59'30"	352°59'30"	+1°03'			S 7°00'E	
Mean	551			+1°03'	10.1	138.0		
19 - 18	545	B 352°59'30"	172°59'30"	-1°03'			N 7°00'W	⊙19 same as ⊙3
20	250	200°13'	20°13'	+2°12'			S 20°30'W	⊙20-same as ⊙3, about 10' W of road
Mean	254			+2°12'	9.8	147.8		and 7' N of sycamore tree
20 - 19	250	A 20°13'	200°13'	-2°12'			N20°00'E	
21	513	165°01'30"	345°01'30"	+0°34'			S14°30'E	⊙21-copper tack in top of 16" oak stump about 20" above ground on W. side of road
Mean	519			+0°35'	5.3	153.1		
21 - 20	513	B 345°01'30"	165°01'30"	-0°36'			N15°00'W	
22	430	208°23'	28°23'	-3°17'			S 28°30'W	⊙22 same as ⊙3
Mean								

Checked 7.7.40 B.

FIGURE 42.—Field notes, stadia control traverse.

(9) Read *A* vernier and call, "Azimuth, so many degrees, etc." to the recorder, who repeats and records (138°41′) as "Control." Read *B* vernier to the recorder, who checks it against the *A* reading and records (318°41′), repeating and adding "Check." Read and record needle (S. 41°30′ E.). Clamp needle. Call in rear rodman.

(10) Read vertical vernier, calling, "Direct, plus (or minus) so many degrees, etc.," to the recorder, who repeats (—0°11′), and checks against the reversed reading (see (5) above). If the instrument is in adjustment, they agree; if not, their mean is correct and is recorded in the notebook (—0°11′).

(11) Make sure that the rear rodman knows the location of this station, unclamp below, and move to next station. Set up and measure *H.I.* Send front rodman forward.

(12) Check vernier settings (*B* vernier should read 318°41′) and reset if necessary. Signal rear rod "Face," reverse telescope, and with lower and vertical motions, set on rear rod at *H.I.* Read *V.A.* to recorder, who repeats and records (+0°12′) on scratch paper as in (5) above. Plunge telescope, set on the rod with lower motion, read and record distance (475). Set on *H.I.* and signal rear rod "Edge." With lower motion, set on edge and signal rear rod "Down." Read *B* and *A* verniers to recorder, who checks and records (318°41′ and 138°41′, respectively). Read needle and record. Read *V.A.*, calling, "Direct, plus (or minus) so many degrees, etc. (+0°12′)" to the recorder, who repeats and checks against the reversed reading recorded on scratch paper. The recorder now records the *V.A.* (see (10) above). Signal front rod "Face." Proceed as in (4) to (11) above, substituting *B* for *A* vernier and *A* for *B*. (They alternate at each change of station which is shown in the notes.) Occupy, read, and record the data for the remaining stations on the traverse.

(13) Bear in mind that only the lower motion is used while sighting on the rear station, and only the upper motion while sighting on the front station.

(14) As the data are obtained the recorder reduces the mean stadia distances to true horizontal distances by applying K, $(c+f)$, and horizontal corrections when necessary; he also calculates the difference of elevation between stations and deduces the elevation of each.

(15) For example, on the first line marked "Mean" he will enter the mean distance, corrected for K and $(c+f)$, in this case $475\times K+(c+f)$ or $475\times 1.01+1.05=(480.8)$ 481 feet corrected distance. On the same line, column *V.A.*, he will enter the mean vertical angle or −0°11′30″. With the Cox stadia computer (or stadia reduction table) the difference of elevations for a distance of 481 feet and the mean vertical angle of −0°11′30″ equals 1.4, which is entered in the "Diff. Elev." column. This subtracted (the sign of the *V.A.* is −) from 135.0, the elevation of ⊙16, gives 133.6, the elevation of ⊙17.

(16) The corrected distances and elevations for the remaining stations of the traverse are obtained in a like manner.

(17) Horizontal corrections (always to be subtracted) are seldom applied to distances if the *V.A.* between the two corresponding stations is less than 5°. These corrections also are obtained with the Cox stadia computer or, more accurately, from stadia reduction tables.

f. Exercise XI.—Run a stadia traverse between two assigned points

by the method described in *e* above and record and calculate corrected distances and elevations for all stations. A diagram, similar to the one in figure 42, must show the location of each station to facilitate easy recovery.

81. Compass traverse with transit and stadia.—*a. Organization of party.*—The party usually consists of—

1 instrument man, who acts as chief of party.
1 recorder.
2 rodmen.
1 or more helpers to clear lines, etc.

NEEDLE TRAVERSE Ft. Belvoir, Va

From 067 To 0131 Azimuth

Chief of party L.B. Brown
Observer M.N. Brack
Recorder C.B. Black
Sketcher L.B. Brown

Rodman: Short, Long
Inst. Berger # 37140
K = 0.99
c + f = 1.07

(13)

From-To	STADIA DIST CORR. DIST	Control	Check	V.A.	Diff. Elev.	Elev.	Needle	Remarks
								Elev. 067 = 129.3
1 - 67	372 / 372	120°17'	300°17'	-0°03'	+ 0.3	129.6	S 60°00' E	
2	620 / 613	313°11'	133°11'	+0°11'	+ 2.0	131.6	N 46°30' W	
3 - 2	432 / 429	130°16'	310°16'	-1°13'	+ 9.0	140.6	S 50°00' E	
4	631 / 626	293°14'	113°14'	+1°27'	+ 15.7	156.3	N 67°00' W	
5 - 4	521 / 517	124°31'	304°31'	-2°03'	+ 18.4	174.7	S 35°00' E	05 is T.B.M. # 92
6	531 / 527	307°47'	127°47'	+3°01'	+ 27.3	202.0	N 62°00' W	
7 - 6	217 / 216	145°35'	325°35'	+0°11'	- 0.7	201.3	S 34°30' E	
8	395 / 392	331°43'	151°43'	-0°43'	- 4.8	196.5	N 28°00' W	08 is T.B.M. # 96
9 - 8	550 / 546	133°19'	313°19'	+2°01'	- 19.2	177.3	S 46°30' E	
10	325 / 323	323°02'	143°02'	-2°31'	- 14.0	163.3	N 37°00' W	
11 - 10	630 / 625	162°11'	342°11'	-0°03'	+ 0.5	163.8	S 17°30' E	011 is cross cut in concrete Surface of road, intersection of Belvoir Rd. and 9th Street
12	420 / 417	340°38'	160°38'	+1°13'	+ 8.8	172.6	N 19°30' W	

Checked: L.B.B.

FIGURE 43.—Field notes, needle traverse.

b. Equipment.—A 1-minute transit, 2 stadia rods, stadia computer or stadia reduction table, *H.I.* stick, stakes, hatchet, brushhook, notebook, and pencil.

c. Form of notes.—Figure 43 shows notes with all observed data reduced to their correct values as the traverse progresses. The method of recording is explained together with the method of observing, etc., in *e* below.

d. Control (starting and closing) data.—This method does not require that the first and last stations occupied with the instrument must be

the starting, respectively terminal, points of the traverse, nor that an azimuth be known. However, knowing the azimuth and occupying these stations will serve as a check of the magnetic declination when in doubt. The elevation of the starting point and terminal point must be known.

e. Field work.—When running a compass traverse, the transit, as a rule, is set up at the first new station of the traverse, usually ⊙1, then at every other station, namely—⊙3, ⊙5, etc. The work, with reference to the example recorded in figure 43, is done as follows:

(1) Set up and "orient" the transit at (⊙1) the first new station on the traverse. Set off on the instrument, if possible, the magnetic declination of the locality in which the work is done. This setting should not be disturbed as it remains the same for all subsequent set-ups. Have rodmen at rear station (⊙67) and forward station (⊙2).

(2) Set *A* vernier to read 0° and clamp upper motion. (*A* vernier is the control vernier on all set-ups.)

(3) Measure *H.I.* with the *H.I.* stick. Unclamp lower motion and turn transit until the north end of the compass needle rests at 0°, and clamp.

(4) Now azimuths, together with their corresponding compass bearings may be taken in any direction. All sights are taken to the "face" of the rod. Unclamp upper motion and sight at rear station (⊙67), bisecting the face of the rod with the vertical wire. Read stadia distance (375) and record in small figures, leaving room for correct distance. Set center horizontal hair at the *H.I.* on the rod and signal rear rodman in. Read *A* vernier (120°17′) and *B* vernier (300°17′) as a check and record. Read needle (S. 60°00′ E.), check same against azimuth, and record.

(5) Read *V. A.* (−0°03′) and record. Send rear rodman to establish new stations (⊙3 and ⊙4).

(6) Unclamp upper motion, turn telescope to front station (⊙2), signal front rodman, and read distance (620).

(7) Set horizontal center hair on rod at *H.I.* and signal rodman down. Read *A* vernier (313°11′) and *B* vernier (133°11′) as a check and record. Read needle (N. 46°30′ W.), check same against azimuth, and record.

(8) Read *V.A.* (+0°11′) and record.

(9) Move transit to new station, passing observed forward station which becomes the rear station, and set up and "orient" the instrument (at ⊙3).

(10) Observe from this station and every alternate station for distance, azimuth, and vertical angles as described in (1) to (9) above.

(11) The recorder will reduce the notes in the field obtaining the values shown in figure 43 as follows: Here $K=0.99$, $(c+f)=1.07$. Multiply first recorded distance (375) by K and add $c+f$, which equals $375\times0.99+1.07=(372.3)$ 372. When K is less than one, it is simpler to deduct $(1-K)\times$ stadia distance from the latter than it is to multiply by K (in the present example $375-3.75+1.07=372.3$). Compute difference of elevation for 372 feet distance and 0°03′ *V.A.*, which is 0.3 foot. As the elevation of the starting point (⊙67) given was 129.3 (see "Remarks" column), and the *V.A.* (from ⊙1 to ⊙67) was minus, the difference of elevation must be added, which makes the elevation of the first station (⊙1) $129.3+0.3=129.6$ feet and is so recorded. The remaining notes are similarly reduced and the values recorded as indicated in figure 43.

f. Exercise XII.—Run a compass traverse between two assigned points by the method described in *e* above and record and calculate corrected distances and elevations for all stations. A diagram showing the location of each station should accompany the notes.

Section XIII

COMPUTATION AND ADJUSTMENT OF TRAVERSES

	Paragraph
General	82
Traverse between two separate points, and closed (loop) traverse	83
Computation and adjustment of azimuths	84
Magnification of scale	85
Computation, adjustment, and checking of coordinates—taped traverse	86
Computation and adjustment of elevations	87
Computation and adjustment of stadia traverse	88
Area computation by double meridian distance method (D. M. D. method)	89

82. General.—*a. Purpose.*—(1) This section treats in detail the use of field notes in the computation and adjustment of rectangular coordinates (horizontal position), azimuths, and elevations pertaining to traverse stations.

(2) Rectangular coordinates are computed from adjusted azimuths and corrected distances in yards. See *b* below.

(3) Adjusted azimuths are obtained as explained in paragraph 84.

(4) Elevations are computed and adjusted as explained in paragraph 87.

b. Rectangular coordinates.—(1) Rectangular coordinates are distances from an assumed or fixed point, called the "origin", to any other point within the boundaries of any one particular rectangular coordinate system. These distances are measured parallel to two

lines intersecting at right angles at the "origin," one line running south-north and the other line west-east. The location of any point in a system of rectangular coordinates is always expressed by two values, *latitude* and *departure*.

(2) In order to avoid duplication and confusion the values of the grid coordinates of the Grid System of the United States, described in section XI, TM 5–230, and given in Special Publication No. 59, United States Coast and Geodetic Survey, should be used for all computations of rectangular coordinates. As grid coordinates are expressed in yards and nearly all military maps have superimposed on them their corresponding grid by rectangular lines, computations for rectangular coordinates of traverse stations should be made in yards. Hence, all measured, corrected distances on a traverse must be converted into yard equivalents, unless they were originally obtained in yards.

(3) Figure 44 shows the standard grid coordinates for zone B, one of the nine grid zones, into which the Continental United States is divided (see Special Publication No. 59, referred to above). The position of the area covered by figure 45 is indicated by the small black square in figure 44.

(4) Figure 45 shows the rectangular coordinates (conforming to the grid coordinates in fig. 44) of a small part of zone B, together with the stations of a short, closed traverse, illustrating the relation of coordinates, distances, azimuths, and bearings between the different stations. In figure 45 ΔY=differences of latitude, ΔX=difference of departure, α=azimuth and β=bearing. The coordinates for ⊙2 are X=1,000,000, Y=1,000,000; distance 2—3=896.76 yards; and the azimuth between them is 156° 18′, making the bearing S. 23° 42′ E. In the computation (par. 86), ΔY is obtained from the distance times the cosine of the bearing, and ΔX from the distance times the sine. After computing (and adjusting) ΔY becomes −821.10 and ΔX, +360.47 making the coordinates of ⊙3: X=1,000,000+360.47=1,000,360.47 and Y=1,000,000−821.10= 999,178.90. Similar relations for the remaining stations of the traverse may be seen by comparing figure 45 with the results of the computation shown in figure 47.

83. Traverse between two separate points, and closed (loop) traverse.—*a.* To compute and adjust positions of stations on a traverse starting and terminating at two separate points, the grid coordinates and elevation of both points, as well as the grid azimuth for both ends of the traverse, must be known.

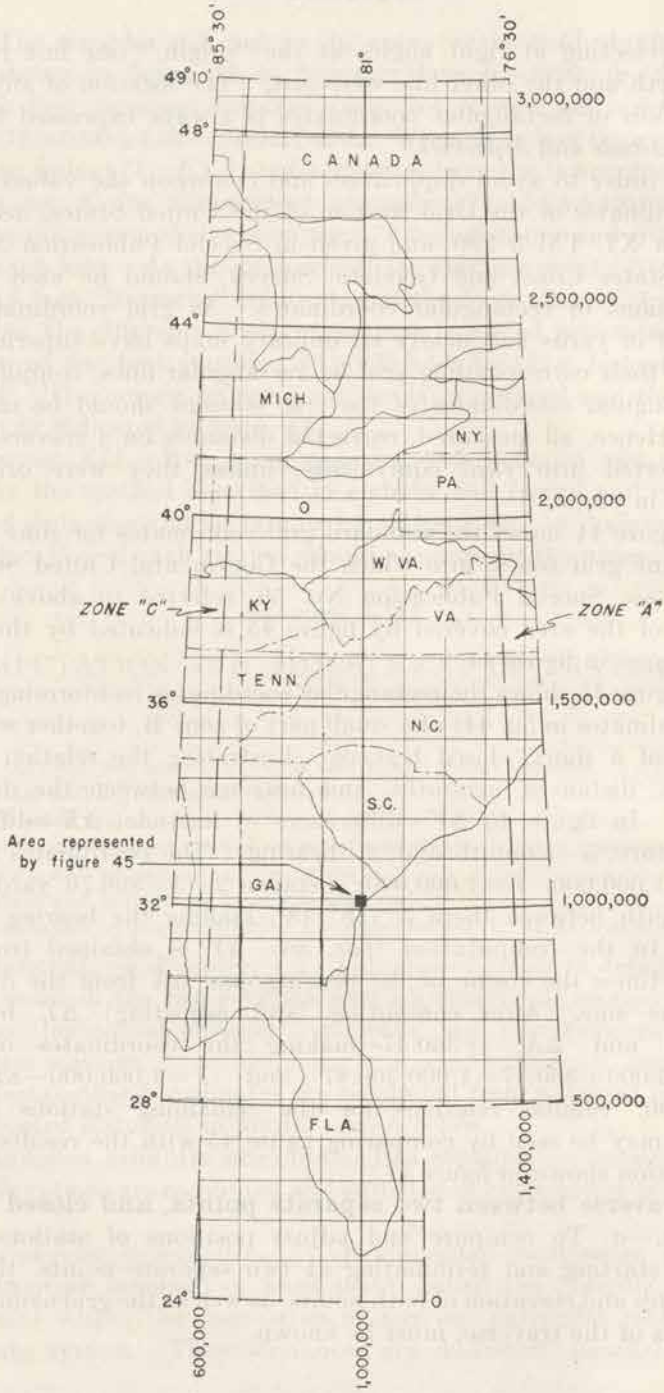


FIGURE 44.—Grid coordinates, zone B.

b. When running a short closed traverse, the coordinates, elevation, and azimuth at the point from which the traverse starts, need only be known, as such traverse closes at the starting point.

84. Computation and adjustment of azimuths.—*a. General.*— To compute grid coordinates, the azimuths of the different courses of a traverse must be first calculated and adjusted. Ordinarily, azimuths are obtained from observation of angles by repetition (see par. 79), or from the method employed in running an azimuth traverse (see par. 80). In the first case, azimuths must be calculated and adjusted

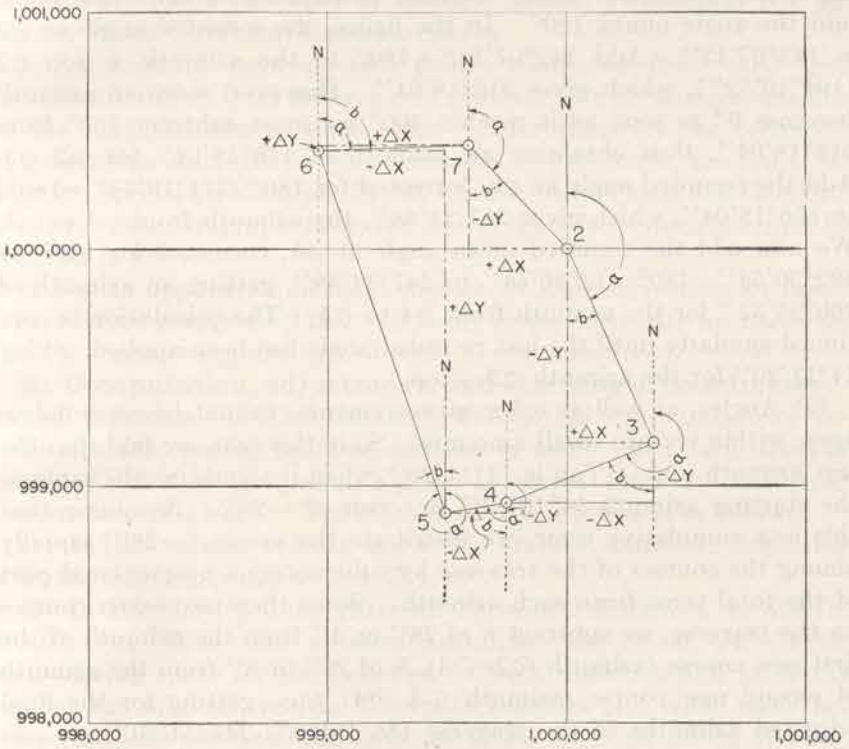


FIGURE 45.—Rectangular (grid) coordinates.

if necessary; in the second case, the azimuths obtained in the field must be adjusted if there is a discrepancy between the check azimuth (recorded in the field) at the end of the traverse and the known or given azimuth.

b. Calculation and adjustment of azimuths from angles by repetition.— (1) To calculate the azimuths of the courses of a traverse add the observed (mean) angle recorded in the field notes, plus or minus 180°,

to the back azimuth (given azimuth minus or plus 180°) of the known course and to the calculated azimuth of each course for each succeeding course. Referring to figure 46 (Form 1), azimuth ⊙2–△ top was given as 14°10′52″; this is a closed traverse and hence the check azimuth on closing should be the same. The back azimuth is 14°10′52″+180° or 194°10′52″ (see third line of form). As the forward azimuth is required in each case, the mean angle plus or minus 180° must be added to each preceding azimuth to obtain the forward azimuth. If the recorded mean angle is 180° or less, add the angle plus 180°, and if the recorded mean angle is more than 180° add the angle minus 180°. In the figure, the recorded angle at ⊙2 is 142°07′12″. Add 142°07′12″+180° to the azimuth △ top–⊙2 (194°10′52″), which gives 516°18′04″. However, since an azimuth becomes 0° as soon as it reaches 360° we must subtract 360° from 516°18′04″, thus obtaining an azimuth of 156°18′04″ for ⊙2–⊙3. Add the recorded angle at ⊙3, corrected for 180° (271°16′34″—180°) to 156°18′04″, which gives 247°34′38″, the azimuth from ⊙3 to ⊙4. We now add the recorded mean angle at ⊙4, corrected for 180°, or 192°50′54″—180°=12°50′54″ to 247°34′38″, getting an azimuth of 260°25′32″ for the azimuth from ⊙4 to ⊙5. The calculation is continued similarly until the last recorded angle has been applied, giving 14°11′20″ for the azimuth ⊙2–△top.

(2) Angles, as well as other measurements, cannot be expected to agree within certain small amounts. So in this case, we find that the last azimuth, ⊙2–△Top is 14°11′20″, when it should be the same as the starting azimuth 14°10′52″, an error of —28″. Assuming that this is a cumulative error, we distribute the excess (—28″) equally among the courses of the traverse by subtracting a proportional part of the total error from each azimuth. Since there are seven courses on the traverse, we subtract ⅐ of 28″ or 4″ from the azimuth of the first new course (azimuth ⊙2–⊙3), ⅞ of 28″ or 8″ from the azimuth of second new course (azimuth ⊙3–⊙4), etc., getting for the final adjusted azimuths of the traverse 156°18′00″, 247°34′30″, etc., as shown in the column "Correction, adjusted azimuths" on the form in figure 46.

c. Exercise XIII.—Calculate and adjust the azimuths for a taped traverse, with angles obtained by repetition, as described in this paragraph and illustrated in figure 46. If a regular form 2 is not available a form similar to form 1A, squared paper, may be used for this and other computations.

85. Magnification of scale.—*a.* Rectangular coordinates computed from azimuths and distances of a traverse between two points more or less widely separated may vary by a small amount from the

standard grid coordinates of these points due to the fact that the Y coordinates based on the polyconic projection contain an error known as the magnification of scale. This error is due to the simple reason that a spherical or spheroidal surface cannot be projected to a plane surface without some sort of distortion, which in the polyconic projection appears as magnification of scale in the Y coordinates, the enlargement varying from zero on the central meridian of the grid zone to 2 yards per thousand yards and over at 4° east or west of the central meridian. The corrections to Y coordinates for magnification of scale are found on pages 31 and 32, Special Publication No. 59, United States Coast and Geodetic Survey.

b. An azimuth between two points computed from their grid coordinates differs from the true grid azimuth by a certain amount due to the magnification of scale error. Corrections for the reduction of geographic azimuths to grid azimuths, or vice versa, are found on page 33, Special Publication No. 59, United States Coast and Geodetic Survey. See paragraph 117*g*, for instructions for reducing azimuths referred to above.

c. For an application of magnification of scale correction for taped traverses see paragraph 118*c*. In many cases, especially for traverses run by stadia, magnification of scale is not considered.

86. Computation, adjustment, and checking of coordinates—taped traverse.—*a. Computation.*—The actual computation of coordinates for control traverse stations is made on a form usually provided for that purpose. Figure 47 shows form 2, considered excellent for this work. The method of computation is as follows:

(1) In the second column enter the station numbers, beginning with the starting point, as 2–3, 3–4, etc.

(2) In the third column enter all horizontal distances, as 896.76, 665.93, etc. Enter the *adjusted* azimuths (156–18–00, 247–34–30, etc.) leaving space for the bearings. Check distances and azimuths against the original record. Compute bearings and enter same between distance and azimuth, as S. 23–42–00 E., S. 67–34–30 W., etc. Check bearings.

(3) In fourth and fifth columns take out logarithms, 5-place or 7-place, of distances and write them in their proper places. Take out log cos and log sin of bearings and enter them under the log distance in the proper column. Add log distance to log cos or log sin of the bearing in each course and check the two completed columns.

(4) In the sixth and seventh columns inspect bearings and mark with a cross the space not to be used; that is, if bearing is S. W., put a cross in the N. and a cross in the E. column, etc. This is necessary,

CALCULATION AND ADJUSTMENT OF AZIMUTH

Control Traverse, from ⊙2 to ⊙2

Project. XIII from Notebook No. 15

	Courses, etc.	First values	Correction Adj. azimuth		Courses, etc	First values	Correction Adj. azimuth
α	⊙2 — △Top	14° 10' 52"	° ' "	α			
Δ	△Top — ⊙2	180 00 00		∠			
α	△Top — ⊙2	194 10 52		α			
∠	⊙2 142-07-12	322 07 12	-4"	∠			
α	⊙2 — ⊙3	156 18 04	156 18 00	α			
∠	⊙3 271-16-34	91 16 34	-8"	∠			
α	⊙3 — ⊙4	247 34 38	247 34 30	α			
∠	⊙4 192-50-54	12 50 54	-12"	∠			
α	⊙4 — ⊙5	260 25 32	260 25 20	α			
∠	⊙5 260-41-54	80 41 54	-16"	∠			
α	⊙5 — ⊙6	341 07 26	341 07 10	α			
∠	⊙6 286-57-04	106 57 04	-20"	∠			
α	⊙6 — ⊙7	88 04 30	88 04 10	α			
∠	⊙7 228-49-24	48 49 24	-24"	∠			
α	⊙7 — ⊙2	136 53 54	136 53 30	α			
∠	⊙2 57-17-26	237 17 26	-28"	∠			
α	⊙2 — △Top	14 11 20	14 10 52	α			
∠				∠			
α				α			
∠				∠			
α				α			
∠				∠			
α				α			
∠				∠			
α				α			
∠				∠			
α				α			
∠				∠			
α				α			
∠				∠			
α				α			
∠				∠			
α				α			
∠				∠			
α				α			
∠				∠			
α				α			
∠				∠			
α				α			
∠				∠			
α				α			
∠				∠			
α				α			

Calculated azimuth ⊙2 to △Top 14°11'20" Correct azimuth 14°10'52"

Error ± +28" No. of courses 7 Correction first course ± -4"

Correction for all other courses = No. of course × (±) -4"

Date: April 26, 1940 Computed by: R.R. Checked by: J.L.P. FORM I

FIGURE 46.—Calculation and adjustment of azimuth.

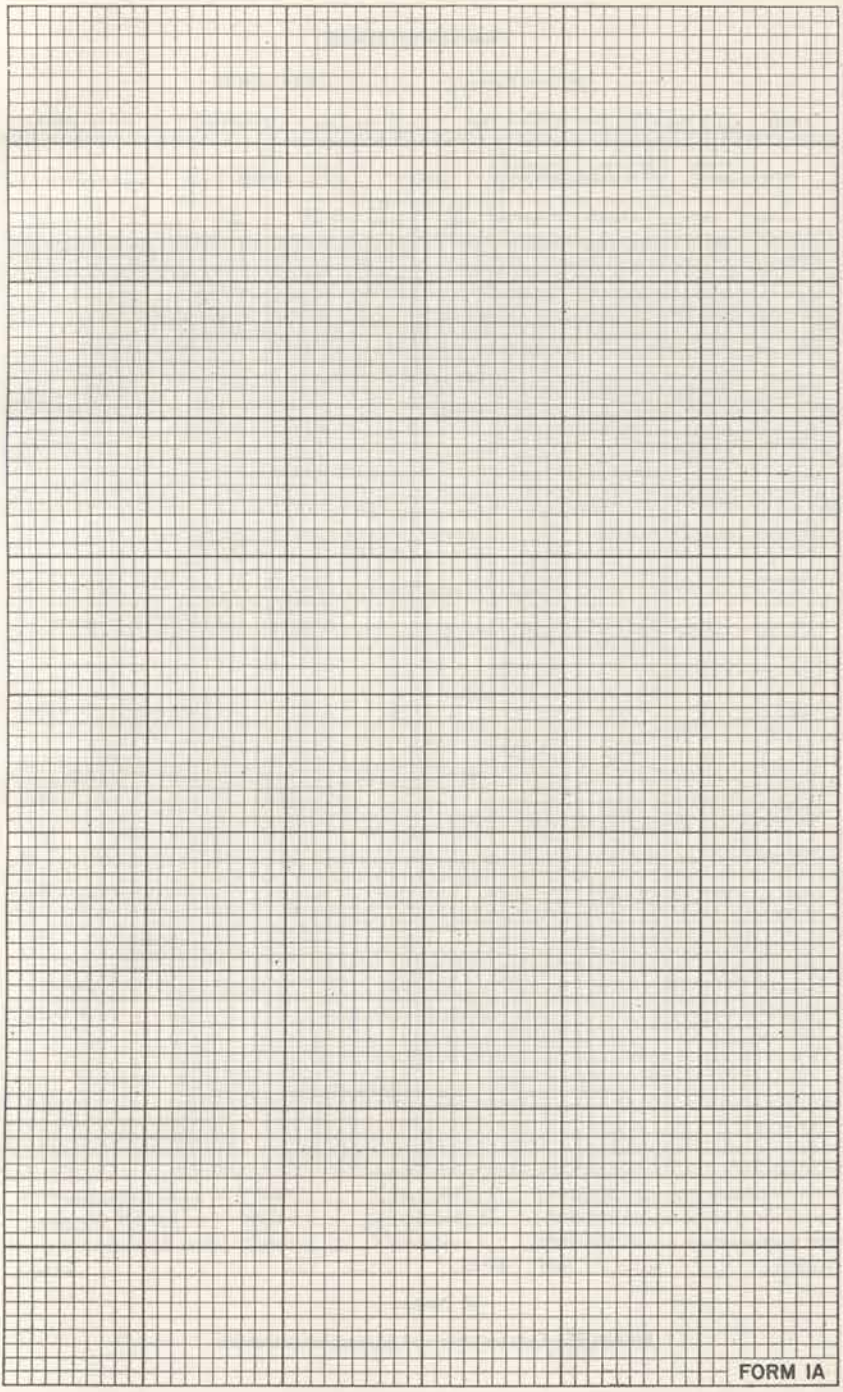


FIGURE 46.—Calculation and adjustment of azimuth—Continued.

TRAVERSE SHEET

Control TRAVERSE

FROM: Sta 2 (B-Fairfax) TO: Sta 2 (B-Fairfax)

I	II STATION	III DISTANCE BEARING / AZIMUTH	IV LOG DISTANCE / LOG COS BEARING / LOG LATITUDE	V LOG DISTANCE / LOG SIN BEARING / LOG DEPARTURE	VI LATITUDE +ΔY NORTH	VI LATITUDE −ΔY SOUTH	VII DEPARTURE +ΔX EAST	VII DEPARTURE −ΔX WEST	VIII COORDINATES Y NORTHINGS	VIII COORDINATES X EASTINGS
47242	2	896.76	2.952676 2	2.952676 2		8.27110	3.60471		10000000.00	10000000.00
		S 23-42-00 E	9.961735	9.604696		−0.3	+0.2		−8.210	+3.6047
	3	156-18-00	2.914411	2.155684		8.271143	2.60472		9991789.00	10003.6047
55252	3	665.93	2.823428	2.823428		2.54403		6.15534		
		S 67-34-30 W	9.958146	9.965890		−0.1		−0.3	−2.5403	−6.1534
	4	247-34-30	2.404893	2.789279		2.54104		6.15537	9989.2487	9997.4493
50182	4	258.52	2.412494	2.412494				2.54900		
		S 80-25-20 W	9.221176	9.993903				−0.2	−4.301	−2.5490
	5	260-25-20	1.633611	2.406397		4.3011		2.54902	9985.1869	9949.0053
44412	5	1622.87	3.210295	3.210318	1.535560			5.26112		
		N 18-52-30 W	9.975980	9.951000	+0.06			−0.3	+1.53560	−5.2512
	6	341-07-10	2.533186	2.720874	+0.03			5.25113	10041.7469	9989.6491
44472	6	624.95	2.795845	2.795845			6.24563			
		N 88-04-10 E	8.627475	9.999975			+0.3		+2.101	+6.2463
	7	88-04-10	1.323122	2.795598	2.101		6.24660		10043.8479	9995.8954
30192	7	600.61	2.778926	2.778926		4.38447	4.10446			
		S 43-06-30 E	9.863360	9.834662		−0.1	+0.2		−4.3847	+4.1046
	2	136-53-30	2.641952	2.613248		4.38448	4.10444		10000000.00	10000000.00
		4669.64 = Total Dist			1.556561	1.555666	1.395491	1.395564		
						1.555656		1.395491		
						.10		.15		
				Check:	1.556611	1.555661	1.395561	1.395556		
		Error = $\sqrt{10^2+.15^2}$ / 4669.64		= .18 / 4669.64 = 1. / 25942			Correction = .10			

COLUMNS I, II, III. COPIED BY Sgt. M.G. Bedell DATE April 2 1940 COMPARED AND FINAL CHECK

COLUMNS IV, V, VI, VII. COMPUTED BY Sgt. M.G. Bedell DATE April 2 1940 BY Sgt T.R. Garrod

COLUMN VIII COMPUTED BY Sgt. M.G. Bedell DATE April 2 1940 DATE April 3 1940

IN 1 SHEETS SHEET NO. FORM 2

① Control.

FIGURE 47.—Computation and adjustment of traverse

TRAVERSE SHEET

Stadia TRAVERSE

FROM: △ Lacey (Fairfax) TO: △ Camp (Fairfax)

I. ELEV.	II. STATION	III. DISTANCE BEARING / AZIMUTH	IV. LOG DISTANCE LOG COS BEARING / LOG LATITUDE	V. LOG DISTANCE LOG SIN BEARING / LOG DEPARTURE	VI. LATITUDE +ΔY NORTH	VI. LATITUDE −ΔY SOUTH	VII. DEPARTURE +ΔX EAST	VII. DEPARTURE −ΔX WEST	VIII. COORDINATES Y NORTHINGS	VIII. COORDINATES X EASTINGS
238.3	△ Lacey	167 9	2.22505	2.22505		158.1	56.0		3737418	986362
223.7		S 19-32-00 E	9.97426	9.52421		- 1	- 1		- 1581	+ 560
	1	160-28-00	2.19931	1.74926		153.2	56.1		3735837	986922
		150.9	2.17609	2.17609		149.7				
309.4	1	S 2-35-00 E	9.99956	8.65391		- 1			- 1497	+ 68
	2	177-25-00	2.17565	0.83000		149.6	6.8		3734340	986990
		238.0	2.37658	2.37658		237.2				
211.4	2	S 4-06-30 E	9.99888	8.85517		- 2			- 2372	+ 171
	3	175-53-30	2.37546	1.23175		237.4	17.1		3731968	987161
		240.6	2.39199	2.39199		245.8				
201.9	3	S 4-01-30 E	9.99893	8.84628		- 2			- 2458	+ 173
	4	175-58-30	2.39092	1.23827		243.6	17.3		3729510	987334
		285.3	2.45530	2.45530		284.1				
169.4	4	S 4-46-00 E	9.99850	8.91959		- 2			- 2841	+ 237
	5	175-14-00	2.45380	1.37489		284.3	23.7		3726669	987571
		239.3	2.37894	2.37894		238.5				
188.9	5	S 4-09-00 E	9.99886	8.85955		- 2			- 2385	+ 173
	6	175-51-00	2.37780	1.23849		236.7	17.3		3724284	987744
		161.2	2.20737	2.20737		82.4	138.4			
172.5	6	S 59-13-30 E	9.70920	9.93409		- 1	- 1		- 824	+ 1384
	7	120-46-30	1.91657	2.14146		82.5	138.5		3723460	989128
		258.6	2.41263	2.41263		258.3				
165.8	7	N 89-03-00 E	8.21958	9.99994			- 3		+ 43	+ 2583
	8	89-03-00	0.63221	2.41257	4.3		258.6		3723503	991711
		172.7	2.23729	2.23729			172.1			
167.1	8	N 85-58-00 E	8.84718	9.99892			- 2		+ 121	+ 1721
	9	85-58-00	1.08467	2.23621	12.1		172.3		3723624	993432
		90.7	1.95761	1.95761			68.7			
161.6	9	S 49-19-30 E	9.81410	9.87990			- 1		- 591	+ 687
	10	130-40-30	1.77171	1.83751		59.1	68.8		3723033	994119
		110.5	2.04336	2.04336		65.5	88.8			
135.9	10	S 53-36-00 E	9.77336	9.90574		- 1	- 1		- 655	+ 888
	11	126-24-00	1.81672	1.94910		65.6	88.9		3722378	995007
		148.4	2.17143	2.17143		130.7	70.1			
143.9	11	S 28-13-30 E	9.94502	9.67480		- 1	- 1		- 1307	+ 701
	12	151-46-30	2.11645	1.84623		130.6	70.2		3721071	995708
		200.9	2.30298	2.30298		130.0	153.0			
147.2	12	S 49-40-00 E	9.81106	9.88212		- 1	- 1		- 1300	+ 1530
	13	130-20-00	2.11404	2.18510		130.1	153.1		3719771	997238
		188.2	2.27462	2.27462		169.5	81.6			
148.5	13	S 25-43-00 E	9.95470	9.63741		- 1	- 1		- 1695	+ 816
	14	154-17-00	2.22932	1.91203		169.6	81.7		3718076	998054
		135.3	2.13130	2.13130		59.4	121.4			
144.7	14	S 63-53-30 E	9.64352	9.95326		- 1	- 1		- 594	+ 1214
	15	116-06-30	1.77482	2.08456		59.5	121.5		3717482	999268
		169.2	2.22840	2.22840		127.1	111.5			
141.8	15	S 41-16-30 E	9.87596	9.81933		- 1	- 1		- 1271	+ 1115
	△ Camp	138-43-30	2.10436	2.04773		127.2	111.6		3716211	1000383
		2962.8 Yds = Total Distance			16.4	2.13818	140.35		△ Lacey 3737418	986362
						164			△ Camp 3716211	1000383
						2.12274			- 212207	+ 140021
						2.12071	140.21		- 212274	+ 140315
						- 1.7	- 1.4		- 1.7	- 1.4
		Error = √1.7² + 1.4² = 2.2					Correction	1.4		
		2962.8	2962.8	1.347				2.2		

CO. UMNS I, II, III. COPIED BY Sgt M.G Bedell DATE March 5 1940 COMPARED AND FINAL CHECK

CO. UMNS IV, V, VI, VII. COMPUTED BY Sgt M.G Bedell DATE March 5 1940 BY Sgt T.R. Garrod

COLUMN VIII COMPUTED BY Sgt M.G Bedell DATE March 5 1940

IN 1 SHEETS SHEET NO. FORM 2

② Stadia

FIGURE 47.—Computation and adjustment of traverse—Continued.

even for experienced computers, to avoid mistakes. In the spaces not marked by a cross write the antilogs of the results obtained in columns IV and V, as 821.13, 360.45, etc.

(5) Add the "North," "South," "East," and "West" columns, each column separately. If the traverse is closing back to the initial point, the sum of the "North" column should equal the sum of the "South" column and the sum of the "East" column should equal the sum of the "West" column. If the traverse is closing on some point other than the initial point, the difference between the sum of the "North" column and the sum of the "South" column should equal the difference between the given Y coordinates (latitudes) of the initial and final stations; and the difference between the sum of the "East" and "West" columns should equal the difference between the X coordinates (departures) of these stations.

(6) In the example shown in figure 47①, a closed traverse, the closing error in latitude is 0.10 (too far to the south) and in departure 0.15 (too far to the west). Compute the closing error, to make sure whether or not the traverse is within the specified allowable error, as shown by the diagram and formula in the figure. Here, no magnification of scale was considered; but if a traverse extends for several miles or more and grid coordinates are used, magnification of scale correction for the distance between the two end points of a traverse should be applied when computing the error of closure, as otherwise the traverse may appear not to come within the allowable error. Draw the diagram representing the correction (0.10 north and 0.15 east). Add distances of all courses. The error of the traverse is expressed by the formula

$$E=\frac{\sqrt{(\delta X)^2+(\delta Y)^2}}{D}$$

in which E=the error ratio of the traverse.

D=sum of distances of all courses in the traverse.
δX=error in X.
δY=error in Y.

Substituting values from figure 47①

$$E=\frac{\sqrt{0.10^2+0.15^2}}{4{,}669.64}=\frac{0.18}{4{,}669.64}=\frac{1}{25{,}942}$$ or 1 part in 25,942.

b. Adjustment.—(1) If the traverse is found to be within the allowable error, the traverse may be adjusted by the following rule: The correction to the latitude of a course is to the whole error in latitude as the corresponding latitude is to the arithmetical sum of

all the latitudes. Similarly for departures. Each correction is applied so as to diminish the whole error.

(2) This rule, altering the length as well as the direction of each course, is based upon purely mathematical considerations and should be applied only when nothing is known as to where the errors probably occurred. (All known errors are corrected before the traverse is adjusted.)

(3) Compute the corrections for each course and enter them on the line above their corresponding latitudes and departures, prefixing the proper signs. Add or subtract the corrections, enter the final corrected latitudes and departures above the corrections, and line out the original differences in latitude and departure.

(4) Calculate the coordinates in the last column (VIII), starting with the known or given coordinates of the first station and obtaining the coordinates of intermediate stations by adding or subtracting the *adjusted* latitude and departure of each course from Y or X of each preceding station.

(5) Enter *adjusted* elevations of stations (see par. 87) in first column.

c. Checking.—The above method of computing and adjusting employs the progressive check system, whereby data and results are checked at each step. Independent computation in duplicate would be the best method of securing a check at each step. This will avoid carrying errors further when newly computed figures serve as initial data for successive steps. The time to check computations can be considerably reduced by using such mechanical labor saving devices as calculating machines, slide rules, special tables, and special diagrams and charts. Most of these devices are designed to be used in checking computations and to make calculations involving figures containing a small number of digits (usually not exceeding four) excepting calculating machines whose construction permits the use of figures having as many as seven or more digits. These are often employed in making the computations. The devices most commonly used are—

(1) Calculating machines of various makes, performing the four basic arithmetical operations of addition, subtraction, multiplication, and division.

(2) The Mannheim slide rule (see sec. XII, TM 5–230) designed for operations involving the use of logarithms of numbers and logarithms of circular trigonometric functions.

(3) Stadia slide rules for computing differences of elevations and horizontal distances between any two points.

(4) Special tables, such as traverse tables, stadia reduction tables, etc.

(5) Special diagrams (see sec. XII, TM 5 230) and charts based on the graphic principle and represented on "coordinate" or "cross section" paper.

Booklets and descriptions accompanying these various devices explain in detail their characteristics so that the average person will find no difficulty in manipulating them.

87. Computation and adjustment of elevations.—*a.* If the elevations of the traverse stations were determined by differential leveling, elevations are adjusted as described in paragraph 54*e*, adjustment of level lines.

b. If elevations are obtained from trigonometric leveling (distances and vertical angles) and come within the allowable error, the adjustment is made by distributing the closing error (the difference between the calculated elevation and the true elevation of the terminal point of the traverse) among the various stations in proportion to the distance between these stations.

c. Elevations for stadia traverses that close within the allowable error may be adjusted by distributing the total error equally among the stations of the traverse. This method saves time and the results are usually acceptable, especially when no further control is based on any of the intermediate traverse stations so adjusted.

88. Computation and adjustment of stadia traverse.—*a. Method.*—The computation and adjustment of a stadia traverse are in all details identical with the procedure outlined in paragraph 86, except that 5-place logarithms are used exclusively. Figure 47② shows complete the computation and adjustment of a stadia traverse on a standard form, similar to figure 47①. All distances are in yards; elevations are in feet.

b. Exercise XIV.—Compute and adjust a traverse from notes furnished or obtained in the field using the methods and form described in paragraph 86 or this paragraph. Complete all marginal data as indicated on the form.

89. Area computation by double meridian distance method (D. M. D. method).—*a. General.*—To compute the area of a piece of land of more or less irregular outline, the boundaries (distances and azimuths) of which have been measured, the D. M. D. method is frequently used. The method is as follows: After all the latitudes and departures have been computed and adjusted in the usual manner (the traverse is a closed figure) compute the D. M. D.'s of all the courses by the following rules, referring to figure 47②:

(1) The D. M. D. of the first course (starting from the most westerly point to avoid negative signs) equals the departure of the course itself.

Springfield Reservoir Survey Jan. 10 1940

COURSE	LATITUDE	DEPARTURE	D.M.D	NORTH AREAS +	SOUTH AREAS −
6 - 7	+ 2101	+ 624.64	624.64	1312.369	
7 - 2	- 438.47	+ 410.46	1659.74		727746.20
2 - 3	- 821.10	+ 360.47	2430.67		1995823.14
3 - 4	- 254.03	- 615.54	2175.60		552667.69
4 - 5	- 43.01	- 254.91	1305.15		56134.50
5 - 6	+1535.60	- 525.12	525.12	806374.27	
				819497.96	3332371.53
					- 819497.96
					2512873.57

2512873.57 = 1256436.78 sq.ft. = 1256436.78 = 259.594 Acres
2 4840

6-7)
624.64
2101
624.64
624640
12492.8
1312368.64

7-2)
1659.74
438.47
11618.18
663896
13277.92
49792.2
663896
727746.1978

2-3)
2430.67
821.10
2430670
243067
48613.4
1944536
1995823.1370

3-4)
2175.60
254.03
652680
8702400
1087800
435120
552667.6680

4-5)
1305.15
43.01
1305.15
39154.50
522060
56134.5015

5-6)
525.12
1535.60
31507.20
262560
157536
262560
52512
806374.2720

Computed: John Heitmann

Checked: J.W. Hathaway

FORM 3

FIGURE 48.—Area computation by D. M. D. method.

(2) The D. M. D. of any other course equals the D. M. D. of the preceding course plus the departure of the preceding course plus the departure of the course itself.

(3) The D. M. D. of the last course should be numerically equal to its departure, but with opposite sign.

The double areas of all the trapezoids (bounded by the courses, their projections on the meridian, and the perpendiculars from the ends of the courses to the meridian) may now be found by simply multiplying the D. M. D. of each course by the latitude of the same course, north latitudes being regarded as plus and south latitudes as minus. Since the sum of all the north double areas minus the sum of all the south double areas, disregarding the sign, equals twice the area of the field, divide their difference by 2, to obtain the actual area of the field. If yards are used for computing the traverse, square yards will be the result; if feet are used, square feet will result. To obtain amount of acreage divide the number of square yards by 4,840, or number of square feet by 43,560. In the example (fig. 48), yards were used for measuring distances.

b. Exercise XV.—Compute the irregularly shaped area (to the nearest hundredth of an acre) lying between the distances and azimuths of computed and adjusted traverse, as described in *a* above, using the form as shown in figure 48. Complete all marginal data indicated on the form.

Section XIV

GENERAL INSTRUCTIONS—TRIANGULATION

	Paragraph
Object	90
Definitions	91
Triangulation, classification	92
Triangulation systems	93
Triangulation figures	94
Existing triangulation data	95

90. Object.—The main object of triangulation is to furnish the necessary control for surveys. This control requires the determination of the positions and elevations of widely separated points on the earth's surface and the directions and lengths of the lines joining these points. These accurate locations provide for correction of the errors of ordinary surveys which otherwise would tend to accumulate. In this country, the bulk of control work of higher orders is done by the United States Coast and Geodetic Survey. This work involves—

a. Reconnaissance and signal erection,

b. Measurement of base lines,
c. Measurement of the horizontal and vertical angles,
d. Astronomic determinations of latitude, longitude, and azimuth,
e. Checking the results at various stages, and
f. Computation and adjustment.

All the above are treated in this manual, except the determinations of latitude and longitude, which are described in United States Coast and Geodetic Survey Special Publication No. 14. In most regions of the continental United States, a geodetic point may be found within 25 miles of any particular location. The need for control in any vicinity should be foreseen so that there may be ample time for triangulating from the geodetic control to the required positions.

91. Definitions.—*a*. Geodetic triangulation includes all those operations required to determine either the relative or absolute positions of different points on the earth's surface when such operations are based on the properties of plane and spherical triangles. By the *relative position* of a point is meant its location with reference to one or more points in terms of such angles and distances as may be necessary. In geodetic work, distances are usually expressed in meters and are always reduced to mean sea level. By the *absolute position* of a point is meant its location by latitude and longitude. Strictly speaking, the absolute position of a point also includes its elevation above mean sea level. Directions are either relative or absolute. Relative directions are indicated by measured or computed angles from other lines. The absolute direction of a line is given by its azimuth, which is the angle it makes with a meridian through either of its ends measuring clockwise from the south point and continuously up to 360°. The azimuth of a line must always be stated in a way that clearly shows to which end it refers.

b. In the actual field work of triangulation, suitable points called *stations* are selected throughout the area to be covered and are definitely marked. The selection of these stations depends on the character of the country and the object of the survey. The stations thus established are regarded as forming the vertices of a set of mutually connected triangles (overlapping or not, as the case may be), the complete figure being called a *triangulation system*. At least one side and usually all the angles in the system are directly measured, using the utmost care. All the remaining sides are obtained by computation of the successive triangles, which (corrected for spherical excess, if necessary) are treated as plane triangles. The line which is actually measured is called the *base line*. It is customary to measure an additional line near the close of the work; this line being con-

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nected with the triangulation system so that its length may also be obtained by calculation. Such a line called a *check base* forms an excellent check on both the field work and the computations. In work of large extent, intermediate bases are often introduced as checks. Lines actually measured on the ground are always reduced to mean sea level before any further use is made of them. It is evident that all lengths computed therefrom will refer to mean sea level without further reduction. In the United States enough existing triangulation stations of an equal or higher order of accuracy can generally be recovered to provide lines suitably located to serve as intermediate and check bases. (See par. 95.)

c. Based upon the accurately determined elevation of some initial point, the elevations of all stations are determined by measurement of the vertical angles between stations, known as trigonometric leveling. These angles, with the calculated distances between stations, determine the differences in elevation. The determinations by trigonometric leveling are checked at reasonable intervals by connection with lines run by instrumental leveling.

d. The stations forming a triangulation system are called *triangulation stations.* Those stations (usually triangulation stations also) at which special work is done are commonly given corresponding names, such as *base line stations, latitude stations, longitude stations, azimuth stations,* etc.

e. Grid triangulation must not be confused with the geodetic triangulation described above, which is employed in advancing the main control system over long distances. Grid triangulation is utilized in breaking down the longer lines of the main control system and increasing the density of fixed points available for fire control, battle map preparation, and other incidental purposes. Initial latitudes and longitudes are converted to military grid coordinates, azimuths are clockwise from grid north, and all computations are accomplished by plane trigonometry. The grid coordinates and azimuths thus determined are sufficiently accurate for the intended purposes but cannot be employed in the further extension of geodetic triangulation.

92. Triangulation, classification.—*a.* There are five classes of triangulation: first-, second-, and third-order, grid, and fourth-order triangulation. The basis of classification is the accuracy with which the length and azimuth of a line of the triangulation are determined. The criteria and methods of applying them will be explained in section XVI. The requirements as to accuracy may be compared as follows:

	First-order	Second-order	Third-order	Grid
Discrepancy between computed and measured length of base or adjusted length of check lines, not to exceed.	1 in 25,000	1 in 10,000	1 in 5,000	1 in 2,000.
Triangle closure:				
Average, not to exceed	1 second	3 seconds	6 seconds	12 seconds.
Maximum, not to exceed	3 seconds	8 seconds	12 seconds	30 seconds.
Usual number of observations:				
Positions with 1-second direction instrument	16	4	2	1.
Positions with 2-second direction instrument			4	
Sets with 10-second repeating theodolite	5 to 6	2 to 3	1 to 2	$\left\{\frac{1}{2}\right.$ $3D/R.$
Sets with 20-second transit				1.
Base measurement:				
Actual error of base not to exceed	1 in 300,000	1 in 150,000	1 in 75,000	1 in 10,000.
Probable error of base not to exceed	1 in 1,000,000	1 in 500,000	1 in 250,000	
Astronomic azimuth, probable error of revolt	0.5 second	2.0 seconds	5.0 seconds	15.0 seconds.

Fourth-order triangulation is used to connect detailed surveying and mapping operations with control of a higher order by graphic methods or transit angles. The sole requirement of accuracy in fourth-order triangulation is that positions must be located without errors that will be appreciable on the resulting map.

b. For any project requiring extensive first- or second-order work, Special Publications of the United States Coast and Geodetic Survey should be consulted, as follows:

No. 26. General Instructions for the Field Work of the United States Coast and Geodetic Survey.
No. 93. Reconnaissance and Signal Building.
No. 120. Manual of First-Order Triangulation.
No. 138. Manual of Triangulation Computation and Adjustment.
No. 145. Manual of Second- and Third-Order Triangulation and Traverse.
No. 158. Bilby Steel Tower for Triangulation.

c. As the first- and second-order systems of the United States Coast and Geodetic Survey are so far advanced, any geodetic triangulation undertaken by the Army will usually be of the third order. No 15-minute quadrangle should contain less than three stations of the third order. In addition, a number of secondary points—such as church spires, windmills, water tanks, trees, and in high mountain regions some of the more prominent summits—must be located by intersection and checked by angles from one or more stations, or located by the "three-point method" of resection, which gives satisfactory results if a fourth point is sighted for a check. Where no such objects are available, at least two points should be flagged for intersection if practicable. These points are intended for supplemental control and should be picture points if possible. Fourth-order triangulation will seldom be useful except for reconnaissance and field location of picture points.

d. While the grid triangulation is designed particularly for use in war, this method should be practiced in all military surveys whenever it is applicable. The larger allowance of error is not due to any inherent inaccuracy in the computation or adjustment, but to reducing the observations to the minimum in order to get results quickly and to the employment of hastily erected signals and poorly conditioned triangles. Another factor making for larger errors is the length of lines, which in war will probably vary from 1 to 5 miles, as one object of this class of work is to provide a fixed point in every 8 to 10 square miles throughout the artillery areas. In war, provision will have to

be made for rapidly carrying the grid triangulation forward in case of an advance.

93. Triangulation systems.—*a.* Figure 49 shows three methods of connecting distant points by triangulation with the same maximum length of sight in each case. System ① consists of a single row of triangles and is usually the least expensive; its use is limited by its lack of accuracy. System ② is formed by triangles in a double row or arranged as hexagons; it secures a moderate degree of accuracy at moderate cost. System ③ consists of a single row of quadrilaterals; it is the most expensive system and gives the greatest accuracy. A quadrilateral is an arrangement of four stations with all the connecting lines observed; thus, successive pairs of stations are connected by six lines, over each of which a pointing is taken from each end.

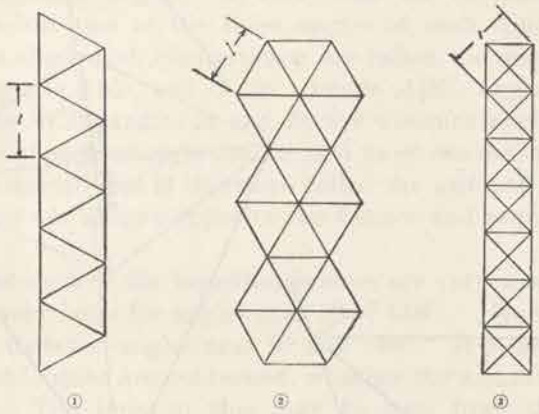

FIGURE 49.—Triangulation systems.

b. Thus, for practically the same distance covered, the number of stations to be occupied is about one-half more for either system ② or system ③ than for system ①; the total length of lines to be cleared out is about 80 percent more for system ② and 55 percent more for system ③, as compared with system ①. The area covered by system ③ is a little less than that covered by system ① and less than half that covered by system ②. But system ③ is superior to the other schemes in affording more "conditions" for adjustment of observed angles. The conditions tabulated are rigid geometrical conditions (as, that the sum of the angles of a triangle equals 180°). Additional conditions increase accuracy of final results.

c. Triangulation of third-order or higher accuracy should be composed of quadrilaterals or central-point figures, unless the work may be closed on adjusted triangulation of equal or higher order with only a

few figures, which may include simple triangles with all three angles observed. The latter are satisfactory for grid triangulation, where the more complicated figures should be avoided in order to shorten both field work and computation.

d. Base lines are generally about one-sixth to one-fourth the length of the average line in the system. It is more important to be able to extend the base by well conditioned figures than it is to have a base of great length. The series of figures connecting the measured base

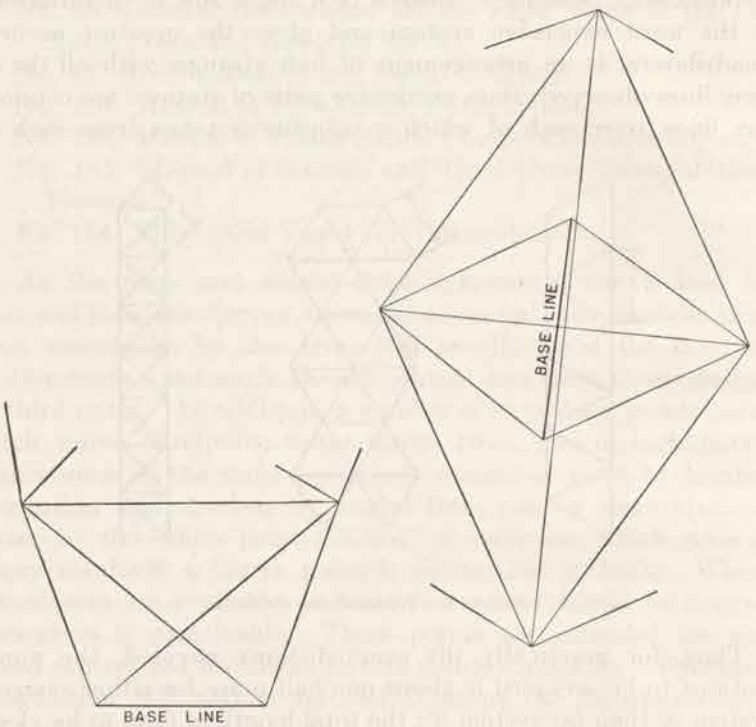

FIGURE 50.—Extension from base.

with the main system is called the base net. For the three higher orders, the base net should be a series of quadrilaterals, and the known side in each figure should be more than 40 percent of the length of the computed side from which further extension is to be made. (See fig. 50.) In favorable terrain, the observance of this rule and that of paragraph 94*d* will keep the conditions of the base extension system fairly close to those of the main system. In grid triangulation, the increase from the known to the unknown side in a triangle of the base net may be as much as one to three.

94. Triangulation figures.—*a.* In order that small errors of measurement shall produce minimum errors in computed values, it is essential that only well shaped figures be used in a triangulation system. The best shaped triangle is equilateral and the best shaped quadrilateral is a perfect square. However, field conditions render the use of perfect figures impractical and it will often be necessary to select, for triangulation, figures which only approximate the ideal. Strong figures should be employed which will maintain the accuracy of the computed lengths of sides in the system from one known or measured line to the next.

b. In section XIX it will be shown that, after all adjustments are made, the length of an unknown side of each triangle is computed by the sine formula—that the sides of a triangle are proportional to the sines of the opposite angles. In every case the computation of the sides involves but two of the three angles of each triangle. These angles used in the length computation are called the distance angles; referring to figures 51①, and ②, in triangle *ABC*, angles *B* and *C*, and in triangle *ACD*, angles *A* and *D*, are distance angles. The remaining angle of each triangle can be used most conveniently to carry forward the azimuth and is therefore called the azimuth angle. The distance angles are always opposite the known and required sides of the triangle.

c. The differences in the logarithmic sines are very small for angles near 90° and very large for angles near 0° or 180°. Hence, it is desirable to avoid distance angles near 0° and 180°. It is immaterial, as far as errors of lengths are concerned, whether the azimuth angles are small or not. The force of this may be seen from the following analysis of figure 51① and ②. In figure 51① and ② the known and required sides are drawn heavy, *AB* being known and *CD* being required. In figure 51①, no matter which pair of triangles is used in the computation of *CD*, no distance angle less than 44° nor greater than 125° is used. The result is a high degree of accuracy in the length. The azimuth angles concerned are very small, being from 8° to 12°, but this fact has no effect upon the accuracy of the length, *CD*, as they are not concerned in the computation of its length. On the other hand, in ②, the distance angles used in computing the required side *CD* from the known side *AB* range from 9° to 125° and the use of two very small angles is unavoidable. The result is a very low degree of accuracy in the computed length.

d. It is desirable to keep the distance angles between the limits 30° and 150° and preferably near 90°, but insofar as accuracy of length is concerned the size of the azimuth angle is immaterial. One station

of a quadrilateral or an outer corner of a central point figure may be left unoccupied without greatly decreasing the probable accuracy of the computed lengths. In a figure in which all stations are occupied, if any interruption (as for example the failure of a signal to show) makes it probable that such a procedure would save considerable time, certain lines to a total of not more than three may be observed over in one direction only.

e. While the above discussion of probable computed accuracies applies particularly to third-order triangulation, the principles also affect grid

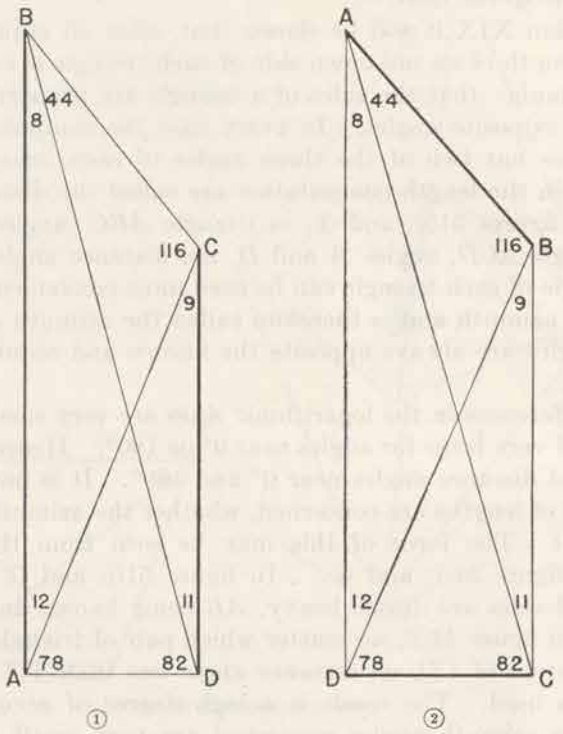

FIGURE 51.—Strong and weak quadrilaterals.

triangulation. In the latter some distance angles of 20° are permissible if the completion of the system is materially accelerated thereby. Around 10° to 15°, the weakness of the triangles is so marked that such small distance angles should be avoided, even in grid triangulation, as part of a system which is to be extended.

95. Existing triangulation data.—*a.* Section XVII will demonstrate that base line measurement should be avoided if connection can be made with previously adjusted stations of an equal or higher order

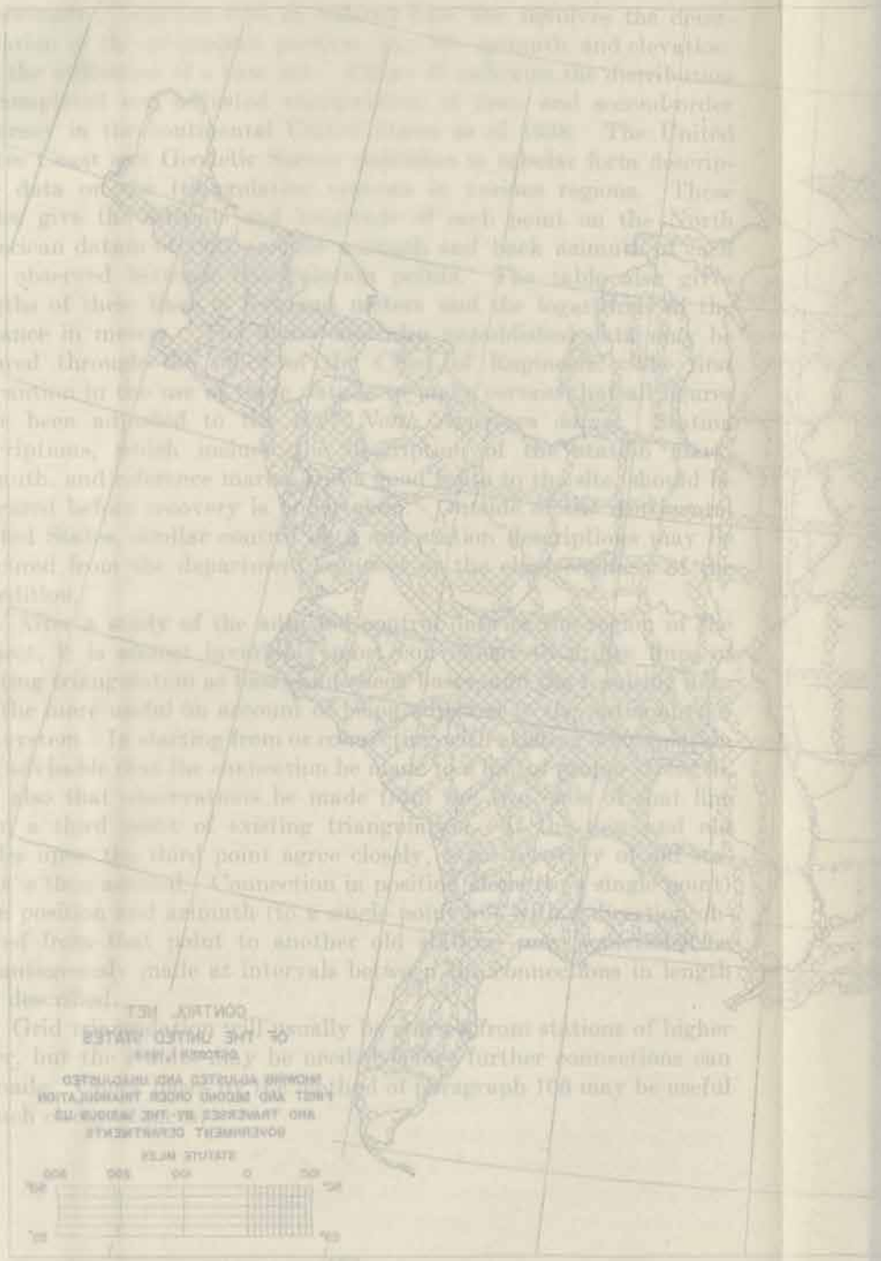

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FIGURE 52.—Map of areas in United States covered by first- and second-order control.

of accuracy. Starting with an isolated base line involves the determination of the geographic position (par. 90), azimuth, and elevation, and the utilization of a base net. Figure 52 indicates the distribution of completed and adjusted triangulation of first- and second-order accuracy in the continental United States as of 1938. The United States Coast and Geodetic Survey publishes in tabular form descriptive data on the triangulation systems in various regions. These tables give the latitude and longitude of each point on the North American datum of 1927 and the azimuth and back azimuth of each line observed between triangulation points. The table also gives lengths of these lines in feet and meters and the logarithms of the distance in meters. The above and also unpublished data may be secured through the office of the Chief of Engineers. The first precaution in the use of these data is to make certain that all figures have been adjusted to the *1927 North American datum*. Station descriptions, which include the description of the station mark, azimuth, and reference marks, and a good route to the site, should be procured before recovery is undertaken. Outside of the continental United States, similar control data and station descriptions may be procured from the department engineer or the chief engineer of the expedition.

b. After a study of the adjusted control data in the region of the project, it is almost invariably most convenient to utilize lines of existing triangulation as bases and check bases, and the resulting data are the more useful on account of being adjusted to the national control system. In starting from or connecting with existing triangulation it is advisable that the connection be made to a line of proper strength, and also that observations be made from the two ends of that line upon a third point of existing triangulation. If the new and old angles upon the third point agree closely, exact recovery of old stations is then assured. Connection in position alone (to a single point) or in position and azimuth (to a single point but with a direction observed from that point to another old station) may sometimes be advantageously made at intervals between the connections in length just described.

c. Grid triangulation will usually be started from stations of higher order, but the results may be needed before further connections can be made. Check bases by the method of paragraph 106 may be useful in such circumstances.

SECTION XV

TRIANGULATION RECONNAISSANCE AND SIGNALS

	Paragraph
Purpose of reconnaissance	96
Organization of party and equipment	97
Station and base line selection and construction	98

96. Purpose of reconnaissance.—Modifications applicable to grid triangulation are described in paragraph 98*o*.

a. The success of triangulation depends upon an intelligent reconnaissance. This reconnaissance should select station locations forming the strongest and most feasible figures for the triangulation and locate the base lines and public roads and other routes. The lengths of sights will often be fixed by the nature of the topography, but where this is not the case there must be a decision as to the average length desirable. The lower limit of length of line is fixed by two considerations. On very short lines it is difficult to get observations of the degree of accuracy necessary to close the triangles within the required limit. Extreme caution is required in centering and plumbing the signals and the theodolite to avoid the errors due to eccentricity. Very short lines also multiply the number of stations and increase the danger of cumulative errors. On the other hand there is no advantage, insofar as accuracy is concerned, in using very long lines. Long lines are apt to introduce delays due to signals not being visible. With long lines, supplementary stations to reach required points in all portions of the area covered are much more apt to be needed than with short lines. In third-order triangulation the sights may range between 3 and 10 miles in length.

b. Experienced surveyors should be assigned to reconnaissance, since much depends on the proper selection of stations. The form of the triangles, the amount of clearing necessary, possible damage to private property, accessibility and cost of stations, the avoidance of possible sources of atmospheric disturbances, and many similar subjects have to be considered in determining the location of stations. The line of sight should not "graze" the ground or any other object so closely as to make lateral refraction possible.

c. The chief of the reconnaissance party must bear in mind that he is selecting points which are to be marked permanently and to serve not only for the work immediately in contemplation but possibly for other surveying projects. The net result of a geodetic survey is a number of scattered monuments and a data sheet showing their locations and elevations. The importance of selecting a place for the station where the mark will be least exposed to disturbance is

evident. If consistent with other considerations, the station should be at a picture point. (See sec. XXVII of this manual and sec. XV, TM 5–230.) The reconnaissance party should keep full notes regarding station locations; these will be supplemented by notes of the signal-building and angle-measuring parties, as their operations afford additional data.

d. Duties of reconnaissance party.

(1) The selection of well conditioned figures.

(2) Making sure that stations are intervisible, without grazing rays.

(3) Siting, flagging, and marking each point temporarily.

(4) Laying out sites and nets for any base lines required.

(5) Directing attention to possible intersection stations.

(6) Preparing an adequate description of the location of each point and the route thereto.

(7) Determining the kind of signal or observing platform, and the clearing required.

(8) Securing the written consent of the property owner for the above. If any clearing is necessary, the value of the timber to be cut should be definitely fixed and agreed upon with the owner before cutting is begun.

(9) Furnishing a plat of the system, and a record of any angles and azimuths as roughly determined by the party.

e. The plat of the triangulation system is copied and used in the field by the construction and observation parties and in the office by the computing section. Another copy in the office is marked with various colors to indicate the progress of station erection, observation, and computation.

97. Organization of party and equipment.—*a.* A chief of party and instrument man, a recorder, and two or more laborers are generally sufficient for the reconnaissance party proper with such additional personnel as necessary for supply, transportation, etc. The triangulator should act as chief of reconnaissance, or his recorder should be a member of the party, thus becoming familiar with the region, the control net, and its stations.

b. The reconnaissance party will find the following instruments useful:

Protractor.
Pocket sextant.
Prismatic compass.
Clinometer.
Aneroid barometer.
Field glasses.
Scale.
Steel tape.
Plane table or sketching board.
Alidade with folding sights.
Notebook and pencil.
Blank forms for station description.

Description of existing stations and bench marks.	Saw.
	Nails.
Best maps of country available.	Wire.
Lineman's climbers, short and long spur, complete with straps.	Signal cloth.
	Crayon.
Ax.	Light line.
Hatchet.	

c. A close study should be made of all available maps. All established stations and bench marks should be plotted on the most suitable medium scale map. A provisional scheme including intersection stations and base line sites, if required, should be lightly penciled on the map, to be revised during the field reconnaissance. Lacking a suitable map, a plane table sheet of suitable scale is prepared showing all existing control, and the new system plotted by intersection and resection during the reconnaissance.

d. The field glasses are needed for picking up distant flags, and for studying remote summits so they may be recognized from other locations. A good prismatic compass supplemented by a pocket sextant in case of local attraction is very valuable for obtaining azimuths and angles from tree tops to distant points. The rays are plotted on the map or sheet and the azimuths and deduced angles are recorded in the notebook. The observations from each point are kept on a separate page. As the scheme progresses some of the figures may be diagrammed on other pages. The latter will be done especially for lines difficult to observe or which are concealed by forest growth. (See par. 98*c*.) As these notes and the station descriptions will be needed for station construction and also by the observing party, the records must be clear and complete.

e. The tools will be needed for minor clearing and for erecting flags. The flags should be varied in size and color if there is any chance of confusion from a distance.

f. Clinometer and barometer notes may aid in the identification of summits. They will be of some use in preliminary determinations of intervisibility on lines that have to be cleared.

98. Station and base line selection and construction.—Special methods applicable to grid triangulation are described in *o* below.

a. General.—(1) Triangulation stations, as far as practicable, should be placed on the crests of ridges and on the highest points of hills and mountains. In a mountainous country it is not necessary to place the stations on the highest peaks, but each one should be on the highest point of the peak selected, and this peak should be the highest one in the immediate vicinity in order that there may be an unobstructed

view in all directions. The most favorable ground for a line of triangles is a valley of proper width, with bold banks or slopes on either side. Stations can then be selected giving well conditioned triangles, with little or no clearing out of lines. In wooded country, clearing must be resorted to or tall stations built, preferably the former if it can be accomplished at reasonable expense. A careful study of the maps will indicate the general lay of the country, the direction of drainage, the approximate location of the highest points, and the route which apparently offers the least resistance. In the actual examination of a region it is necessary to consider only the topographic features that may come within range of the work in hand, such as the crests of the highest ridges, hills, and peaks, which may serve as observation points in laying out the scheme.

(2) When visiting one of these commanding points, the first step is to locate its approximate position on the map or sketch. Trained powers of observation and a habit, natural or acquired, of self-orientation, by which is meant the faculty of knowing at any point visited for the first time just where to look for other points requisite to the work in hand, will best help the observer to secure a full knowledge of the country and will suggest the best method of laying out the work.

(3) The next step is to study closely with the aid of binoculars the prominent objects that may come within the area of the scheme for the purpose of fixing them in the memory and estimating their appearance from other directions. The positions of objects observed may be closely determined by magnetic compass bearings and by estimating the distances. Each point should be plotted on the work sheet or map in the position fixed by the bearings and estimated distances. Any distinctive features of the objects observed which will assist in identifying them when seen from other points should be noted.

(4) If in wooded country the observer, having selected from the ground what appears to be the best location, must assure himself by climbing trees or otherwise that previously located stations are visible, or can be made so by reasonable construction or clearing, and that there is a proper field of view in the direction of progress of the system. If maps are not available, the man on reconnaissance must from each located station pick out points in advance which appear to be suitable for stations, determine their location as nearly as possible by intersection, and proceed thereto as best he can. Certain natural landmarks are apparent in any landscape which will determine some of the locations; others will be more difficult.

b. Selection of base sites and nets.—The necessity for any base line measurement should be foreseen, if possible, so that a suitable site

may be chosen, and a net of comparable strength arranged for the connection to the main systems. A material part of the accuracy of the base measurement may be lost through a very weak base net. A base may be measured with tapes with the required accuracy over fairly rough ground and moderate slopes. It is of more importance to locate the base so as to secure well conditioned figures in the base net than to locate it in the smoothest place.

c. Direction of stations not intervisible.—As to intervisibility of stations, the man on reconnaissance must recognize that sometimes one station will be concealed from another merely by forest growth which can be cleared out. He must, before extending his system in such manner as to depend upon a doubtful station, assure himself that it is practicable to render it visible at reasonable expense. In

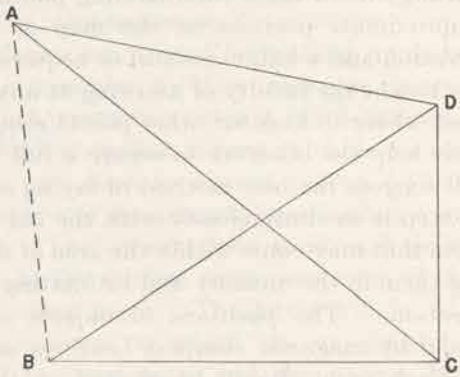


FIGURE 53.—Determination of direction of stations not intervisible.

some cases ground reconnaissance along direction lines determined approximately from one or both stations will suffice. In other cases it may be necessary to determine the direction from one station to another by some method such as the following, which is applicable if two other points can be found from each of which both stations are visible: In figure 53, AB is the line to be cleared, and C and D are two points from which both stations A and B can be sighted. Call CD unity. Measure angles from C and D. Solve triangle BCD for side BC and triangle ADC for side AC. Two sides and included angle of triangle ABC are now known. Solve for other angles (par. 119*d*). These angles will give the direction for line AB from either A or B using C and D as reference points. In such cases, additional personnel and additional equipment, such as a transit, may be temporarily required.

d. Height of stations.—(1) The height that it is necessary to build a station for observation between two points depends on the relative heights of the ground at the stations and of the intervening ground. Two factors enter into the problem: The curvature of the earth's surface and the refraction of light by the earth's atmosphere. These factors involve corrections of opposite sign.

(2) Curvature equals the ratio of the square of the distance to the mean diameter of the earth, or $\frac{K^2}{2R}$ where K and R are, respectively, distance and the earth's radius, in the same units. Refraction equals the ratio of the square of the distance to the semidiameter of the earth times a coefficient m, or $\frac{K^2}{R}m$. The value of m is taken as 0.07. The formula for the combined effect of curvature and refraction is the difference of the two expressions, or $(1-2m)\frac{K^2}{2R}$. Calling the height of the station in feet h, and the distance in statute miles between two stations K, the following approximate formulas result:

$$h=0.574\ K^2$$
$$\text{and } K=1.32\sqrt{h}$$

The values of h may be obtained from table XVI, TM 5–236. However, the values of $h=0.574\ K^2$ are so close to the tabular values that reference to the table is unnecessary if the formula is at hand.

(3) The calculated heights of stations should be increased by at least 10 feet in order to avoid grazing rays, unfavorable refraction, and other atmospheric disturbances.

e. Intervisibility of stations can be determined from the above formula or table, which gives the height of one station for visibility when observed from another at the ground level in open, level country. Some examples are given.

(1) In figure 54①, two stations are at the water level on opposite shores of a lake 8 miles wide. The line of sight should be at least 10 feet above the water. At 8 miles $h=0.574\times8\times8=36.7$ feet. The tripod heights will be 10 feet at A and 47 feet at B.

(2) In figure 54②, if the towers can be made of equal height, the line of sight would meet the 10-foot height above the water halfway between C and D. At 4 miles $h=\frac{36.7}{4}=9.2$ feet. It is seen that both tripods need be only 19 feet high.

(3) In figure 54③, two stations D and F, respectively, at 100 and 200 feet elevation, are 8 miles apart. A bare ridge E at an elevation

of 120 feet is 2 miles from D on line toward F. What is lowest tripod that can be used if the target at the other station is 8 feet? As the obstruction at E is 2 miles from D, the effective elevation at D is reduced by curvature and refraction, $0.574\times2\times2=2.3$ feet, to 97.7 feet. Similarly at F, $0.574\times6\times6=20.7$ feet deducted from 200 gives an effective elevation of 179.3 feet. Adding 10 feet to avoid grazing ray at E, the elevation becomes 130 feet. In figure 54④ the three

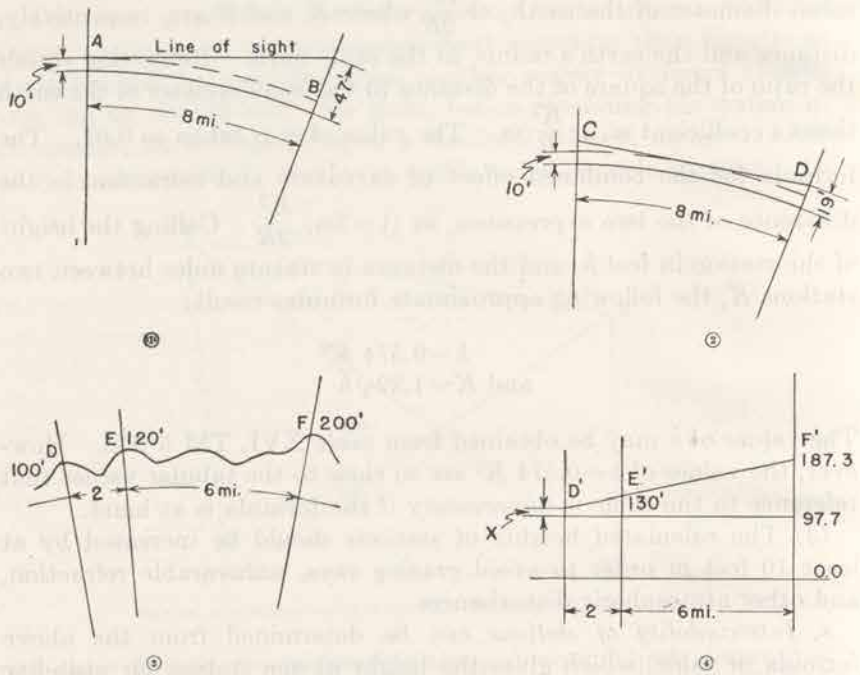


FIGURE 54.—Intervisibility of stations above the ground.

points are considered as based on a plane surface, with all elevations corrected as above for curvature, refraction, and grazing. The height of the necessary tripod may be found by proportion. A target 8 feet above the ground at F' would have 187.3 feet elevation. Then

$$\frac{187.3-130}{6}=\frac{187.3-(97.7+x)}{8} \text{ or } x=13.2 \text{ feet},$$

which is the height of tripod necessary at D. Inspection of figure 54④ shows that a higher tower would be required at F. Other things being equal, it is always more economical to build at the station nearest the obstruction, for the height necessary to clear the obstruction increases in direct proportion to the distance from it.

f. Types of stations.—(1) Triangulation, because of the constant availability of the checks described in section XVI, and because its stations are usually better distributed and more advantageously located for most purposes, is generally preferred to traverses of cor-

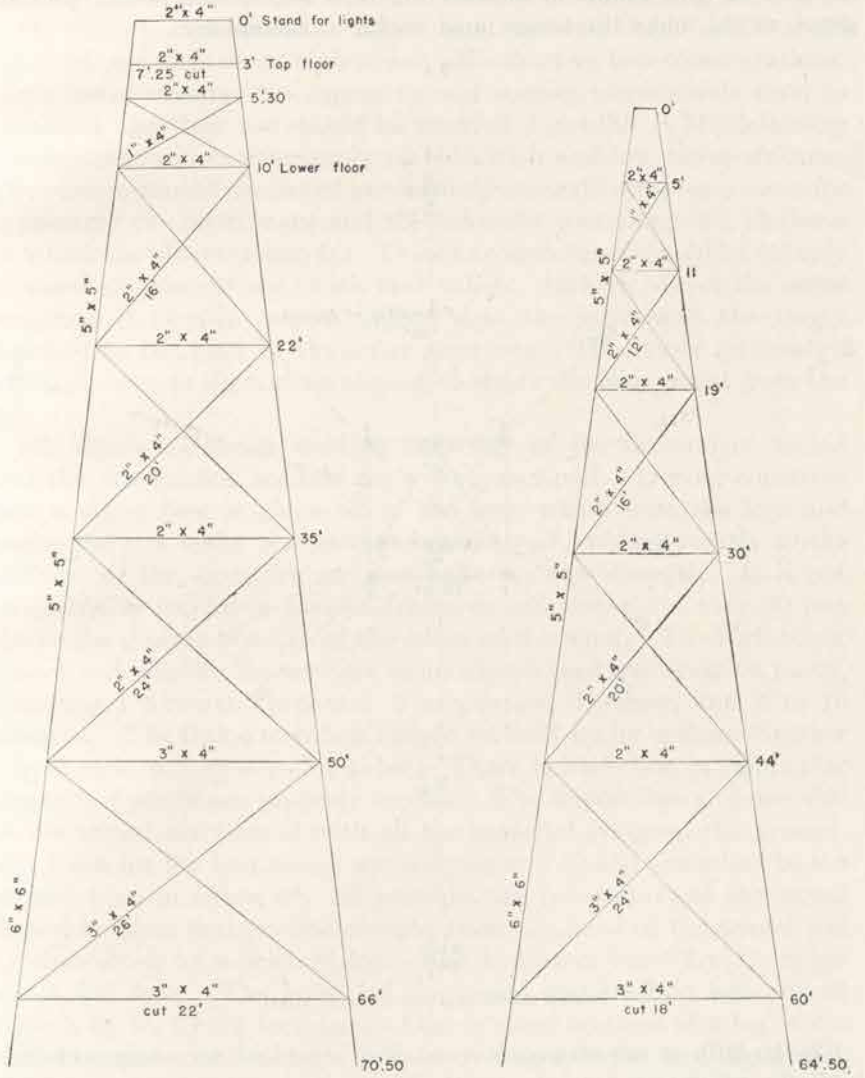


FIGURE 55.—Side views of scaffold and tripod for 60-foot signal.

responding accuracy. Traverses were formerly employed in heavily wooded, almost flat country, due to the expense and time consumed in the construction of high wooden towers. The Bilby steel towers

are extensively utilized by the United States Coast and Geodetic Survey. In the Army the steel towers are less common, since they have to be procured beforehand, and they occupy considerable truck or trailer capacity. The use of a steel tower near the front would be ill-advised, as a couple of intersecting rays and a "pin-point" photograph might make the tower most useful to the enemy.

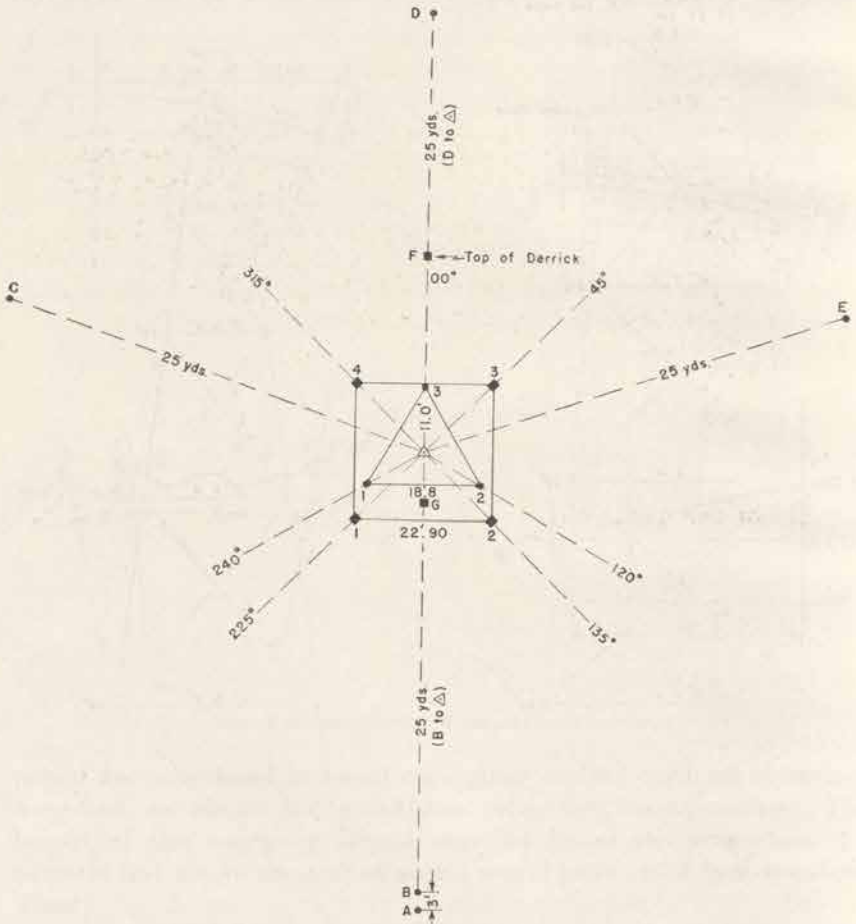


FIGURE 56.—Ground plan for 60-foot signal.

(2) In hilly or mountainous country it is seldom necessary to build high observing towers to elevate the instrument, as it is only necessary to select prominent intervisible points that satisfy the requirements of a strong scheme that will give proper control over the area covered.

(3) Rolling country requires very careful reconnaissance, if towers are avoided while strong figures are laid out. Occasional selection of

a church spire, flagpole, or other similar object which cannot be occupied may help in advancing the system without much construction. The use of towers for instrument supports should be avoided wherever possible because of the expense and delay, and because usually the station cannot be used for future work without rebuilding the tower.

(4) *Station construction.*—Instrument stations which are over 20 feet high are classed as high tower, all others as low tower stations. High tower stations are expensive and require considerable time to construct and their use should be avoided if possible. The following governing considerations apply to both high and low tower stations. The station should consist of two entirely separate structures—one for supporting the instrument and the other for sustaining the platform on which the observer stands. These two structures should be entirely of wood and should not touch each other. In high towers the outer structure is usually carried higher than the inner and the target fastened to the apex of the outer structure. The upper framework will also serve to support an awning to shade the instrument from the sun.

(5) Figure 55 shows working drawings of the instrument tripod and the observation scaffold for a 60-foot signal. During construction a slight bow is given all of the legs, which puts the legs and horizontal ties under a moderate initial strain, adding greatly to the stiffness of the structure and somewhat to the strength. It is not advisable to employ a wooden-frame instrument stand over 60 feet above the ground because of the effect of the wind. The high tower station will require the services of an experienced construction party, including 1 foreman carpenter, 2 carpenters, 1 rigger, and 2 to 10 laborers. The frame members should be built up by spiking together 2-by-6-inch or 2-by-4-inch lumber. There is little loss in rigidity or strength if joints are properly broken. The tripod legs and one side of the tripod are framed with all the material lying on the ground. The holes for the foundation are laid out and staked according to the ground plan in figure 56. If possible, the orientation of the signal should be such that no line of sight from the head of the tripod will be obstructed by a scaffold leg. The footplates are 2 by 12 inches and 3 feet long. The holes for the tripod and scaffold legs are all made 3 by 3½ by 3½ feet deep. One or more sections of a leg of the scaffold may be used as a gin pole for raising side No. 1 of the tripod. The gin pole should be about two-thirds the height of the tripod. Four-inch posts for attaching guys should be set as shown at *A*, *B*, *C*, *D*, and *E* in figure 56. *G* represents the foot of the gin pole. When the tripod is completed, spike the feet to the footplates and put on anchors, making the anchor platform about 3 feet square. Fill the

hole to the top, keeping the earth well tamped. Opposite sides of the scaffold are framed completely and all four legs are cleated before raising. In the case of an obstructed line on which the obstruction is not discovered until the towers have been built, the difficulty can be overcome by building a superstructure on the signal at each end of the line.

(6) The low tower tripod and a scaffold, as shown in figure 57, can readily be constructed by a carpenter and two helpers.

(7) Another type of instrument support is a tree trunk cut off at a suitable height and stripped of all branches. This will save building a tripod at least, and may save much clearing. A separate platform should be built for the observer. The tripods furnished with the

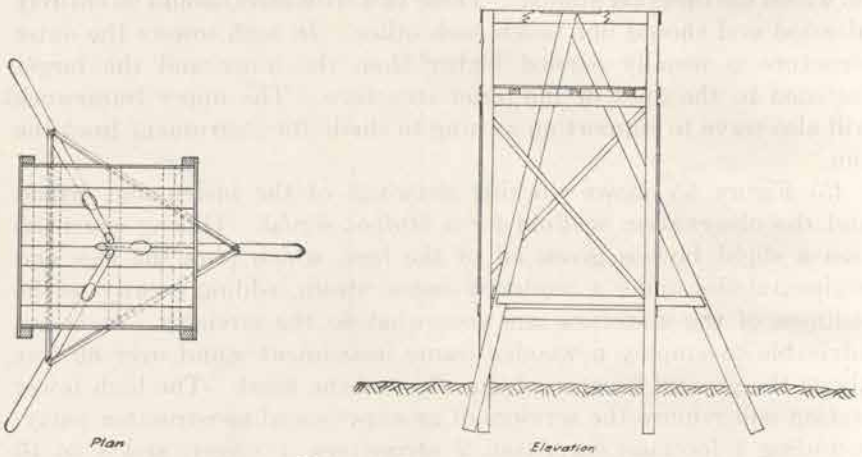


FIGURE 57.—Low tower station.

modern instruments are rigid enough for any but the most accurate work. If the ground is soft or spongy, the tripod shoes should be supported on 2- by 4-inch stakes driven solidly into the ground.

g. Targets.—(1) A good target should be clearly visible against all backgrounds, readily bisected, rigid, capable of being accurately centered over the station, and so constructed that the center of the visible portion, whether in sun or in shade, shall coincide with its vertical axis.

(2) To make a target visible against both light and dark backgrounds it should be painted in alternating black and white belts. For ready bisection it should be as narrow as possible without sacrificing distinctness. This is accomplished by making the width subtend an angle of from 2 to 4 seconds of arc. Since the arc of 1 second is 0.3 inch for a mile radius, an angle of 4 seconds would re-

quire a target 0.1 foot in diameter for 1 mile distance, or 1 foot in diameter for 10 miles distance. A flag or panels of appropriate size should be added so that the signal may be readily picked up through the telescope.

(3) When the signal must be seen from a distance, and especially when it is to be used also for topographic or hydrographic purposes, a tripod signal is used of the type shown in figure 58①. As there is space for observer and instrument below the braces, this signal does not have to be disturbed during occupation. One objection to this type is that it is difficult to plumb over a mark already in place. The upper portion of the legs will often be covered with cloth or painted boards to attract attention. Each leg should be secured to the ground by a stake or other means. Wire guys to hold the center poles are necessary if the pole has a considerable length. The largest tripods usually have a height of about 20 feet to the apex of the tripod and from 30 to 35 feet to the top of the center pole. Where cut lumber is available, timbers 4 inches square are generally used for the legs and center pole of large signals and 2- by 4-inch lumber for those of smaller tripods. Ordinarily the center pole should be large enough in diameter to be visible through the theodolite from the adjacent stations, as otherwise observations will be made upon the tripod or banners and errors due to phase and eccentricity will develop. One-inch boards are ordinarily used for dressing signals and should be rough or should have only one side planed, as a rough surface is superior for holding whitewash.

(4) A convenient method of constructing a large tripod signal is indicated in figure 58②. The drawing shows the legs and center pole as assembled on the ground. An identifying letter is placed at the lower end of each part. This assembly is arranged so that the lower ends of the legs *A* and *C* are in the approximate positions they will occupy when the signal is completed. The tripod is then erected by using leg *B* as a lifting pole and prop. When the tripod is erected and secured in the proper location the lower end *D* of the center pole, which pivots on the bolt, is brought within reach by means of a line attached at *D*, and the pole is then adjusted to a vertical position and secured by cross braces to each leg of the tripod. The targets on the center pole may be attached before or after this operation. The sides of the tripods are then boarded up as desired. The spaces between boards need not be less than the width of the boards, and wider spacing may be desirable in order to cover as much surface as possible with a limited amount of material.

(5) Figure 58③ shows a pole signal held in a vertical position by wire guys with the foot of the pole resting on a low bench. The

bench may be made of two stakes driven in the ground on either side of the station mark, with a piece of scantling placed across on top

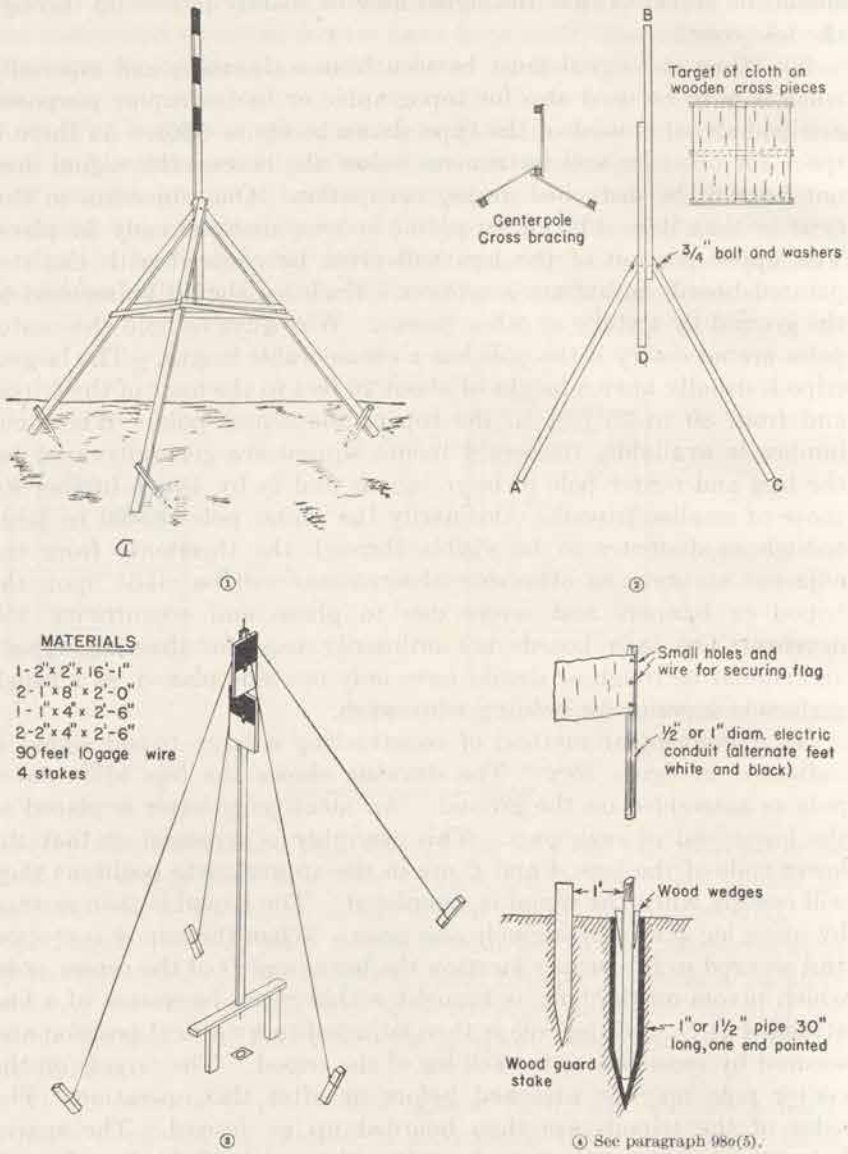


FIGURE 58.—Types of targets.

and nailed to them. The foot of the pole should have a spike driven at its center projecting about an inch, and when the pole is erected

this spike should be placed in a hole bored in the crosspiece of the bench directly over the station. Each set of guys should consist of four wires of No. 12 smooth galvanized wire. The number of sets depends upon the height of the pole. The pole is easily lowered when the station is occupied by loosening the guy or guys on only one side. The guys on the other three sides are not loosened from their anchors. To replace the pole it is only necessary to stand it up on the bench and fasten the loosened guy or guys on the one side. The centering of the pole or that part on which observations are made should be tested after the pole has thus been replaced, but it will usually be found that it has not been disturbed.

(6) While a target should be comparatively narrow at the part on which the pointings are made, something more conspicuous is desirable at longer distances or wherever the background may be unfavorable. A white flag might be satisfactory if the wind always exposed it. A flag attached to a bar of wood or metal at the top of the pole, and with a tack or two to the pole near the bottom, allowing the lower corners to flutter, is very good. The cloth should be slashed if the station is near a settlement. If the background makes a pole difficult to distinguish, a piece of white signal cloth hung behind the pole will make it easy to bisect, especially if the pole is dark in color.

h. Heliotropes and signal lamps.—(1) Either of these types has to be procured in advance, and requires a group of trained light tenders to set and keep them adjusted for good observation. The opaque targets as described above are best for daylight work if they are clearly visible.

(2) A heliotrope (fig. 59①) is an instrument to reflect the sun's rays by a mirror from the station sighted on to that occupied by the observer. The heliotrope should rarely be used as a signal for distances less than 20 miles, unless the atmosphere is very smoky or hazy, or because it is difficult to make an opaque signal visible in a dense wood; nor should it be used at a greater distance if an opaque signal can be seen. Opaque signals give relatively good definition, while the beam from the heliotrope is too large at short range and it is difficult to arrange satisfactory understanding between the observer and the distant heliotroper.

(3) An emergency heliotrope, shown in figure 59②, may be made by driving two nails vertically about 2 feet apart into a board, the heads of the nails to be used as sighting points for the beam of reflected sunlight. Place the board on the stand and aline the heads of the nails with the station of the observing party. Next fit a narrow strip of paper to the front side of the forward nail, the strip

projecting slightly above the nailhead. With a common mirror a few inches in diameter throw the reflected rays of the sun along the line of the nailheads. This will be accomplished when the shadow of the head of the rear nail falls on and exactly covers the head of the forward nail. The paper strip mentioned above enables one to make this exact contact. The center of the mirror should be held approximately in line with the nailheads to avoid eccentricity of the light shown the observer. If the direction to the observer is nearly opposite the direction to the sun, it may be necessary to use an auxiliary mirror to reflect the sun's rays onto the mirror which is in line with the nails.

(4) When daytime observation between two stations is impeded by dense smoke, fog, or haze, signaling at night gives the best results. Electric signal lamps operated on dry cells are now used by the United States Coast and Geodetic Survey almost exclusively. Aside from the electric connections, only two adjustments are needed for the electric lamp, one for focus and the other for the sighting devices. The focusing adjustment is made by the screw socket into which the bulb fits. Since it frequently differs for different bulbs of the same apparent size, it should be made each time a new bulb is used. It is best done by directing the light upon a flat surface, such as a tarpaulin about 100 feet or more away, and varying the adjustment until the brightest part of the disk is but little larger than the lens of the lamp. After this has been done the sighting device should be adjusted to point exactly above the center of the brightest part of the illuminated surface and as far above it as the sighting tube is above the center of the lens of the lamp. As the transportation of the lamp from station to station is apt to disturb both of these adjustments, they should be tested before the lamp is posted at a new station.

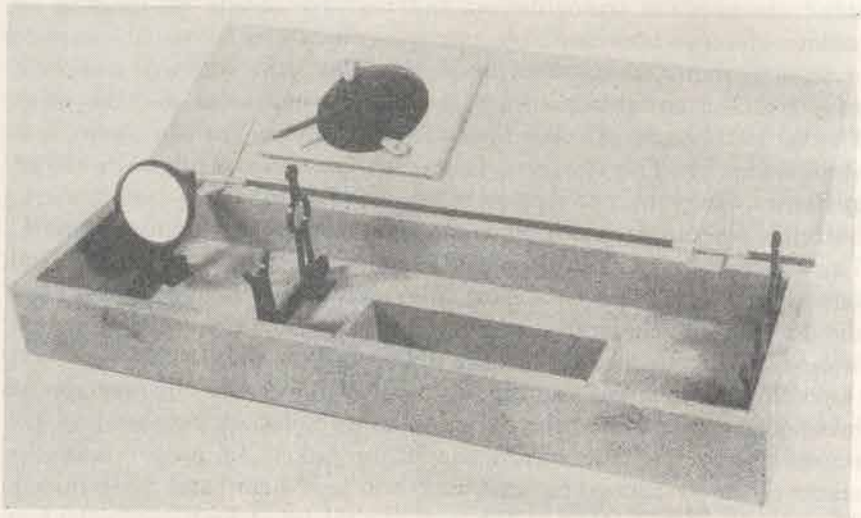
(5) For azimuth shots around a mile, a flashlight bulb, after removing the lens and reflector, is suitable. The bulb is pointed toward the instrument and secured while plumb over the mark. If the light seems too bright at the shorter distances, some partially exhausted cells should be employed. If it is pointed exactly toward the instrument, a flashlight with lens and reflector may be observed at distances considerably over a mile.

i. Searchlight stations.—A truly vertical searchlight beam makes a fairly good substitute for a highpole signal, and can be observed over 20 miles with a telescope of large objective. These beams are excellent for intersecting crossroads from high towers in level, heavily wooded country. They can be used to advantage in resection. Past results indicate that they will not yield close enough figure closures for advancing even third-order work. Close to the front lines, they

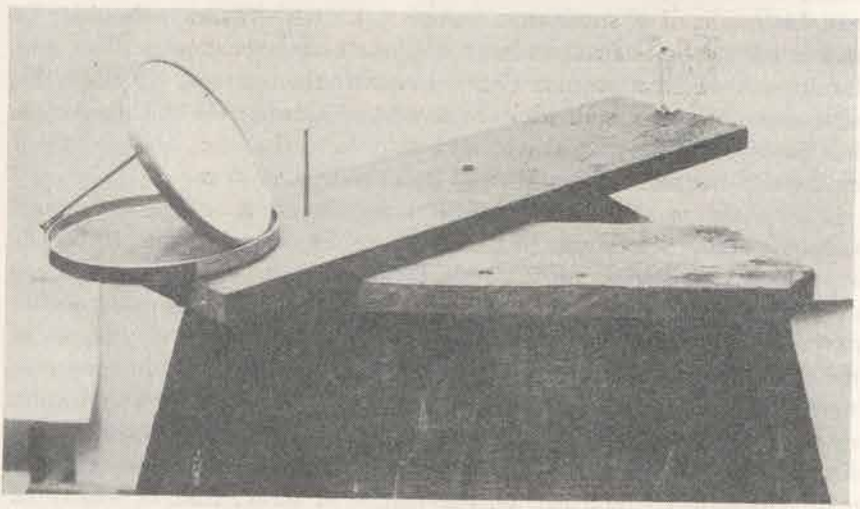
might afford the enemy the opportunity of obtaining intersection points in inaccessible territory.

j. Station marks.—(1) Stations of either first-, second-, or third-order accuracy on triangulation and traverse should be marked by tablets of some noncorrodible metal set firmly in posts of concrete or in large boulders or outcropping bedrock, but where a station is on a building suitable marks of a different character may be used. The tablets (see fig. 8) may be set in place by means of cement, sulphur, or lead. The Corps of Engineers survey control mark shown in figure 8 may be appropriately stenciled to serve as bench mark, triangulation or traverse station, or reference or azimuth mark. The concrete posts should be not less than 8 inches in diameter and should extend 30 inches below the surface of the ground. They should have the shape of a truncated prism or cone, in order that the lower end may be larger than the upper and thus better able to resist the lifting effect of frost action. For the same reason the post should be smoothly molded for the upper 10 or 12 inches of the part beneath the surface. Particular care should be taken to insure that the materials used in making the concrete are clean and well mixed. The top 12 or 15 inches of the post should be at least equivalent in strength to a 1–2–3 mixture of cement, sand, and stone; the base may be made of a somewhat leaner mixture. Where a boulder is used it should be at least as large as the concrete post prescribed and should extend to a similar depth beneath the surface. Where the tablet is to be set in bedrock care should be exercised in selecting rock that is of suitable durability, and also to make sure that what is apparently bedrock is not a small detached mass of rock.

(2) *Special marks.*—Under certain conditions special marks may be used. Where no larger boulder or bedrock is available at a station and where by its location it would be unduly expensive to construct a concrete mark, a metal pipe of suitable size and of noncorrodible material may be used. The base of this pipe should be so shaped as to resist extraction of the pipe and should preferably be set in concrete. In swamps a long metal pipe set inside a drain tile filled with hydraulic cement may be used. Where a station mark must be set on land subject to cultivation it is better to have the top of the post entirely below the depth which can be reached by a plow; that is, about 12 inches below the surface. Where a mark of this type is set it is necessary that measurements to the center of the roadways, section lines, blazed trees, corners of buildings, etc., be made in sufficient number to enable one seeking to recover the mark in the future to determine its location within a few feet. The mark itself can then be found by digging or by prodding with an iron rod.

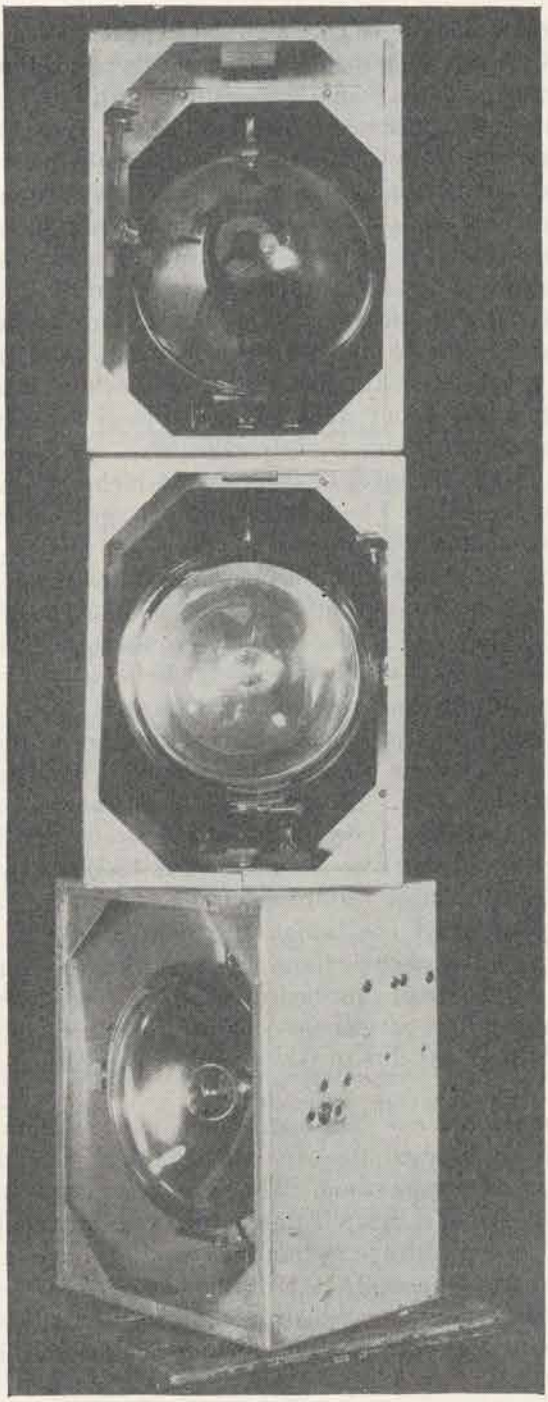


① Heliotrope, box type.



② Emergency heliotrope.

FIGURE 59.—Heliotropes and signal lamps.



③ Signal lamps.

FIGURE 59.—Heliotrope, and signal lamps—Continued.

(3) *Subsurface marks.*—Where a concrete post is used, a subsurface mark should be set if possible. This mark should preferably be made of concrete, not less than 6 inches thick and 10 inches in diameter, with the station point marked by a metal tablet, copper bolt, or other durable substance. The subsurface mark should be 4 or 5 inches below the base of the concrete post, and extreme care must be taken that the subsurface mark is directly underneath the surface mark.

(4) The procedure in making the standard concrete mark is as follows: A hole is dug to a depth of 3½ feet or more. It should be 16 inches in diameter for the first 2½ feet, and 10 inches in diameter at the lower end. Concrete made of good cement, sand, and gravel or broken rock is placed in the lower part of the hole to a depth of 6 inches. A standard tablet station mark is then set in the concrete, with the top of the tablet slightly depressed. This completes the underground mark. A layer of from 4 to 6 inches of sand or dirt is then put into the hole. The hole is then enlarged about 2 inches in radius near the bottom in order that the lower end of the block of concrete for the surface mark will be mushroomed, and then the hole is filled with concrete to within 9 inches of the surface of the ground. Next a mold or frame 12 inches on a side at the top, 13 inches at the bottom, and 12 inches in depth is set in the hole on top of the concrete and filled in around the outside with dirt tamped firmly. The frame is then filled with concrete level with its top and a standard tablet station mark is set in the center of the concrete, with the top of the tablet slightly depressed. The tablet must be centered exactly over the underground mark. The top of the concrete should be smoothed with a trowel and the frame should be left in place to protect the concrete until it becomes firmly set. Care must be taken not to disturb the position of the tablet in the underground mark when placing the layer of sand or dirt and when pouring the concrete for the surface mark. A piece of thin board should be placed over the lower mark or other suitable means used to insure against any horizontal movement of the tablet due to the impact or pressure of the material above.

(5) *Selection of names.*—The name of a triangulation or traverse station should be stamped upon the metal tablet, preferably before it is set into the concrete or stone. The name should not be duplicated within the confines of a State if duplication can be avoided. Names for triangulation and traverse stations should have a geographic significance wherever possible. Care should be taken by the chief of party to ascertain the name which is most prevalent for a particular geographic feature, for frequently a mountain or stream will have

different names in the same region. Station names should be short, for it is important that the same name be used for the same station throughout the records and computations.

(6) As an aid to a reconnaissance party in the recovery of established stations from the United States Coast and Geodetic Survey descriptions, their standard notes (extract from pages 112–114, Special Publication No. 145) on the marking of stations are given below. Their descriptions do not describe the marking used at a station but simply give the numbers of the standard notes which describe the station, underground, reference, and witness marks. The notes were made as general as possible in order that it might not be necessary in the field to describe small and unimportant variations.

DEPARTMENT OF COMMERCE

U. S. Coast and Geodetic Survey, 1929

"MANUAL OF SECOND AND THIRD ORDER TRIANGULATION AND TRAVERSE"

(If a later edition is available it should be used instead)

Surface marks

Note 1.—A standard disk triangulation station mark set in the top of (*a*) a square block or post of concrete, (*b*) a concrete cylinder, (*c*) an irregular mass of concrete.

Note 2.—A standard disk triangulation station mark wedged in a drill hole in outcropping bedrock (*a*) and surrounded by a triangle chiseled in the rock, (*b*) and surrounded by a circle chiseled in the rock, (*c*) at the intersection of two lines chiseled in the rock.

Note 3.—A standard disk triangulation station mark set in concrete in a depression in outcropping bedrock.

Note 4.—A standard disk triangulation station mark wedged in a drill hole in a bowlder.

Note 5.—A standard disk triangulation station mark set in concrete in a depression in a bowlder.

Note 6.—A standard disk triangulation station mark set in concrete at the center of the top of a tile (*a*) which is embedded in the ground, (*b*) which is surrounded by a mass of concrete, (*c*) which is fastened by means of concrete to the upper end of a long wooden pile driven into the marsh, (*d*) which is set in a block of concrete and projects from 12 to 20 inches above the block.

Underground marks

Note 7.—A block of concrete 3 feet below the ground containing at the center of its upper surface (*a*) a standard disk triangulation station mark, (*b*) a copper bolt projecting slightly above the concrete, (*c*) an iron nail with the point projecting above the concrete, (*d*) a glass bottle with the neck projecting a little above the concrete, (*e*) an earthenware jug with the mouth projecting a little above the concrete.

Note 8.—In bedrock (*a*) a standard disk triangulation station mark wedged in a drill hole, (*b*) a standard disk triangulation station mark set in concrete in a

depression, (*c*) a copper bolt set in cement in a drill hole or depression, (*d*) an iron spike set point up in cement in a drill hole or depression.

Note 9.—In a bowlder 3 feet below the ground (*a*) a standard disk triangulation station mark wedged in a drill hole, (*b*) a standard disk triangulation station mark set in concrete in a depression, (*c*) a copper bolt set with cement in a drill hole or depression, (*d*) an iron spike set with cement in a drill hole or depression.

Note 10.—Embedded in earth 3 feet below the surface of the ground (*a*) a bottle in an upright position, (*b*) an earthenware jug in an upright position, (*c*) a brick in a horizontal position with a drill hole in its upper surface.

Reference marks

Note 11.—A standard disk reference mark with the arrow pointing toward the station set at the center of the top of (*a*) a square block or post of concrete, (*b*) a concrete cylinder, (*c*) an irregular mass of concrete.

Note 12.—A standard disk reference mark with the arrow pointing toward the station (*a*) wedged in a drill hole in outcropping bedrock, (*b*) set in concrete in a depression in outcropping bedrock, (*c*) wedged in a drill hole in a bowlder, (*d*) set in concrete in a depression in a bowlder.

Note 13.—A standard disk reference mark with the arrow pointing toward the station, set in concrete at the center of the top of a tile (*a*) which is embedded in the ground, (*b*) which is surrounded by a mass of concrete, (*c*) which is fastened by means of concrete to the upper end of a long wooden pile driven into the marsh, (*d*) which is set in a block of concrete and projects from 12 to 20 inches above the block.

Note 14.—A conical mound of earth surrounded by a circular trench.

Note 15.—A tree marked with (*a*) a triangular blaze with a nail at the center and each apex of the triangle, (*b*) a square blaze with a nail at the center and each corner of the square, (*c*) a blaze with a standard disk reference mark set at its center into the tree.

k. Reference marks.—At least one and preferably two reference marks should be set at each station. They should be metal tablets set in concrete posts, boulders, or bedrock. The metal tablets used for the reference marks should have a different inscription upon them than the station tablets and preferably should bear an arrow pointing toward the station. Where more than one reference mark is used at a station, they should be stamped and numbered serially, clockwise as viewed from the station. Particular care should be taken in selecting sites for these reference marks where they will not be subject to disturbance. Fence lines or section lines are suitable sites. The azimuths and distances from the station mark are accurately determined. Witness marks are used primarily to recover the general locality of a station. They may be indefinite, such as a stream junction, fence corner, blaze on tree, etc. The distances and directions to witness marks need be only moderately accurate.

l. Azimuth marks.—At each station where a high or low tower is needed to enable the observer to see the adjacent stations of the scheme an azimuth mark should be established. This mark should

be placed over 1,000 feet away, at some point visible from an instrument mounted on an ordinary tripod set on the ground over the mark. An engineer recovering the station will then not only have a geographic position but will be able to obtain an azimuth as well. The azimuth mark should be of the same size and character as a reference mark. Azimuths should also be obtained to several other objects, such as water tanks, stacks, and church spires if they are in view from the ground.

m. Intersection stations (see par. 107*d*) are of two kinds—features of the terrain, such as towers, finials, flagpoles, sharp peaks, etc.; and ground stations, which have to be monumented. For the latter, a pole signal or flag has to be erected plumb over the station mark. The terrain features are preferable, as no mark or signal has to be constructed, and the object remains in view long after the triangulation party has moved on. It is essential that the records of observations on any intersection station from several occupied stations should have the name of the intersection station exactly the same. For example, in case of a church spire, the name and denomination of the church should be given. Either the reconnaissance or erection party should measure accurately and report the vertical distance from the top of an intersection station to the ground, as such stations need not be visited by the observation party.

n. Description of station.—(1) The description furnished by the reconnaissance party contains all information needed for construction such as route, exact location, height of signal, necessary clearing, etc. and also any items which might be useful during observation. The construction party adds the details and measurements of the station and other marks and pertinent items. The observation party checks the description, makes necessary changes, and adds the prescribed distances and directions to reference and azimuth marks, etc., using a main scheme station as the initial direction (par. 108). As soon as the above data have been obtained, form 4, Description of Station (fig. 60), is prepared, forwarded to, and filed at the headquarters of the organization making the survey. A copy should be retained for field use.

(2) The purpose of the description is to enable one who is unfamiliar with the locality to find the exact point determined as the station, and to know positively that he has found it. The essential information which should be included in a description is as follows:

(*a*) *Locality* (general and particular).

(*b*) *How marked.*

(*c*) *Distances* (by tape) and *directions* (by theodolite) from center of station to reference and witness marks.

(3) If a feature of the terrain is intersected, and the name is sufficiently descriptive (see *m* above), no further description need be furnished. However, the "Description of Station" should be filled out showing the elevation of the top and the elevation of the ground or station mark, if any.

(4) Where search is made for a triangulation or traverse station established in a previous year, a note to that effect must be entered on an appropriate form. If the station is found, the recovery note should state the condition in which the mark was found and should give any modifications or additions to the description which would make the station more easily found in the future. If the station is not found, the note should indicate the thoroughness of the search made and give recommendations as to whether or not the station should be marked "lost" in the records.

(5) *Report of recovery of any Federal survey mark.*—Member organizations of the Board of Surveys and Maps should instruct their field officers to report upon the condition of station and bench marks visited by them which were established by another member organization. If a properly equipped field party of a member organization finds in poor condition a Federal triangulation or traverse station mark of a third or higher order of accuracy and if its proper location can be determined with certainty and accuracy, either by a recovered underground mark or by measurements from two or more reference marks, the party should re-mark the station if practicable. If the tablet marking the original station is recovered, it should be reset. If an underground mark exists, due care must be exercised to insure that the new surface mark is exactly centered over the subsurface mark.

o. Grid triangulation, special methods.—(1) Those who are concerned with grid triangulation should be thoroughly familiar with the specifications and methods of executing triangulation of higher accuracy, which are designed for extending more precise control over considerable distances. Grid triangulation is intended only for the most rapid fulfilment of two urgent needs as follows:

(*a*) To establish horizontal and vertical control which will aid in the preparation or revision of battle maps. (See sec. XIX, TM 5–230.)

(*b*) To furnish control for the combat troops, more particularly for the Field Artillery, who have their own survey organizations, whose duties include the adaptation of the grid triangulation data to the needs of the firing batteries and the Field Artillery intelligence service. Echelons of the Engineer survey organization should cooperate with the Field Artillery whenever practicable without neglecting other assigned missions.

3rd Order

DESCRIPTION OF STATION

Station Name Daly
State Va.
County Howard
Locality Marlboro, Va.
Year 1940 Recovered
Chief of party 1st Lt. J. D. Bell, 66th Engrs
Observer T. Sgt J. Doe
Described by Cpl T. Cane
Marked by Cpl Q. Pine
Elevation of mark 248 ft. Trig Method
Height of tower 15 ft.

Distances and directions to reference marks and prominent objects which can be seen from the ground			
Object	Distance	Direction	Azimuth
△ Webb		00° 00' 00"	
R. M. No. 1	79.45 ft.	17 33 27	
R. M. No. 2	103.07 ft.	125 08 19	
Potter W. T.	5 miles	158 58 11.1	
A. M. No. 1	0.2 m.	337 57 10.8	

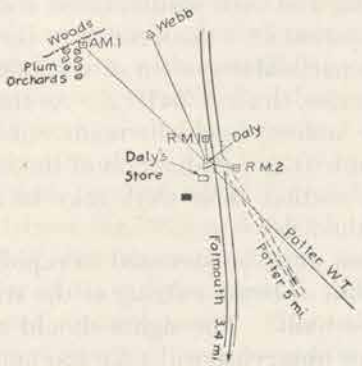
Detailed description From highway (No. 10) at Falmouth, 3.4 miles N. and W. on gravel road direct to Daly's Store opposite road junction. Station is monument 3" above ground 30 ft. N. of N.E. corner of store, 24 ft. west of 12" oak and 40 ft. W. of centerline of road.

R.M. No. 1 is N. of station, a monument 4" above ground, 15 ft. W. of centerline of road, and 15 ft. N.W. of culvert

R.M. No. 2 is E. of station, a monument at edge of field, 12 ft. E. of centerline of road running S.E., 4 ft. S. of S. row fence post, and 30 ft. S. of telephone pole

A.M. No. 1 is N.W. of station, a monument 6" above ground in edge of woods at N. corner of plum orchard, 1.3 ft. E. of 10" twin oak tree.

Potter W.T. is S. and E. of station, an aluminum painted, spheroidal bottom, tank near the village of Potter.



FORM 4

FIGURE 60.—Description of station.

262341°—40——12

(2) Reconnaissance for grid triangulation is a specialty requiring continual practice over varied terrain. The deliberate triangulator can reconnoiter the ground, select stations, and erect signals before he starts observing. The grid triangulators have to try to fix points in inaccessible territory and to prepare for the rapid extension of their system with the advancing forces. The solution of map and terrain problems without troops is helpful. Most valuable of all are practical exercises and maneuvers in which control schemes are actually worked out and rapidly extended as the combat units move forward. The survey troops should receive notice of such problems early enough to permit the collection of data and the establishment of stations in the concentration area. This provision is entirely justified, as reconnaissance proceeds continuously in actual operations.

(3) Some stations must be marked before observations are begun. Time permitting, the reconnaissance should be conducted with a plane table, rays being drawn from each station to several likely points where other stations might be located. As each station is marked, the lines to be observed from it are drawn in. If time is too short for the above procedure, the tentative positions of the stations are marked on a map or diagram, and the lines drawn in as they are found to be feasible. However brief the reconnaissance, each station to be occupied must be visited to make sure that the points to be observed from it are visible from the height of the instrument. In suitable terrain, several observers may accompany the reconnaissance and begin observing the first triangle as soon as a few stations have been selected. To avoid confusion each net or series of grid triangulation stations may be assigned a letter, and each station name composed of the sign for grid triangulation station (▽) the number of the organization, the letter designating that particular system of stations, and the number as established in that series, thus: ▽64D12. At the conclusion of the reconnaissance definite orders should be made out for the observers, as described in paragraph 113*e*. A diagram of the scheme is furnished the computing section so that their work may be started on receipt of the first data from the field.

(4) Grid triangulation must be executed so rapidly that clearing is limited to a small amount of brush-cutting at the station site, and not even low towers can be built. The sights should not average under 2 or 3 miles in length or observing will take too long. All considerations emphasize the importance of skill in reconnaissance. The simple triangles are easier to locate than any figure having more lines of sight, and other requirements are reduced in order to speed the progress. Occasionally some form of intersection or resection may

have to be used in advancing. In such cases a check must be made on other work in order to detect possible gross errors. Frequently the signal will be a flagged pole above a house or tree, necessitating an eccentric station for the instrument. The reduction to center (par. 109) is quite simple, provided the eccentricity is minimized, and the record shows its amount and direction so clearly that misinterpretation is impossible. Ideally, all stations should be located at picture points (see sec. XXVII), on the highest ground in the vicinity, convenient for Field Artillery use, but away from command or observation posts and defensive works, and should be in a position protected from traffic of any sort. While choosing the exact spot for the station mark, consideration should be given the utilization of any station mark already established in the vicinity. Confusion may thus be avoided, and a valuable tie to other work may be made. Two witness marks should be blazed, or witness stakes 3 inches out of ground should be set for each station and the direction and distance to nearest tenth of foot recorded.

(5) Concrete monuments will not be established by grid triangulation. A 3-inch stake, 30 inches long, may be used in an emergency. All station marks should be driven flush with the ground, no matter how sheltered, and a substantial guard stake, 6 inches out of ground, will be driven 1 foot away from the station. A 1-inch pipe, 30 inches long and pointed at one end, would be the most suitable station mark. If the troops disturb the driven pipes, 2-inch pipes with two ½-inch steel rods, each 1 foot long, inserted at right angles through holes drilled through the pipe near the bottoms will be more difficult to extract, if the earth is well tamped over the rods. The use of pipes is advised as a piece of 1- or ½-inch round electric conduit, with alternate feet painted black and white, with a small flag or red and white panel fastened to the top through small drilled holes, would make a good signal if the conduit were secured in a vertical position with several wooden wedges at the rim of the pipe (fig. 58④).

(6) Perhaps the best type of signal for grid triangulation would be a quadrupod or tripod (fig. 58① and ②) which permits observing at the station without disturbing the signal, but even a light frame would require lumber and a carrying party. A fairly high guyed pole might have to be used in some cases. Flag poles on buildings or in trees should be fastened in a truly vertical position. In grid triangulation all elevations are deduced from vertical angles. Any party erecting a signal or flag must measure and report the vertical distance to the nearest tenth of a foot from the ground to the top, and to any other part on which a vertical angle might be observed. Should a

signal have to be lowered during observation, the adjacent observers may point at the instrument if the recorder and others keep out of the way. Night observations are easily made and give good accuracy if the light tenders are well trained.

(7) Reconnaissance never ceases. Before the observations for one net are completed, preparations must be under way to move on. In war, the most forward stations have to be located out of enemy view, often so situated as to make forward extension impracticable. Larger signals should be prepared for several of the rear stations located on high ground so that they may be quickly erected as the advance begins, and resected on from the new stations which become available in the forward areas. One or two observers might remain in rear long enough to observe on the new signals and so simplify the computations and minimize the possibility of error.

(8) The Field Artillery will need new grid triangulation points as they move into new positions. Every effort must be made to press the fringe of advance intersection points further into inaccessible territory. If the advance should develop so rapidly that the grid triangulation lags, one or two observers may be assigned to carry an azimuth forward with the artillery, as it is highly important that they know grid azimuth on going into position. To assume an azimuth (par. 137) necessitates great delay and labor in converting all of the data to the correct grid azimuth. With the proper azimuth from the start, the coordinates may be corrected by simple addition or subtraction. Astronomic azimuths are quickly obtained, weather permitting, but unfavorable conditions have to be expected.

(9) The engineer survey troops should maintain liaison with similar organizations in their own sector and on the flanks. Frequent ties help to keep all of the engineer surveys on a common datum.

SECTION XVI

CHECKING METHODS—TRIANGULATION DATA

	Paragraph
General	99
Closing on measured or established bases	100
Triangle closures	101
Horizontal angles	102
Checks in grid triangulation	103

99. General.—*a.* Though triangulation is the preferred method of establishing control, there are sources of large and small errors, almost too numerous to mention, including—

(1) Base line not accurate.

(2) Basic data erroneous.
(3) Established station not truly recovered.
(4) Poorly conditioned figures.
(5) Too many figures between checks.
(6) Signal not centered over station mark.
(7) Instrument not plumbed over mark.
(8) Poor condition of instrument.
(9) Rough handling of instrument.
(10) Instability of instrument support.
(11) Too much sun or wind on instrument.
(12) Instrument unsuited for program of observation.
(13) Parallax of cross hairs.
(14) Plate levels out of adjustment.
(15) Failure to turn tangent screws against the springs.
(16) Too much time between pointings.
(17) Too few observations for the accuracy sought.
(18) Poor visibility.
(19) Observing on false station.
(20) Careless bisection of target.
(21) Lateral refraction.
(22) Crooked signal poles.
(23) "Phase" of target.
(24) Errors in reading micrometers or verniers.
(25) Partiality to previous readings or to expected results.
(26) Mistakes in recording.
(27) Unjustifiable alteration of notes.

b. Remedies for many of the above items are obvious. Some of them are governed by the order of accuracy which is prescribed. Others are difficult to detect or recognize even when the results are materially affected. Increasing the number of observations might reduce the effect of some conditions but could not correct an error in centering over the mark, or faulty bisection of the target due to the imperfect construction or design of the latter, etc. Prevention of error, resulting from constant care and attention to detail, makes possible maximum speed in obtaining triangulation data of the required accuracy with the minimum of men, material, and equipment. Strict compliance with instructions, the employment of standard methods, and the formation of orderly habits in every procedure will limit the amount of work which has to be repeated. The persistent elimination of conditions likely to cause errors will make their occurrence so rare that the causes can be isolated and remedied as soon as the effects become apparent.

100. Closing on measured or established bases.—The principal criterion in classifying the accuracy of a triangulation system is that the discrepancy between the measured length of a base line and its length as computed through the scheme from the next preceding base, after the usual adjustment of the angles and sides, shall not exceed the small ratios given in paragraph 92*a* for each order of accuracy. The word "base" implies either a base line measured to the proper order of accuracy as described in section XVII, or a monumented line of previously adjusted triangulation of an equal or higher order of accuracy.

101. Triangle closures.—Coupled with the above gage of the length agreement on bases and almost coordinate in importance are the requirements limiting the error of angle measurements, for the limits imposed on angular errors serve to maintain a uniform accuracy along the chain of triangles. The triangle closures allowed for each of the orders in paragraphs 92*a* are the amounts the sums of the three angles (mean values after deduction of spherical excess, if any) may differ from 180°. As soon as observations have been completed at the three corners of a triangle, this check is applied. The difference should usually be much less than the maximum so that the average of all triangle closures in the scheme will be within the prescribed limit. In order to maintain the required accuracy through an extensive system, the specifications make use of other criteria, such as the number and strength of the geometrical figures between adjacent bases, the observation of an astronomic azimuth at specified intervals, and the accuracy of measurement of base lines. In the triangulation generally executed by the Army, keeping the distance angles over 30° and securing average triangle closures within the prescribed allowance of error should bring about the requisite agreement of the computed lines with the measured bases, or lines previously established. Thus it is that triangle closure is the first and only reliable check which can be applied during the observations. Any unusual differences, or upward trend of the average error of closure, should be at once investigated and corrective measures applied. To insure that the necessary accuracy is maintained throughout the triangulation, it is essential to give careful consideration to the instrumental equipment and the methods of observing, in order that the systematic and accidental errors may be kept within the prescribed limits and that no part of the system will exhibit undue weakness.

102. Horizontal angles.—*a. Agreement.*—The close agreement of different measurements of an angle is not a check on the triangulation. In the direction method (par. 107*b*) any readings which give values materially different from the other values for the same line indicate

errors in pointing or reading, but uniform results are no proof that the system will finally close. With the repetition method (par. 107*c*) close agreement between the first angle and the mean of the set, between the two halves of the set, or between sets of the same angle is no assurance that the triangles adjacent to the station will have good closures. Even the horizon closure, the difference of the sum of all the mean angles at a station from the correct 360°, is no check to prove that the angles are correct. A difference more than that allowed for a single triangle might indicate some sort of error, but an exact horizon closure of 360° might be made without a single angle's being close to its correct value.

b. Astronomical azimuth checks.—These should be made between bases of extensive systems in order to detect any "swing" or deviation from true aximuth. See paragraph 92*a* for the allowable probable error of the check azimuth. The observations for second and third order are usually made on Polaris at any hour (par. 177), making about 50 percent more observations than would ordinarily be taken on triangulation with the same class of instrument. The observed value is not corrected for the deflection of the vertical before being used in the adjustment of the triangulation. In regions with no high mountains and no great differences in the density of the nearby subsurface geological structures, the error in the observed azimuth due to the deviation of the plumb line will not usually exceed 1 or 2 seconds of arc, but in other regions may reach several times that amount.

103. Checks in grid triangulation.—Under peace conditions such checks are similar to those for the higher orders, but with the larger allowances given in paragraph 92*a*. Such practice is good training, as the relative speed and accuracy of the various methods may be tested by frequent closures on more accurate work. In maneuver and war, the work must be pressed forward so rapidly that little time can be spent on checks. Usually the grid triangulation will be somewhat ahead of the main triangulation, and the data must be published as soon as obtained. Circumstances will seldom be favorable for check base measurement, even by the short base methods. Advancing through a double chain of triangles, or ties with colateral chains might afford a check or merely show a disagreement. During lulls in the advance, and whenever personnel can be spared from more urgent work, provision must be made for checks. Every opportunity must be made for securing intersections on conspicuous features on high ground everywhere they may be found, in inaccessible territory, on both flanks and in the rear. Especially favorable points which are

out of enemy view should be provided with better signals than are needed for the immediate purpose. If all of the observers are trained to observe on and describe such prominent marks as can be identified, checks by resection and intersection should be obtained at such intervals that check base measurement becomes unnecessary.

SECTION XVII

BASE LINES

Paragraph
Types of base lines 104
Base lines with invar tapes 105
Base lines with steel tapes 106

104. Types of base lines.—*a. General.*—The measurement of an accurate base may occasionally become necessary. Base lines for triangulation of the first and second orders should be measured with a probable error (see par. 105*d*(12)) not greater than one part in 500,000, instead of 1,000,000 as shown in paragraph 92*a*. For military purposes, the gain in accuracy is not worth the additional time spent in securing the lower limit. Special Publication No. 145 of the United States Coast and Geodetic Survey gives complete details and some special tables, which are not included in TM 5–236.

b. Base lines with invar tapes.—The same methods are used in measuring second- and third-order bases, and the precautions against error are the same. The only differences are in the permissible errors. The allowances mentioned in paragraph 105 are for third-order accuracy. Invar tapes ("invar" from invariable) are made of an alloy of nickel and steel, characterized by a coefficient of expansion which is one-tenth or less that of steel. This low coefficient is of great advantage as temperatures of the tape are difficult to obtain exactly. If steel tapes have to be used for this class of work, the measurements should be made either at night, or when the sky is heavily overcast. An invar tape should not be reeled or unreeled rapidly or under a heavy tension, or wound upon a reel having a small diameter, or dragged over the ground, or shaken violently, or stretched beyond its yielding point, nor should it be subjected to sudden large changes in temperature. When reeling an invar tape, the marked side should be inside or outside, whichever it was originally. Kinking a tape may render it useless as a precise measuring instrument for whether the tape is allowed to remain kinked or is straightened, its effective length will be different from its previous standardized length. In fact, invar tapes require such good care that they are not used for third-order traverses and such purposes.

c. The base lines with steel tapes (par. 106) are of much higher probable error, as the special equipment employed with the invar tapes will not be available. However these steel tape bases are intended only for grid triangulation.

105. Base lines with invar tapes (third order).—*a. Apparatus.*—(1) The following instruments and appliances will be useful:

2 awls, marking.
2 dividers, pairs.
1 level, with rod.
2 plumb bobs.
2 range poles.
2 scales, one-tenth meter, boxwood, reading to millimeters.
1 stretcher apparatus for tape, complete, consisting of two staves with loops and tape attaching clips, two balances, and an apparatus for testing balances.
Strips, copper, for stake tops, of same thickness as tape, 20 per kilometer.
1 tape, steel, 30-meter (standardized or tested in field), for measuring set-ups and set-backs.
1 tape, steel or invar, 50-meter unstandardized, for marking out base.
3 tapes, invar, 50-meter, standardized.
1 theodolite.
3 thermometers, backed, for tapes.

Tools for staking line, as follows: hammer, hatchet, hand saw, 2 mauls, 2 machetes, 10d nails, posts and stakes, and movable tripods or chaining bucks if ground is so rocky that stakes cannot be driven.

(2) All new invar tapes should be calibrated at the United States Bureau of Standards, and a copy of the certificate should be issued with the tape. As the length of an invar tape seems to change in course of time, one of them should be sent each year to the Bureau of Standards for recalibration. The Bureau certificate states the absolute length of the tape under standard conditions of temperature, tension, and support, in addition to the coefficient of expansion per degree centigrade, and the weight in grams per meter. Great care must be exercised to use a tape under the conditions of tension and support for which its standardization equations are given. One of the three tapes is used exclusively as a reference tape with which the two field tapes are compared both before and after measuring each base (*d*(3) and (4) below). The base should be measured in sections about 1 kilometer in length, with the exception of one section which may be longer or shorter than this. Each section should be measured at least twice, one measurement being made forward, and the other backward. If the discrepancy in millimeters between two measurements of a section exceeds $25\sqrt{K}$ (where K is the length of the section in kilometers), additional measurements of that section should be made until two values made in

opposite directions are secured which agree within this limit. The two field tapes will be alternated in use.

(3) *Tape-stretching apparatus* is shown in figure 61. It consists essentially of two staves of steel tubing, pointed at the bottom and with wooden tops. A loose-fitting leather loop with an attachment to receive the looped end of the tape slips over the staff used at the rear end of the tape. The leather loop can be easily slipped up and down on the staff according to the height of the rear stake. A frame for holding the spring balance is attached to the forward staff by means of a spring friction grip. The tape is fastened to the balance by the attachment shown. There is a finger bar by which the forward contact man carries the end of the tape when moving forward.

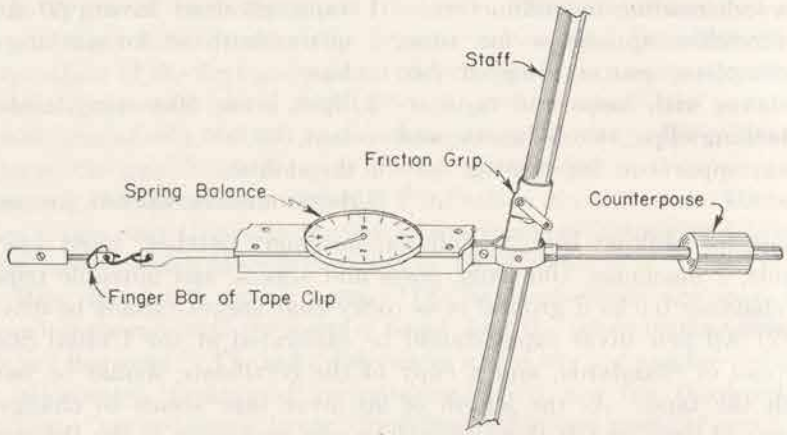


FIGURE 61.—Tape stretcher and spring balance.

(4) The *spring balance* is also shown in figure 61. The counterpoise shown in the figure may be so adjusted as to prevent any drag of the frame of the balance on the drawbar when tension is applied. Errors in base measurements have frequently been due to spring balances which do not read correctly. For this reason a testing apparatus for the spring balances is sent with the balances. This should be used in testing before and after each day's work. The form of tester is usually a weight which is applied to the balance held vertically, after which the pointer is adjusted exactly to 15 kilograms. When this is done the balance will indicate true tensions when it is used in a horizontal position. In other words, the weight is of 15 kilograms mass, as weighed by the spring of the balance at the Bureau of Standards, minus the weight of the drawbar and other movable parts of balance below the spring. The most common injury to a spring balance used on base measurement results from the tension being suddenly released,

allowing the drawbar to snap back. This may change the position of the dial pointer by several hundred grams and even result in injury to the spring. For this reason the tension on the tape should always be released gradually. If through accident the drawbar of the balance is allowed to snap back, the balance should be tested before measuring is resumed.

(5) *Thermometers.*—These are special and rather expensive thermometers, correct to within 0.5° C., within the ordinary range of temperature. They are tested at the Bureau of Standards before being sent to the field. Field computations need not take into account the graduation errors of the thermometers found by the standardization. The glass tube is supported in a metal holder to prevent the breaking of the tube when the tape is being handled or flexed. During standardization a thermometer of the type described above is fastened at each end of the tape, at a point 1 meter toward the center from the terminal mark, the distance being measured from the mark to the nearer end of the thermometer. On base measurement the thermometers should always be fastened in this same position by narrow bands of adhesive tape. When moving the tape, care will be taken to keep these thermometers on the upper side and thereby avoid getting any twist in the tape.

b. Preparation of base.—(1) The first step is to clear the base line so that the ends will be intervisible and the work of making the measurements will not be impeded. All grass, weeds, and brush should be cut close to the ground so that there is no possibility of any touching the tape. Set up a transit at one end of the base and arrange pole at the other; also set intermediate range poles if necessary. Now drive 4- by 4-inch posts on the line accurately spaced 50 meters between centers. Before staking is begun, the extra tape used for spacing the posts should be compared with a standard tape, so that intervals may be correct and frequent *set-ups* or *set-backs* avoided. If a steel tape is used, it may save time to make an approximate allowance for the change in length due to temperature. The 25-meter point of the tape may be marked by a strip of adhesive tape to make it readily recognizable while staking. The tops of the posts should be not less than 10 inches nor more than 24 inches above the surface of the ground. Each post should be driven to a firm bearing. Halfway between posts drive a 2- by 4-inch stake offset from the line of sight so that the face of the stake nearest the line of sight will be about 1 inch from it. All stakes should be offset on the same side of the line of sight. Sighting along the tops of each two of the 50-meter posts, locate and drive a tenpenny nail horizontally into the face of the

intermediate stake so that the nail will be on line with the tops of the posts. These nails serve as intermediate supports between the 50-meter posts during measurements. The tape is brought into position with its rear mark in contact with the mark on the base station or on the bench above it, and the proper tension is applied, with the edge of the tape touching the final alining mark on the forward stake. A copper strip is then nailed in position flush alongside the tape on the alining side, and at the same time a pencil mark is made on the top of the stake opposite the forward mark on the tape. This mark is used as a rear contact mark in setting the stakes for the next tape length. The tape is then moved forward and the process repeated. All posts at kilometer ends should be braced. If for any reason the intermediate support between the tape ends cannot be placed so it will be

FIGURE 62.—Chaining bucks.

down to grade with the terminal posts, it should be numbered with colored chalk and marked with a piece of cloth so it may be noticed and touched upon by the levelman. All stakes are numbered with colored crayon as they are driven, and intermediate supports above grade should be given a fractional number. An intermediate support which is not on grade makes what is known as a broken grade. At ravines or streams it is frequently necessary to place a stake on the edge of the bank and begin a tape length from that point. If it is less than half a tape length from the previous tape-end post, the number of which, for instance, is 34, the fractional-length post should be numbered "34 set-up", the next terminal post being numbered 35. If the fractional-length post is more than half a tape length from the next preceding tape-end post, a 2- by 4-inch stake should be driven in at the half-tape mark and be given the number 34½, and the post on the edge of the obstruction should be numbered "34½ set-up." By

following this system there will be no uncertainty in the interpretation of records. Where the ground is so rocky that stakes cannot be driven, "chaining bucks" may have to be utilized. See figure 62.

(2) *Base and monuments.*—The concrete monuments at the ends of the base have their copper hubs hair-lined in the direction of the base and at right angles thereto, accurately marking the base-end points. If the permanent mark at an angle station is low, a bench should be built over it and a scratch made on a copper strip fastened to the bench directly over the station mark to which the measurements may be made.

c. Procedure for measurement.—(1) When prepared to begin the actual measurement of the base, check the alinement of the stakes by means of the engineer transit and range poles. If any of the stakes are out of line, aline them before proceeding further. Next a line of levels should be run over the line to determine the relative elevations of all 50-meter posts and broken grade stakes. (See fig. 64.)

(2) *Organization of party.*—Six men are ordinarily required, as follows:

Front contact man.
Rear contact man.
Recorder.
Front stretcher man.
Rear stretcher man.
Middle man.

The chief of party makes the forward contact unless there is another experienced front contact man, in which case the chief may act as recorder. If any of the men are inexperienced, it is better to measure 1 or 2 practice kilometers before the recorded measurements are begun.

(3) *Method of measuring.*—(*a*) The middle man carries the middle of the tape high off the ground when moving forward, places the tape on the middle support when the tension is to be applied, sees that the tape is not in contact with weeds, brush, or other obstructions, notifies the recorder of all middle supports marked "broken grade," carries and places the tape so that there is no twist in it, and each time makes sure that the middle support is not more than 10 centimeters distant from the middle mark on the tape. If a nail is used as a middle support for the tape, he must rapidly and lightly tap the under side of the tape near the support with a stick somewhat larger than a pencil until the front contact man calls, "Ready," in order to lessen the friction over the nail.

(*b*) The *rear stretcher man* (fig. 63) with the rear tape stretcher holds the tape in position during the time the tension is on, so that the rear terminal mark on the tape is opposite to, or slightly forward of, the mark on the copper strip on the rear stake. As he comes up to the rear stake he must place the rear staff firmly in the ground at the proper distance back of the rear stake directly in line with the stakes, and at the same time he must slip the leather loop bearing the tape link to the proper height on the staff so that when the full tension is applied the tape will be a few millimeters above the top of the stake. The tape must not drag over the top of the stake at any time. In order to maintain a steady position of the staff, the rear stretcher man should have the top of the staff back of one of his shoulders, his body being forward of the staff. As soon as the front

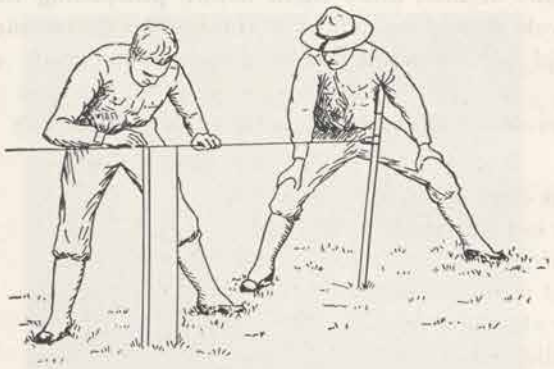


FIGURE 63.—Making contact with tape, rear end.

contact man calls, "Mark," and the thermometers are read, the tension is slackened off. The tape is then carried forward without being detached from the rear stretcher, the rear stretcher man maintaining just enough tension on the tape to keep it from touching the ground.

(*c*) The *front stretcher man* applies the proper tension to the tape as measured by the spring balance attached to the front stretcher. In moving forward he carries the front stretcher and balance detached from the tape. As the tape is brought forward into position the front stretcher man with one hand holds the balance out horizontally with the hook in such position that the tape can be quickly attached. As the tape is attached he places the staff in line with the stakes at the proper distance from the front stake and applies the tension smoothly, rapidly at first, but gradually more slowly as the 15-kilogram point is neared. Jerking motions must be avoided as they may injure the balance or tape. With the staff held in the same manner as described

for the rear stretcher man, and with one hand steadying the balance so the drawbar swings free, the front stretcher man can quickly bring the tape into equilibrium under the proper tension, at which time he informs the front contact man that the tension is correct. When under tension the tape must just clear the top of the forward stake and must not drag over it, otherwise the full tension will not be transmitted throughout the tape. Care must be taken that the stretcher staff is moved in the vertical plane through the stakes, for otherwise the balance will be twisted and friction exerted on the drawbar. The tension must be kept constant at 15 kilograms and watched closely, for if the dial pointer indicates more than 100 grams from 15 kilograms when the front contact man calls, "Mark," the front stretcher man should immediately tell the front contact man in order that the marking may be repeated. If the tension is satisfactory at the call of "mark", the tension is quickly but smoothly slackened off while the front contact man is reading the forward thermometer, the balance is held out for the detachment of the tape by the front contact man in the same manner as for its attachment, and the advance begun to the next position.

(*d*) The *recorder* should be an experienced man. If he is not, the chief of party must frequently inspect the record, especially where broken grades or set-ups and set-backs are to be recorded. It is the duty of the recorder to be sure that no blunders are committed, such as the dropping or adding of a tape length, or recording a half tape length as a full one, or a set-up as a set-back. He should check the chalked numbers on each stake as it is reached and make all notes necessary to a definite and correct interpretation of the record.

(*e*) The *rear contact man* (fig. 63) makes the rear contact and reads the rear thermometer. As the tape is brought up to a new position he steadies the tape as the rear staff is being placed in position and the tension applied, taking care that the tape does not drag over the rear stake. As the tension is applied he advises the rear stretcher man whether to ease off or take up on the tape. Standing directly opposite the mark on the copper strip nailed to the top of the stake, with one hand he firmly grasps the tape between the staff and the mark and with the other hand he lightly touches the tape on the opposite side of the stake to steady it. He can then flex the tape with his rearward hand and bring exactly opposite each other the marks on the tape and copper strip. Before flexing the tape it is necessary, of course, that the mark on it shall be slightly forward of the mark on the strip. The marks are thus held in coincidence until the front contact man calls, "Ready," when the rear contact man calls, "Right," and the front

contact man answers, "Mark," denoting the completion of the marking of the tape length. Immediately following the call, "Mark," the rear contact man reads the rear thermometer and the tape moves forward to the next position, unless someone at the forward end of the tape calls, "Hold," or "Tension."

(*f*) The front contact man must decide when all the conditions which affect the tape as a measuring unit are complied with, satisfying himself that the tape is in proper equilibrium under proper tension and support before he makes the forward mark. He also carries the forward end of the tape when moving forward and thus to a large extent sets the pace for the entire operation. The sequence of his movements during the measurement of a single tape length is as follows: As the forward stake is reached he lowers the front end of the tape from its position above his shoulder and attaches the link to the hook of the balance, grasping and guiding the hook to make sure that the attachment is made with a single movement. He then steps quickly back to a position alongside the forward stake, where he steadies the tape into its proper position just clear of the top of the stake, alongside the copper strip and between him and the strip. As the tension is perfected and the tape approaches equilibrium he places the point of the sharp, symmetrically pointed awl on the edge of the copper strip next to the tape and keeps it opposite the terminal mark on the tape until he is satisfied that conditions are right. After glancing at the balance to check the tension and down the tape to check the alignment he calls, "Ready," as above described. When he hears the response, "Right," from the rear contact man, he marks the copper strip with the awl, calling, "Mark," as the marking is completed. In making the mark several precautions must be taken. The awl must be very sharp; it should at no time touch the tape in the region of the terminal mark; the eye of the man making the forward contact, the terminal mark of the tape, and the axis of the awl should be kept in approximately the same vertical plane; and the mark should be made by the contact man's moving the awl away from him in order to keep constant any error due to parallax. The mark should begin at the very edge of the copper strip in order to make it easier for the rear contact man to make contact. Immediately following his call of "Mark" the front contact man reads the forward thermometer as the tension is released, then detaches the front tape link from the balance as he starts to pull the tape forward to the next position. If the tape is always carried in the hand which is toward the rear when making the contact, there is no danger of causing a half twist in the tape, for when detaching the tape from the balance

the finger clip is caught in the rear hand, fingers pointing down, then the end of the tape is elevated above the shoulder without changing the grip, and at the next position is again attached with the hands in the same relative position.

(*g*) Sometimes it will happen that when the tape is advanced 50 meters its forward end will fall short of the next metal strip. When this occurs, make a scratch on the rear metal strip as close as possible to its forward end and make the rear contact there. Then measure with a pair of dividers and millimeter scale the distance between the two scratches on the rear metal strip and add this to the length of the base line. This operation is known as setting forward. The opposite condition obtains when the forward graduation overshoots its post when a set-back becomes necessary. The set-backs and set-ups should be recorded in special columns and their algebraic sum (set-backs are minus) should be added to the length of the base line. Small set-ups and set-backs should be measured with an error not greater than 0.1 or 0.2 millimeter. As an additional safeguard the recorder may check the measurement of set-ups and set-backs and have the contact man in turn check his entry in the record book.

(*h*) Upon arriving at the end of the base, if the distance from the last 50-meter stake is less than 25 meters, measure this distance by means of the steel tape. If the distance is greater than 25 meters, proceed as follows: Determine approximately the 25-meter point on the 50-meter tape and mark it on the tape; place a stake approximately 25 meters from the last 50-meter stake, aline it, and nail a copper strip on it. Now stretch the 50-meter tape and mark on the strip the point where this approximate 25-meter point comes. Then reverse the tape end for end and mark where the 25-meter point comes again. If the distance between the two approximate 25-meter points is then bisected, the resulting point will be 25 meters from the last 50-meter stake regardless of the accuracy with which the 50-meter tape was bisected. The distance forward from this 25-meter point to the end of the base may then be measured with the steel tape. Set-backs of more than 10 centimeters should never be used. This method will avoid in large measure the possibility of a large error in length due to an error in the sign of a partial tape length distance on both measurements. Some base tapes have marks at approximately 5-meter intervals along the tapes, the lengths of the intermediate intervals having been determined with secondary accuracy. Where such tapes are available long set-ups are sometimes so staked as to enable the invar tape to be used for the greater part of the set-up—that is, for some multiple of 5 meters, the remainder of the set-up being measured with a standardized steel tape.

(*i*) In order to avoid confusion, the marks placed on the copper strips during the second measurement are usually distinguished from the first markings by a bar across the scratch, while a third measurement would have a second bar. On the first measurement of each kilometer section the end mark of the section is given a distinguishing mark on the copper strip. On the second measurement, as the stake marking the end of each section is reached, a set-up or set-back is taken to the original section mark on the strip, and the measurement

LEVELING FOR BASE LINE
From ΔE.Base to T.E.20
Obs: Sgt. R. Roe
Rec: Cpl. T. Crane
Inst: Berger Dumpy #8923
July 5, 1924 - 2 P.M.
Clear and hot
Gentle breeze
(2)

POST	B.S. Meters	F.S. Meters	DIFF. Meters	DIFF. Feet	Back- B.S. Feet	ward F.S. Feet	DIFF. Feet	DISTANCE Meters	MEAN DIFF. Feet	INCLIN. CORR. mm.	ELEV. Feet
ΔE.B	2.754				9.035						779.76
ΔE.B SET-UP		2.438	+ 0.316	+ 1.037		7.999	+ 1.036	20	+ 1.036	2.5	
1		2.004	+ 0.434	+ 1.424		6.575	+ 1.424	50	+ 1.424	1.8	
2		1.937	+ 0.067	+ 0.220		6.354	+ 0.221	50	+ 0.220	0.0	
3		1.540	+ 0.397	+ 1.302		5.052	+ 1.302	50	+ 1.302	1.6	
4		1.309	+ 0.231	+ 0.758		4.290	+ 0.762	50	+ 0.760	0.5	
5	2.511	1.110	+ 0.199	+ 0.653	8.270	3.648	+ 0.642	50	+ 0.648	0.4	
6		2.438	+ 0.073	+ 0.240		8.005	+ 0.265	50	+ 0.252	0.1	785.40
6½ (B.G)		2.440	- 0.002	- 0.007		8.005	+ 0.000	25	- 0.004	0.0	
7	0.471	2.779	- 0.339	- 1.112	1.550	9.121	- 1.116	25	- 1.114	2.3	
8		0.799	- 0.328	- 1.076		2.625	- 1.075	50	- 1.076	1.1	
9		0.976	- 0.177	- 0.581		3.202	- 0.577	50	- 0.579	0.3	
10		1.273	- 0.297	- 0.974		4.176	- 0.974	50	- 0.974	0.9	
10 SET-UP		1.300	- 0.027	- 0.089		4.265	- 0.089	4.6619	- 0.089	0.1	
11	1.647	1.738	- 0.438	- 1.437	5.404	5.712	- 1.447	50	- 1.442	1.9	
12		1.962	- 0.315	- 1.033		6.437	- 1.033	50	- 1.033	1.0	779.09
13		1.437	+ 0.525	+ 1.722		4.714	+ 1.723	50	+ 1.722	2.8	
14		0.537	+ 0.900	+ 2.953		1.765	+ 2.949	50	+ 2.951	8.1	
15		1.734	- 1.197	- 3.927		5.692	- 3.927	50	- 3.927	14.3	
16	1.247	1.469	+ 0.265	+ 0.869	4.100	4.820	+ 0.872	50	+ 0.871	0.7	
17		1.837	- 0.590	- 1.936		6.031	- 1.931	50	- 1.933	3.5	
18		2.004	- 0.167	- 0.548		6.575	- 0.544	50	- 0.546	0.3	
19		2.540	- 0.536	- 1.759		8.330	- 1.755	50	- 1.757	2.9	
20		2.486	+ 0.054	+ 0.177		8.149	+ 0.181	50	+ 0.179	0.0	776.65

Checked by A.X.Ba. Computed by J.D. Checked by L.Y. Mean Elevation = 47.1 + 780.2

FIGURE 64.—Leveling for base line.

of the next section is begun at the section mark. By this method a comparison is obtained between the forward and backward measures of a section.

(4) *Record data* will be recorded in a separate field notebook for each base line measured. One double page or more will be devoted to each of the following:

Leveling for base line (fig. 64).
For each kilometer section.
Comparison with reference tape (fig. 65).
For each field tape before measurement.
For each field tape after measurement.

Measurement of base line.
For each kilometer section, forward.
For each kilometer section, backward.

A base of 3 miles (about 5 kilometers) would require a total of 19 pages if no work had to be repeated.

(*a*) *Leveling for base line.*—The sample form (fig. 64) shows the level record when using a rod graduated in meters on one side and feet on the other. When such a rod is used the levels are run in only

COMPARISON WITH REFERENCE TAPE
Front Contact: T Sgt J. Doe
Rear Contact: Sgt R Roe
Field Tape: # 2916 Invar
Ref. Tape: # 3861 Invar
(15)
Cloudy and still
March 1940
Before 1st measure
21°C
2.04 mm
20°C
F.T. # 2916
R.T. # 3861
21°C
2.92 mm
21°C

1.8	2.8
2.4	3.2
2.4	3.4
2.0	2.6
1.6	2.6
5)10.2	5)14.6
2.04	2.92

F.T. @ 20.5°C
0.88 mm
longer than
R.T. @ 21.0°C

Computed J. D.
Checked R. R.

FIGURE 65.—Comparison of notes with reference tape.

one direction, but both sides of the rod are read at each rod point, the reading in meters being recorded as the forward running and the reading in feet as the backward running. If the rod used is graduated on only one side, forward and backward running is necessary. The numbering of the stakes in this record should correspond to the numbering on the "measurement of base line" form (fig. 66). Extreme care should be taken to get readings on all broken grades and partial tape lengths, and these should be plainly indicated in the record. All columns must be marked meters or feet, whichever is applicable.

(*b*) Before measurement is begun, a pair of posts on especially favorable ground are selected and braced for utilization in all comparison of the field tapes with the reference tape. To avoid possible confusion, the metal strips on these two posts will have to be renewed from time to time. The mean of five separate measurements should be obtained by taking set-ups or set-backs of about a centimeter each. (See fig. 65.)

(*c*) *Measurement of base line* (fig. 66).—In recording the tape measures two thermometer readings indicate a full 50-meter tape length and one thermometer reading a half tape length or a set-up. Each half tape length or large set-up should be recorded on a separate

MEASUREMENT OF BASE LINE
From Δ E.Base to Station T.E. 20 Forward

FRONT CONTACT: T SGT. J. DOE
REAR CONTACT: SGT. R. ROE
HEAD TAPE: SGT. J. ADAM
REAR TAPE: CPL. T. CANE
RECORDER: SGT. L. VANE

JULY 7, 1924
CLOUDY, NO WIND.
TAPE No. 922
BALANCE No. 302
THERMOMETERS No 51818 & 33260
(12)

Section From		Section To		Temp. °C Forward	Rear	Set-up meters	Set-back meters	Tape Supports		Remarks	
Δ E.B.		Δ E.B.	set-up	20.0	\	20.0000		2	0-20		8.10 A.M.
Δ E.B.	set-up	1		21.3	21.0			3			
1		2		21.2	21.0			3			
2		3		20.6	20.0	0.0714		3			
3		4		21.2	21.0			3			
4		5		21.0	20.5			3			
5		6		20.0	19.8			3			
6		7		20.0	20.0		0.0214	3	B.G. at $6\frac{1}{2}$		
7		8		20.2	20.0			3			
8		9		20.4	20.5			3			
9		10		20.6	20.6			3			
10		10	set-up	\	(21.5)	4.7000	0.0381	2	←Steel Tape 872		
10	set-up	11		20.7	21.0			2	←Crossing Gully		
11		12		20.8	21.0	0.0027		3			
12		13		20.8	21.0		0.0732	3			
13		14		21.0	21.0			3			
14		15		21.0	20.0			4	Supp. 0-12$\frac{1}{2}$-25-50		
15		16		20.8	20.5			3			
16		17		20.5	20.3			3			
17		18		20.7	20.5			3			
18		19		20.8	20.4			3			
19		20		21.0	20.5			3			
			Mean =	20.7	20.5						

Computed by L.V. Checked by J.D.

FIGURE 66.—Measurement of base line.

line, and not on the same line with a full tape length. The numbering of the stakes should plainly indicate the full tape lengths and the partial lengths. (See *b*(1) above for method of numbering.) Notes in the "Remarks" column should clearly explain any unusual conditions. At the beginning of the day's work, and as often as changes occur, an entry should be made in the "Remarks" column giving the names and duties of the chief of party, recorder, and the two contact

men, also a statement as to the results of the comparison of the balances and the dial reading being used on the balances. All marginal notes and entries at the top of the page should be made as measurement progresses. The insertion of notes after leaving the section is very dangerous to accuracy.

d. Computations.—(1) *General.*—Figure 67 is compiled from the field records (figs. 64, 65, and 66) shown on earlier pages of the notebook, and also from subsidiary computations performed on later pages of the notebook, where a double page should be allotted for each single measurement of a kilometer section. The circled numbers

COMPUTATION OF Exemplar BASE LINE (22)

SECTION	DATE 1924	DIR OF MEAS.	TAPE NO.	TAPE LENGTHS NO	TAPE LENGTHS METERS	TEMP	CORRECTIONS TEMP	CORRECTIONS TAPE AND CATENARY	CORRECTIONS SET-UP SET-BACK	CORRECTIONS INCLINATION	CORRECTIONS SEA-LEVEL	REDUCED LENGTH	ADOPTED LENGTH	v	v^2
A.E.B. TO TE 20	7/7	F	922	–	–	°C 20.0	METERS −0.0001	METERS −0.0009	METERS +20.0000	METERS −0.0471	METERS −0.0382	METERS 1024.4744	METERS		
	"	"	922	20	1000	20.6	−0.0064	−0.0756	−0.0586						
	"	"	872	–	–	21.5	0.000	+0.0013	+4.7000		COMPUTED BY: CHECKED BY:				
PAGE (12)			(12)	(12)	(12)	(12)	(32)	(32)	(12)	(2)	(32)	(32)			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)

FIGURE 67.—Computation of base line.

in figure 67 indicate from what page of the notebook the data above it were taken. The same system should be followed on the subsidiary pages, marking a circled number opposite each item from another page. Each part of the computation should be labeled so that the work can be readily checked by another.

In this manner the entire record of the work and computations, with the names of the computer and checker of each item, is kept in one place. The only inconvenience is that only one pair of computers can use the book at one time. To aid the explanation, the columns of

figure 67 have been numbered in parentheses at the bottom. The corrections commonly applied to base line measurements are as follows:

Temperature (8).
Tape (9).
Catenary (9).
Set-ups and set-backs (10).
Inclination (11).
Mean sea level (12).
Tension.
Alinement.

The results show the adopted length of each kilometer section, length of the base line, and the probable error. Columns (1) to (7), inclusive, of figure 67 are obtained from figure 66, "Measurement of base line," 20.6 in column 7 being the mean of all the temperatures for the 20 whole tape lengths.

(2) The "Leveling for base line" form should be completed as shown in figure 64, the distances in column (9) are obtained from the "Measurement of base line" form (fig. 66). If the rod readings are in both meters and feet, the differences of elevation of one of the columns are converted to either unit by table IX or X, TM 5–236, and written in column (8) and the mean differences are put in column (10). Column (11) is for the inclination corrections in millimeters with the sum at the bottom. For 50-meter tape lengths and measurements made with the intermediate 5-meter marks these corrections can be obtained from table XIV, TM 5–236. These tables are made out for differences of elevation in both feet and meters, so either feet or meters may be used in column (10). The corrections for other lengths and also for differences of elevations outside the limits of the tables must be computed. If l is the inclined length and h the difference of elevation of the two ends, the correction is

$$C_g = -(l - \sqrt{l^2 - h^2}) = -\frac{h^2}{2l} - \frac{h^4}{8l^3} - \frac{h^6}{16l^5} - \ldots$$

Since for a 50-meter tape length the second term $\frac{h^4}{8l^3}$ is less than 0.1 millimeter where h is less than 3.1 meters, on ordinary grades the correction will vary directly as the square of the difference of elevation. Since the correction varies inversely as the distance, fractional tape lengths are likely to a larger error in the grade correction. It is very important that all broken grades and partial tape lengths be indicated on this form and that the grade correction be computed for the corresponding distance. The most frequent mistake made in computing

grade corrections arises from using a 50-meter length instead of the real length. In column (12) several of the elevations are computed and the mean elevation of the section, correct within 100 feet, is written at the bottom.

(3) *Standardization.*—The data secured from the "Comparison with reference tape" (fig. 65) are used to derive new standardization data for each of the field tapes, invar or steel. Assuming that the certificate gives 49.99595 meters at 26.4° C., when supported at the 0-, 25-, and 50-meter points, and a coefficient of expansion of 0.00000104 per degree C. for the reference tape No. 3861, and a coefficient of expansion of 0.00000110 and a weight of 25 grams per meter for the field tape No. 2916, the length of No. 3861 at 21° C. is $49.99595-5.4\times0.00000104\times50$ or 49.99567 meters. No. 2916 at 20.5° C. was 0.00088 meters longer or 49.99655 meters. At 26.4° C. (assumed as the temperature of standardization for No. 2916) it would be $49.99655+5.9\times0.00000110\times50=49.99687$ meters, which should agree closely with the original calibrated length of No. 2916, and will be used as the length of tape until another comparison with the reference tape.

(4) *Temperature correction* is computed as follows: The correction $=(T-Ts)\times$coefficient of expansion$\times50\times$number of tape lengths, in which T is the mean temperature for the section and Ts is the temperature of the tape at standardization. The value of Ts is given in the standardization data for the tape. (See (3) above.) The coefficient of expansion may be considered as the change in length per meter for each degree centigrade change in temperature and is given with the standardization data. The number of tape lengths is given in the column headed "Tape lengths" and is the number of full tape lengths recorded in (5) in figure 67. For tapes with a positive coefficient of expansion the temperature correction is, of course, +or—, according to whether the mean temperature is greater or less than the standard temperature. There are a few tapes which have negative coefficients of expansion, and for these corrections would have the opposite signs.

(5) *Tape and catenary corrections* are combined in column (9). The correction is obtained from the standardization data (see (3) above) for the tape or by computing the catenary correction when the tape is supported in an unusual manner. The tape correction per tape length is the difference between 50 meters and the length of the tape as given for the proper method of support. For instance, in the sample form (fig. 66) there are 18 tape lengths supported at three points, 1 at four points, and 1 at two points. The correction for a tape

supported at three points is obtained directly from the standardization values, and this is multiplied by 18 for the 18 tape lengths. Referring again to the data on the form, the correction for the one tape length supported at four points is obtained by combining the proper fractional parts of the corrections for the three- and the two-point supports. The correction for the tape supported at two points may be computed by the catenary formula below. The algebraic sum of these corrections is entered in the column headed "Tape and catenary," the sign depending on whether the length of the tape is greater or less than 50 meters. The effective length of the tape when suspended between supports under tension is affected by the shortening due to the sag and to the stretching due to the tension. The correction due to the sag is given by the formula

$$C_s = -\frac{n}{24}\left(\frac{w}{t}\right)^2 l^3$$

where n=number of sections into which the tape is divided by the equidistant supports.
l=length of each section in meters.
w=weight of tape in grams per meter.
t=tension in grams.

To illustrate by an example: For tape No. 2916, supported at 2 points under a tension of 15 kilograms,

$$n=1$$
$$l=50$$
$$w=25$$

and

$$C_s = -\frac{1}{24}\times 25^2 \times 50^3 \times \frac{1}{15{,}000^2} = -0.01447 \text{ meters.}$$

(6) *Set-ups and set-backs.*—In the column headed "Set-up and set-back" is entered the algebraic sum of the set-ups and set-backs recorded on the form in figure 66, the set-ups being plus and the set-backs minus. In the sample shown in figure 67 the two large set-ups and the corrections to them are recorded separately. All set-ups, however, could have been combined, and also the temperature, tape, and catenary corrections for the large set-ups could have been combined with the same corrections for the full tape lengths, and then the entire computation of a section would have been on one line. It simplifies the checking of the computation somewhat, however, to enter each large set-up on a separate line, as shown on the sample form.

(7) The *inclination correction*, always negative, is the sum at the bottom of column 11, "Leveling for base line," figure 64.

(8) *Reduction to mean sea level.*—Since the lines of a scheme of triangulation are reduced to their equivalent lengths at sea level, the length of any base must be likewise reduced to sea level before it can be used in adjusting the triangulation to which it is connected. This requires the connection of the base line levels to a bench mark and the computation of the elevation above sea level of the tape supports in order to obtain a mean elevation for the base. The formula used in reducing a base to sea level is

$$C=-S\frac{h}{r}+S\frac{h^2}{r^2}-S\frac{h^3}{r^3}+\cdots\cdots$$

in which C is the correction to reduce to sea level a section of length S, of a mean elevation h, with r the radius of curvature of the earth's surface for that section. Only the first term of the formula need be used for any field reduction.

The computation of the sea level correction, shown on sample in figure 67, is given below, assuming a value for log r (mean radius of curvature in meters) of 6.80421. The mean elevation as obtained from "Leveling for base line" (fig. 64) is 780.2 feet.

log 1,024.5005 meters	=3.01051
log 780.2 feet	=2.89221
log feet to meters	=9.48402
colog r	=3.19579
log C	=8.58253
C	=0.0382 meter, always negative

(9) *Corrections for erroneous tension* are seldom needed. Variations in tension change the effective length of the tape in two ways: First, by changing the shape of the catenary (correction for sag), and second, by changing the length of the tape due to stretching, both changes tending to increase the effective length when the tension is increased. Since it is easy to keep the error in the applied tension within a 100-gram limit, the errors in length attributable to incorrect tension are negligible on third- or second-order base measurement. In order that a true tension of 15 kilograms may be applied, the dial pointing on the spring balance should be adjusted to the proper reading whenever the tests show it to be appreciably in error. If it cannot be adjusted, the proper allowance should be made in the dial reading when applying tension to the tape and a remark giving the dial reading used should be inserted in the record book immediately following the statement giving the results of the testing of the balances.

(10) The *alinement correction* should more properly be called the alinement error, for although the same correction formula and tables apply to differences in alinement of the tape as to differences of grade the alinement can usually be made sufficiently exact to require no correction. It should be borne in mind, however, that alinement errors are always of the same algebraic sign, tending to make the measured length greater than the actual length, and for that reason they should be kept much smaller in magnitude than the inaccuracies in the grade corrections. The section describing the staking of the base gives details of the precautions to be taken in alining the stakes. In addition, some member of the taping party, usually the rear contact man or the front stretcher man, should check each tape length as the measurement progresses to see that the tape does not change its horizontal direction at the middle support, and also that the forward stake has not been disturbed in alinement.

(11) Finally the algebraic sum of the uncorrected length and all corrections gives the reduced length for the section, and the mean of the reduced lengths from the forward and backward measurements gives the adopted length. The columns headed "v" and "v^2" are used in computing the probable error of the measurement of the base.

(12) (*a*) The *probable error* is usually computed by the method described below. This method is based on the theory that the errors of standardization and of the determination of the coefficients of expansion are either largely included in or are masked by the discrepancies in the measured lengths of the sections. The probable error of each section is computed by the formula

$$p.\ e. = 0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}}$$

where v is a residual (the result of subtracting the mean from a single value) and n the number of measures of the section. Where a section is measured only twice the probable error will be 0.6745 times one-half the difference between the two measured lengths. The probable error of the entire base is the square root of the sums of the squares of the probable errors of the component sections.

(*b*) The "probable error" of the measurement of a physical quantity is obtained by mathematical formulas applied to the differences between the two or more measured values of a quantity and their mean value. The probable error is a measure of the accidental errors only—that is, of those small errors which have no marked tendency to be predominantly either plus or minus. The probable error is simply a measure of the closeness of agreement among the several

values of a quantity obtained by successive measurements. It will give no indication of the presence of systematic errors—for example, if a steel tape graduated at 30° C. is used at 0° C. and no temperature correction is applied, the measurements may agree within a very small limit and give a very small probable error, yet the result would be in error about 0.4 inch in every 100 feet or about 1 part in 3,000. Neither will the probable error give any indication of blunders, for a tape length dropped from each measure will not affect adversely the probable error of the measurement of a base.

(13) (*a*) The "actual error" is the difference between the true value and the measured value of a physical quantity. It is the sum of all the systematic and accidental errors which have not been eliminated from the final adopted measured value. As the absolute value cannot ever be known, the actual error cannot be exactly determined, but its maximum value can always be estimated. The accuracy of the estimation depends directly and entirely upon knowledge of the maximum uncorrected effect of each source of error. After the error to be expected from each source is evaluated an estimate can be made of the "total actual error", which is one of the criteria for base measures.

(*b*) To illustrate again by the measurement of a base: suppose that the error in marking and the error in correcting for temperature of the tape are the only ones affecting the measurement. The error in marking the ends of a tape is partly systematic and partly accidental; the systematic error will be eliminated by taking the mean of an equal number of forward or backward measures if the person marking remains always on the same side of the base line, and experiments show that with proper methods the accidental error in marking a single tape end is about 0.1 millimeters, or 1 part in 500,000. For a kilometer section of 20 tape lengths, the probable error from marking errors alone would be 1 part in $\sqrt{20} \times 500{,}000$ or 1 part in 2,235,000. In correcting for temperature there are three principal component sources of error—namely, the error in the calibration of the thermometer, error in reading the thermometer, and the undetermined difference between the true thermometric reading and the mean temperature of the tape. Suppose standardizations and tests show that the probable error of calibration is half a degree centigrade and that the probable error of reading is of the same magnitude. Also, that the average difference between the true thermometric readings and the mean temperature of the tape under the conditions of measurements would not exceed 2° C., but that this difference is always of one sign. The probable divergence in temperature between the tape and the thermometer readings would certainly not exceed 3°. If the tape to be used were made of invar,

with a coefficient of expansion of 1 part in 1,000,000 per degree centigrade, the maximum error to be expected would be 3 parts in 1,000,000, or 1 part in 333,000.

106. Base lines with steel tapes.—*a. General.*—If possible, the measurement of base lines for grid triangulation should be avoided by some of the expedients of paragraph 103. However, the methods described below are intended to yield an accuracy of 1:10,000, which is consistent with grid triangulation, without the procurement of special equipment, and at the minimum expenditure of time and men. This method of base measurement should be considered as a refinement of the 1:5,000 taping described in paragraph 47*c* by utilizing standardized tapes, thermometer, accurate spring balance, Abney level, and perhaps stakes or tripods over rough ground and on heavy slopes; by measuring the base once in each direction; and by applying corrections consistent with the required accuracy. An emergency might justify the short base method described in paragraph 132, at the expense of accuracy.

b. Apparatus.—(1) *Standardization of tape.*—Each organization should reserve at least one standardized tape of each length with standardization certificate, to be utilized solely for calibrating purposes. To insure proper calibration the reference tape should be made from the same metal as the field tape being calibrated. If possible the calibrating course should be selected on smooth ground, either level or uniformly sloping, and sheltered from sun and wind, so that a true comparison can be made in whatever position of the tape is desired, and without reduction for difference in temperature. Expose both tapes for about one-quarter of an hour. Then lay off the length of the standardized reference tape with the tension and supports specified in the certificate accompanying this tape. (The end marks may be recorded by fine pencil lines on patches of adhesive tape stuck to some smooth, fixed surface.) Immediately afterward, the field tape is stretched over the course with the tension and method of support which will be employed in measuring the base, and the length is recorded. These marks should be carefully checked by several trials. The marked length of the reference tape is then corrected for any deviation from true length as shown by the certificate, to establish marks for the true length. If the field tape is longer than the true established length, the excess is the error and the correction is plus and must be added to all measurements. If the field tape is shorter, the correction is minus and must be subtracted from the recorded measurements during the computation of the base line. As it is assumed that both reference and field tapes are at the same

temperature, no account is taken of temperature during the calibration but the field tape measurements have to be corrected for the average difference in temperature from the original standardization temperature shown in the certificate. On smooth, even ground, the most rapid progress can be made with a 300-foot tape, supported throughout. Over rough, broken ground, a 100-foot tape supported at both ends is more satisfactory. One of each should be compared with reference tapes before and after the base measurement if both are used.

(2) *Tape-stretching apparatus.*—Heavy rawhide thongs, long enough to wrap around the hand, make the most satisfactory tape handles. The spring balance, which is kept attached to the front end of the tape, should be of ample capacity and frequently compared with one reserved as a standard. As substitutes for tape stretchers, a stout staff may be inserted through a turn of the rawhide at either end of the tape, but utmost precautions must be taken that the tension is applied slowly and steadily while observing the spring balance. A steel tape may be ruined by stretching it beyond its yielding point.

(3) *Accessories* include flagged range poles which should be lined up with a transit, hand and Abney levels, a backed thermometer which should be attached to the rear end of the tape with adhesive tape in such a way that the bulb is in contact with the steel and the whole protected from breakage, a few 2- by 4-inch stakes, and chaining bucks or light tripods which may be employed to support the ends of the tape over rough, broken ground. For the last, instrument tripods with a 4- by 6-inch block of wood rigidly secured to the top will be adequate. A pin or pencil mark may be used to show the end of the tape, and "breaking the tape" may be thus avoided.

c. Preparation of base.—This is limited to selecting the most suitable site and cutting the brush. If the measurement is along a steel rail or hard-surfaced road, the measurement should not be made during a sunny day, as the temperature readings will not be reliable.

d. Procedure for measuring—(1) *Organization of party.*—While shortness of personnel might limit the party to two men, the increase in accuracy and speed would justify the following:

Front contact man (in charge).
Head tape man.
Rear tape man.
Recorder.
Stakeman or tripod carrier.

(2) The method of measuring is in general similar to the 1:5,000 taping described in paragraph 47*c*. Measuring the line both ways

and taking the other precautions mentioned here and making the corrections as outlined in *e* below should give results of 1:10,000 accuracy, provided both tapemen are skilled and in practice. However the division of duties in the larger party is at least an insurance against mistakes. The front contact man levels the tape or reads the slope with an Abney level, besides marking the length, which leaves the front tapeman free for alinement and tension. The recorder keeps all the notes, reads the thermometer for every whole tape length, and assists the rear tapeman set on the mark when necessary. If the stretcher staffs are utilized the rear tapeman quickly sets the end to the mark and holds fast while the head tapeman applies the tension. If the tripods are used much, time will be saved by using three with a man to carry each. On remeasurement, a short length should be measured at the beginning, if necessary, in order to avoid confusion which might be caused by having the old and new marks together.

(3) *Record data.*—Figure 68 shows notes of the field work and the results of the computation for one measurement of the two that are required. If they do not agree within 1:10,000, the measurement will be repeated until there are two, in opposite directions, which are within the allowance. In the heading the data for the two tapes used are the results from the comparison with the reference tapes with the field tension and the method of support immediately before measuring the base.

e. Computations.—These may include corrections for tape length, temperature, sag, slope, and mean sea level. The corrections are shown at the bottom of the field notes (fig. 68).

(1) The *correction for tape length* is positive if the field tape is longer than the true distance, and negative, if shorter.

(2) The *correction for temperature* is made at the average of the temperatures recorded for each tape length. The coefficient of expansion of steel tapes averages 0.0000065 for each degree Fahrenheit. The correction equals 0.0000065 times (the standardization temperature minus the mean temperature of the field measurements with that tape). This correction is negative if the recorded temperatures average over 68° and positive if less.

(3) The *correction for sag* is always negative, but is unnecessary if the tapes have been calibrated under field conditions.

Let

n=number of sections into which each tape length is divided by its equidistant supports.

l=length of each section in feet.

w=weight of tape in pounds per foot.

t=tension in pounds.

Then the correction for each tape length is

$$C_s = -\frac{n}{24}\left(\frac{w}{t}\right)^2 l^3$$

(4) The *correction for slope* is always negative, and equals $\frac{d^2}{2l}$, where l=the inclined length, and d=the difference of elevation of the two ends. The correct horizontal distance equals the length of tape times

BASE LINE △E BELV. TO △W BELV.
300 ft. tape no. 1608 corr – 0.19 ft. @ 68°F
100 ft. tape no. 1617 corr + 0.07 ft. @ 68°F
300 ft. tape on ground 30 lb. pull

Front contact: T Sgt J. Doe
Head Tape: Sgt R. Ross
Rear Tape: Cpl T. Cane
Recorder: St Sgt L. Vane
100 ft tape supported at both ends.

Jan. 21, 1940
gentle breeze
very cloudy
20 lb. pull

15

Sta.	300' tape	temp° F	V.A.	slope corr.	100' tape	temp° F	V.A.	slope corr.	Support	Sun	Remarks
△E. Belv.	300.00	80							gravel	cloudy	8:20 a.m.
					100.00	79			ends		
					100.00	78	3°40'	−0.20	do		
					100.00	78	3°00'	−0.14	do		
	300.00	77	1°20'	−0.08					concrete		
	300.00	77	1°00'	−0.05					do		
	300.00	76	2°10'	−0.21					do		
	300.00	76	2°00'	−0.18					do		
	300.00	76	1°40'	−0.13					do		
					100.00	76	4°10'	−0.26	ends		
					56.76		3°00'	−0.08	do		
	300.00	78	0°20'	−0.01					concrete		
	300.00	79	0°20'	−0.01					do		
					100.00	78	5°00'	−0.38	ends		
△W. Belv.					76.56		2°20'	−0.06	do		9:30 a.m.
Total	2400.00	619		−0.67	633.32	389		−1.12			
Mean		77				78					
Tape corr.	−1.52				+0.44				300 ft.	Tape	2397.67
Temp. corr.	−0.14				−0.04	log L =	3.48148		100 ft.	Tape	632.60
Slope corr.	−0.67				−1.12	log H =	3.00000		Total		3030.27
	2397.67				632.60	colog R =	2.67969		M.S.L.		−0.14
							9.16117		one	way	3030.13
								Computed		L. Vane	
								Checked		Y. Bet	

FIGURE 68.—Notes and computation for base line with steel tape.

the cosine of the angle of slope, which may be obtained from Abney level or transit readings.

(5) The *reduction to sea level* is always negative.

Let

L=length of line in feet.
H=average height above sea level in feet.
R=mean radius of curvature of the earth in feet.
(colog R=2.67981−10)

Then the correction is

$$-L\frac{H}{R}$$

f. Exercise XVI.—Measuring a base line with steel tape.—By the methods of this paragraph, calibrate the field tapes, measure an assigned base line, keeping the notes as in figure 68, and obtain the final length as the mean of two measurements in opposite directions which agree within 1:10,000.

SECTION XVIII

MEASURING ANGLES—TRIANGULATION

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107. Measuring horizontal angles.—*a. General.*—(1) Only specially trained men using suitable equipment can observe angles with the accuracy specified for triangulation without wasting time. The observing party proper consists of two men, both of whom should be skilled in every phase of their work, as an uncertain recorder can seriously hamper an expert observer. Light tenders should be reliable men imbued with the importance of properly displaying their lamps. Transportation and other assistants are detailed as required.

(2) Even the best 20-second transits would cause marked decreases in accuracy and speed. On the other hand, the excess bulk and the delicate refinements of the large, heavy instruments made for first- and second-order work interfere with the more rapid manipulation fitting for third-order and grid triangulation, for which any of the theodolites described in section X are suitable. Particular instructions in the proper handling, care, and adjustment of these instruments should be imparted to the men using them.

(3) In measuring angles with either the direction or the repeating instrument, certain errors are reduced or eliminated by—

(*a*) Taking a number of readings to diminish the error in pointing.

(*b*) Taking half of the readings with the telescope direct and half reversed. This eliminates errors due to lack of adjustment of the line of collimation and of the horizontal axis.

(*c*) Taking initial readings at different points on the graduated circle to reduce errors due to faults in its graduation.

(4) The program of observation must be arranged to secure triangle closures within the limits required with the instruments issued for the work. The number of observations usually required for each order of

accuracy as given in paragraph 92*a* is based upon the experience of the United States Coast and Geodetic Survey, whose observers are experienced and equipped with instruments designed for the different orders. Partially trained army observers may require 50 percent more observations to obtain the specified triangle closures.

b. The *direction method* of measuring angles consists of measuring the direction to each station from some one station taken as an initial. The directions are the angles measured clockwise from the initial station to each of the other stations. The angle at a station between any two observed stations is the difference of their directions. A *direction theodolite* does not usually have a slow motion screw for the lower motion, though the direction method of observing may be used with a theodolite arranged for repetitions by keeping the lower motion clamped. A direction theodolite is almost invariably read by means of micrometer microscopes. In observing, a pointing is made on the initial station and then upon each station around the horizon in a clockwise direction, closing on the initial station; the telescope is then reversed and readings made counterclockwise, the double set of readings constituting one *position*.

(1) *Circle settings*.—When two or more sets of observations with either a direction or a repeating theodolite are made on the same angle, the initial setting for each set should differ by an amount depending upon the number of positions to be observed and the number of verniers or micrometers on the theodolite. The interval in degrees between successive settings with a 2-micrometer or a 2-venier theodolite is given by the formula $I=\frac{360°}{mn}$, where I is the interval in degrees, m the number of verniers or micrometers, and n the number of positions or sets. In addition, an increment represented by the value of one division of the circle divided by the number of sets to be observed should be added to the difference in degrees between settings in order to eliminate the error of graduation of the verniers or the run of the micrometers. For instance, with a circle graduated to 10 minutes and with two sets observed on an angle, the settings would be approximately 0°00′00″ and 90°05′00″.

(2) The *notes* in figure 69 show the results for two positions, with the readings for each station grouped for ready comparison. The degrees and minutes for the counterclockwise readings are recorded as a check only in case of observing a single position. On the right-hand page each direction is entered separately and then meaned for the final values. The angles may be entered in the "Remarks" column.

(3) With any direction instrument when a broken series is observed the missing signals are to be observed later in connection with the chosen initial, or with some other one, and only one, of the signals already observed in that series. With this system of observing, no local adjustment is necessary. Little time should be spent in waiting for a doubtful signal to show. If it is not showing within, say, 1 minute when wanted, pass to the next. A saving of time results from observing many or all of the signals in each series, provided

HORIZONTAL DIRECTIONS — Obs: T Sgt J. Doe — April 17, 1940 — (16)
At △ Gunter — Clarke Co., Ohio — Rec: Cpl T. Cane — Clear and warm
Inst 1" Berger 87961 — Moderate wind

Objects obs	Dor R	Reading	A Backward B		Mean "	° '	"	Direction ° '	"	Remarks	
△ Bane	D	0°00'	22	22	22					4:01 p.m.	
			24	28	26		24				
	R		26	28	27					4:13	
			21	25	23		25				
	R	270 05	39	42	40.5					4:21	
			33	37	35		37.8				
	D		31	33	32					4:30	
			29	32	30.5		31.2				
										∠=45	40 00.1
△ White	D	45 40	23	29	26	45 40	02				
	R		20	22	21	39	56				
	R	315 45	35	38	36.5	45 39	58.7				
	D		33	37	35	40	03.8	45 40	00.1		
										∠=30	55 03.3
△ Staunton	D	76 35	28	30	29	76 35	05				
	R		24	26	25		00				
	R	346 40	38	41	39.5	76 35	01.7				
	D		36	40	38		06.8	76 35	03.4		
										∠=10	29 38.2
△ Glen	D	87 05	04	08	06	87 04	42			Signal	in shade
	R		01	06	03.5		38.5				
	R	357 10	17	20	18.5	87 04	40.7				
	D		15	18	16.5		45.3	87 04	41.6		
										∠=272	55 18.4
									Checked J. Doe		

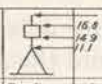
FIGURE 69.—Notes of horizontal directions.

there are no long waits for signals to show, but not otherwise. When the elevations of the stations differ greatly it is necessary to keep the horizontal axis of the instrument level in order to avoid large and troublesome errors. Any releveling should be done between positions.

(4) With an instrument of the Wild type, reading directly to 1 second, the two micrometers are meaned by the process of reading and the tenths of seconds should be recorded leaving the fifth and sixth columns blank. Another advantage of the Wild type is that all readings are made without moving from the eye end of the telescope.

c. The repetition method.—(1) The 10-second vernier theodolite is a *repeating instrument* on which successive observations of a horizontal angle are added by repetition and finally meaned to secure an accurate measure of the angle. A set of observations consists of six repetitions of the angle with the telescope in the direct (or reversed) position, followed immediately by six repetitions of the explement of the angle with the telescope in the reversed (or direct) position. ("6 D/R" is the abbreviation for such set.) Set the A vernier at or near zero,

Horizontal Angles
State: Ohio County: Clarke
At △ Knoll



H.I 4.6 ft

Obs. T Sgt J. Doe
Rec: Cpl T. Cane
Inst: 10" K & E 60806

April 18, 1940
Cloudy and cool
15 mile wind

(4)

Objects observed	Tel	Rep.	Readings	A	B	Mean	Mean of 6		Set Mean	Hor. Adj.	Remarks
△ Shoulder	D	0	0° 00′	00	10	05″					10:35 A.M.
△ Knob		1	30 15	30							
		6	181 32	40	50	45	30° 15′	26.7″			
	R	6	359 59	50	00	59 55		28.3	27.5	26.6	30° 15′ 26.6″
△ Knob	R	0	00 00	20	30	25					10:55 A.M.
△ Top		1	82 41	00							
		6	136 06	30	40	35	82 41	01.7			
	D	6	0 00	40	40	40	40	59.2	41 00.4	40 59.5	82° 40′ 59.5″
△ Top	D	0	0 00	25	30	27.5					11:10 A.M.
△ Shoulder		1	247 03	40							
		6	42 21	50	60	55	247 03	34.6			
	R	6	0 00	20	30	25		35.0	34.8	33.9	247° 03′ 33.9″

		Vertical Angles						
		±	Readings	C	D	Mean	Angles	
Shoulder	D	+	5° 22′	00	00	00		
	R		5 21	30	60	21 45	+5 21	52.5
Knob	D	+	2 41	00	30	15		
	R		2 41	00	00	00	+2 41	07.5
Top	D	−	3 16	00	00	00		
	R		3 16	30	30	30	−3 16	15.0

1:00 P.M.
1:10 P.M.

Checked T. Sgt J. Doe.

FIGURE 70.—Notes of horizontal and vertical angles.

read and record both verniers and their mean as in figure 70. Set on left-hand station, unclamp above, set on right-hand station, read and record A vernier only. From this the recorder subtracts the initial reading, multiplies the result by 6, and notes the product on scratch paper so that he can give the degrees and approximate minutes of the sixth repetition at the request of the observer. Make 5 more repetitions, setting on the left-hand station with the lower motion and tangent screw, and with the upper motion for the right-hand station. Read and record both verniers and their mean. From this subtract the initial mean, divide by 6, and record as the "Mean of 6." With-

out disturbing the verniers, reverse the telescope, and take 6 repetitions of the explement, measuring the angle from the former right-hand station to the other, recording the final readings of both verniers and their mean, which should be nearly the same as the initial mean. Subtract the final mean from the sixth repetition mean, divide by 6, and record as a second value of the mean of 6. The mean of the set is recorded in the next column. Before beginning another set, the circle reading should be changed in order that an error in reading may not affect 2 angles.

(2) With any repeating theodolite, measure only the single angles between adjacent lines of the main scheme and the angle necessary to close the horizon. Releveling may be done between sets or between the separate angle measures of a set; that is, when the lower clamp is loose. After observing a set for each of the angles, the horizon is adjusted to make the sum of the adjusted angles equal 360°. Record under "Hor. adj." A second set of the same angles may be recorded on another page and the mean of the two horizon adjustments recorded in full, bold figures in the "Remarks" column. If it is certain that several sets will be required, a page of the notebook may be allotted for all the sets of each angle. The initial circle settings should be as described in *b*(1) above.

d. Observations on intersection stations.—(1) An intersection station is one which is not occupied, the position of which is determined by observations upon it from stations of the main scheme or from supplementary stations. It may be a signal over a marked point or it may be a well defined natural or artificial object, such as a tank, church spire, or sharp mountain peak. Here the term "intersection station" is used in a restricted sense to mean a station located by intersections with fewer observations than are specified for the main scheme. A line to such a station must not be used as a base for new triangulation.

(2) In selecting intersection stations it should be kept in mind that the geographic value of triangulation depends upon the number of points determined, the size of the area over which they are distributed, and the permanence with which they are marked. The geographic value of a triangulation is lost for a given area when stations cannot be recovered within that area. The chance of permanency is made greater by increasing the number of stations as well as by thorough marking. For the reasons stated there should be determined as intersection stations many artificial objects of a permanent character, such as lighthouses, church spires, cupolas, towers, chimneys, and standpipes. Make the description definite whenever practicable. It is advisable to observe upon each intersection station from at least

three stations in order to obtain a check upon the position, and the directions from at least two of the stations should, if practicable, form a good angle of intersection at the object to be located. A possible intersection station should not be disregarded if only two directions to it can be secured.

(3) The direction method of observation should be used in observations upon intersection stations even if the theodolite is a repeater. Each series of observations on intersection stations should contain some one line, and only one, of the main scheme (or a line used in fixing the position of a *supplementary station*, which may be established for the purpose of making more intersection readings possible). This main scheme line is always made the initial. The notes for the repeating instrument are similar to figure 69, except that the page is headed "Intersection Stations." Two positions should be observed with a 10-second instrument, one with a 1- or 2-second direction instrument. When only one position is observed, the degrees and minutes of the counterclockwise readings are recorded to provide a check on the clockwise readings. The vertical angles are recorded as in figures 70 and 72, and may be recorded on the same page as the above directions.

108. Description of station.—After checking the data received from the reconnaissance and station construction parties (see par. 98*n* for other details), the original description should be written in the angle or direction record book or in a separate notebook carried by the recorder for that purpose. Taking one of the main scheme stations as an initial, the directions to the nearby azimuth and reference marks are observed to the nearest 10 seconds. These marks are observed separately so as to avoid changing the focus during other observations. As soon as possible after leaving the station, while the topography of the vicinity is fresh in mind, the written notes should be transferred to Form 4, "Description of station" (fig. 60). A single copy only need be sent to headquarters, but it is a good plan always to make one carbon copy to be retained in the field for reference until the end of the season, when the duplicates can be transmitted to the office if no longer needed. The form, after being completely filled out, should be read over carefully to see that there are no reversed directions and that no part of the description is vague, ambiguous, or erroneous. The name of the station given in the description should correspond to that given in the triangulation records and computations. On account of inadequate descriptions, there has been considerable confusion in identifying such objects as flagpoles, which are sometimes replaced by another pole in the immediate vicinity. The description

of such objects should state the particular part of the grounds, building, or other structure where located. While the written description must be complete without a sketch, one may be added if desired.

109. Eccentric stations.—*a. General.*—(1) When such an object as a church spire or flagpole is used as a triangulation point, the instrument is set up over a point called the *eccentric station*, which should be as close to the true station as possible. Occasionally a target or signal pole may be out of plumb, due to heavy wind or other disturbance, thus becoming an *eccentric object*. Any eccentricity of the instrument or object should be avoided when possible because of the added computing necessary to reduce the observations to center and because of the greater danger of angular errors. Eccentricities of more than a meter or two are especially apt to cause poor triangle closures because of the difficulty of securing a true value of the horizontal distance to the eccentric point.

(2) The importance of the proper recording of the eccentricity of signals or lights and theodolites must be emphasized for the reason that many serious mistakes and ambiguities in triangulation records are traceable to that source. The computation necessary to correct the directions and angles to make them refer to the true station is called the *reduction to center*.

b. Eccentric station.—The horizontal distance between the eccentric station and the center of the station must be carefully measured. The notebook should contain a sketch which shows the relation between the two stations, and the line extending to that station of the main scheme which was used as the initial station in making the horizontal angle observations. Form 6, "Transcript of results" (figs. 82 and 83), as observed at the eccentric station, should contain the direction to the nearest minute only from a main scheme station as an initial to the center of the station, and the distance should be shown in a sketch at the bottom of the sheet.

c. Eccentric object.—The notebook and transcript of results should show definitely the amount of eccentricity of the pole or target, and the direction in which the signal is eccentric. Frequently the eccentric pole is in the line from the center of the station to one of the other stations. In such case the fact should be noted, with the eccentric distance, and a statement as to which side of the center of the station the center pole lies. From the eccentric distance and direction, corrections are computed for any observations made upon the eccentric pole from other stations.

d. Reduction to center.—With approximate distances to the objects observed as scaled from the map or plat of the triangulation system,

the observing party makes a preliminary computation of the reduction to center, so that the observer may have a record of all the triangle closures. Form 5, "Reduction to center," bears full directions and the copy in figure 71 shows examples worked out. As this reduction to center is based upon approximate distances, the resulting corrections are not made on form 6, but the completed form 5 should be attached for the further guidance of the computing section. The instructions which should appear on the form are printed in full below.

Eccentric Station } Δ Chase

d = 36.06 feet

(ft. to yds.) log = ~~9.52288~~ } cross out one
(ft. to m.) log = 9.48402 } or both
log d = 1.55703
colog sin 1" = 5.31443
(1) sum = 6.35548

Diagram

(2) STATIONS	Center	Δ Trail	Δ Bossing	Δ Little River	Δ Lyons		
(3) s (if log unknown)	—	—	—	—	—		
(4) A (direction)	0° 00'	124 27	179 18	242 47	249 02		
(5) log sin A		9.91625	8.08696	9.94904	9.97025		
(6) log s (meters ~~or yards~~)		4.40254	4.49198	4.51928	4.30616		
(7) log (sin A/s)	(5)-(6)	5.51371	3.59498	5.42976	5.66409		
(8) log C (correction)	(7)+(1)	1.86919	9.95046	1.78524	2.01957		
(9) C in seconds	+ or -	+ 74.0	+ 0.9	-61.0	-104.6		

Date: January 12, 1940 Computed by J.A.S Checked by J.W.H

Eccentric Object Observed at Station } ▽ 64H21

d = 4.00 feet

(ft. to yds.) log = 9.52288 } cross out one
(ft. to m.) log = ~~9.48402~~ } or both
log d = 0.00000
colog sin 1" = 5.31443
(1) sum = 4.83731

Diagram

(2) STATIONS	Center	Δ Kent	▽ 64H18	▽ 64H20	▽ 64H22		
(3) s (if log unknown)	——	8000	6000	4000	2000		
(4) A (direction)	0° 00'	0 00	61 10	122 20	244 35		
(5) log sin A		——	9.94252	9.92683	9.95579		
(6) log s (~~meters or~~ yards)			3.77815	3.60206	3.30103		
(7) log (sin A/s)	(5)-(6)		6.16437	6.32477	6.65476		
(8) log C (correction)	(7)+(1)		1.00168	1.16208	1.49207		
(9) C in seconds	+ or -	——	+ 10.	+ 15.	-31.		

Date: January 12, 1940 Computed by J.H.B Checked by R.A.H

FORM 5

FIGURE 71.—Reduction to center.

REDUCTION TO CENTER

INSTRUCTIONS

Third-order triangulation is computed in meters. Grid triangulation is computed in yards.

$$c = \frac{d}{\text{Sin 1 second}} \times \frac{\text{Sin } A}{s} = \text{the required correction in seconds}$$

d is the distance in feet (to hundredths) from the eccentric station to the true station.

s is the distance in meters (or yards) between the true stations involved.

Log s is taken directly from the computation of the triangle sides. Before the sides are computed, log s is obtained from s, which may be scaled from the map or plat.

A is the direction of the distant station reckoned clockwise from and referred to the direction from the eccentric to the true station, or center, taken as zero. This definition of A is true also for the case in which the object pointed on is eccentric.

The sign for any correction is the same as for the corresponding sin A, and is therefore negative if A is over 180°.

Obtain A to nearest minute and c to nearest 0.1 second (nearest second for grid work); use five-place logarithms.

Reductions for an eccentric instrument.—Insert station name in upper heading and cross out the unnecessary conversion log.

Line (2) should contain the names of the stations observed from the eccentric position of the instrument.

Lines (3) and (5) may need to be derived by successive approximations from the side computations if the eccentric reductions are large.

To correct a direction of a list of directions as on form 6, "Transcript of results," change the sign of c for the initial direction and algebraically add it and c for the required direction to the observed direction.

To correct an angle, change the sign of c for the first line observed and add it algebraically to c for the second line and to the observed angle.

Reduction for an eccentric object observed.—Insert station name in lower heading and cross out the unnecessary conversion log.

Line (2) should contain the names of the stations from which the eccentric object was observed.

No distinction need be made between the direction from the eccentric object to the distant station and the direction from the true station to the distant station, except when the eccentric reduction is more than 1 minute.

The remainder of the computation on this form is made as for an eccentric instrument. However the reductions to directions are to be applied to observed directions, at the stations named in line (2), to the eccentric object at the station named in the heading. The directions to which these reductions are to be applied are to be found on various sheets of form 6, "Transcript of results," and not on one sheet, as is the case when the instrument is eccentric. A note should be made on each of the sheets opposite the direction or angle concerned, referring to the reduction to center.

e. The application of the corrections is explained on the form. The directions of the main scheme stations observed from Chase eccentric, the corrected directions from Chase, and the corresponding angles for figure 71 are as follows:

	Directions observed from Chase eccentric			Corrected directions from Chase			Corrected angles from Chase		
	°	′	″	°	′	″	°	′	″
△ Trail	0	00	00. 0	0	00	00. 0			
							54	50	10. 5
△ Bossing	54	51	23. 6	54	50	10. 5			
							63	27	26. 3
△ Little River	118	19	51. 8	118	17	36. 8			
							6	14	40. 9
△ Lyons Salt Works	124	35	16. 3	124	32	17. 7			
							235	27	42. 3
△ Trail									

The angles in the third column would have been the same, had they been computed directly from the angles at Chase eccentric.

110. Vertical angles.—*a. General.*—(1) Grid triangulation increases the importance of trigonometric leveling, as sufficient elevations of an accuracy suitable for making battle maps and for fire control purposes can thus be obtained with a minimum amount of spirit leveling. In order to provide starting and check elevations, a complete scheme of vertical angle observations should be carried through all triangulation. This complete scheme should consist of a continuous series of vertical angle measures through the main scheme of the triangulation, observing each line over which horizontal angles are observed (the observations over each line to be made in both directions if both ends of the line are occupied), and should also include observations of vertical angles upon all supplementary and intersection stations corresponding to the horizontal angles measured upon such stations. Connections should be made with elevations accurately determined by spirit leveling as frequently as possible.

(2) If some of the stations are within sight of the ocean, the elevations of the stations, as determined by the vertical angle measurements carried through the chain of triangles, can be checked and made more exact by observations upon the sea horizon. Elevations determined in this manner are not as accurate as when frequent connections can be made to bench marks, for the observations are nonreciprocal and an arbitrary value must be used for the coefficient of refraction, m, which may vary for daytime observations on the sea

horizon from 0.7 to 0.13. The approximate formula for computing the height of station from the observed angle of depression is

$$h=\frac{1}{2}\rho \sin^2 1 \text{ second } \frac{\theta^2}{1-2m},$$

where h=elevation of station above sea level.

ρ=radius of curvature of the earth (in the same unit of length as h).

m=coefficient of refraction.

θ=observed angle of depression in seconds of arc.

Using mean values of ρ and m, the approximate formula becomes simply

$$\log h \text{ (in meters)} = (5.95005-10)+2 \log \theta$$

$$\text{or, } \log h \text{ (in feet)} = (6.46603-10)+2 \log \theta$$

b. Trigonometric leveling.—This consists essentially in determining the difference of elevation between two points by using the principles of triangulation. The vertical angle between two stations, that is, either the altitude or zenith distance of one as seen from the other, is measured. The horizontal distance between the stations having been found by triangulation, the difference of elevation may be computed by solving the triangle formed in the vertical plane containing the two stations. The difference of elevation between two stations whose distance apart is known can be determined by either—

(1) Observation at one station only, known as the method of nonreciprocal observation.

(2) Observations at both stations either simultaneously or not simultaneously, known as the method of reciprocal observations. From the nature of the case the effects of curvature and refraction are involved in any form of trigonometric leveling. The method of reciprocal observations is preferred to the one-station method because the effects of curvature and refraction tend to balance out when the observations are taken in both directions. This balancing is more nearly complete if the observations in both directions are taken simultaneously. But on triangulation work where each station is occupied consecutively and additional observers are rare, the method of reciprocal observations is usually used but no attempt is made to observe in both directions simultaneously.

c. Corrections.—In computing from nonreciprocal observations (sec. XIX), the vertical angle has to be corrected for curvature and refraction. The curvature, varying with the square of the distance, is always positive, while the refraction is nearly always negative and

usually about one-seventh of the curvature. As the grid triangulation lines are short, no objectionable error is likely to be introduced by assuming that the coefficient of refraction is constant at 0.07. Table XVI, TM 5–236, for correcting nonreciprocal vertical angles is based upon the above assumption. The correction K is always positive and applied so as to increase positive and decrease negative vertical angles. As vertical angles are generally read on any intersection station from at least three main scheme stations, the mean of the three elevations will tend to balance out any errors due to variable refraction.

d. Hours for observing.—In geodetic triangulation the vertical angle observations should be made between noon and 3 p. m., while the coefficient of refraction is most constant; and in any event not before 10 o'clock in the morning nor later than 1 hour before sundown. Early morning and late afternoon observations are of lower accuracy, owing to rapidly changing vertical refraction.

e. Program for observing vertical angles.—It is essential for accurate measurement of vertical angles to have the vertical axis of the instrument truly vertical, or to have the errors of verticality eliminated by the method of observation. Great care must be exercised in leveling and adjusting the instrument. Residual errors in verticality are then eliminated by reading the vertical angle with the instrument in both direct and reversed positions. Either one of two observing programs may be used. When using the first method, level the theodolite, then with the circle right and the object near the vertical wire—

(1) Bisect the object with the horizontal wire, using the telescope-clamp slow-motion screw.

(2) Bring the bubble to the center of the level vial.

(3) Read both verniers.

(4) Turn the instrument 180° in azimuth and transit the telescope.

Then repeat (1) and (3) in the same order. Do not change the adjustment of the level on the vertical circle between the two pointings of a set. By the second method, level the theodolite and, with circle right, perform operations (1), (2), and (3), as described above, on each object around the horizon; then transit the telescope and again perform the same operations upon each object. In the second method, the bubble in the vertical circle vial is centered for each pointing, both direct and reversed. Both methods avoid any necessity of computing corrections for the bubble being off center. With most theodolites two direct and two reversed readings should be made for each station; with the Wild type reading directly to 1 second, one direct and one reversed are enough.

f. The *vertical circles* are variously graduated, some showing vertical angles and others zenith readings. Of the latter, some vertical circles are graduated clockwise and others counterclockwise. As with any other surveying instrument, the observer should study all the divisions and decide upon the correct procedure for rapidly obtaining correct results. In any case, the readings are recorded as they are shown by the instrument and afterward converted to vertical angles, if necessary. Many of the vertical circles on the 10-second instru-

VERTICAL ANGLES — Collins Co, Texas — At Δ Cone — H.I 4.8 ft — Obs: T Sgt J Doe — Rec: Cpl T Cane — Inst: 1" V.C. Wild 7923 — June 18, 1940 — Clear and hot — Some mirage — (5)

Objects observed	Tel	Readings	± Vertical Angles	Remarks
Δ Fulton	D	89° 56' 06"		1:07 P.M.
	R	270 03 45	+ 00 03 50	
Δ Sell	D	90 12 21		
	R	269 47 32	− 00 12 24	
Δ Vann	D	90 23 38		Top of signal
	R	269 36 12	− 00 23 43	
Δ Brink	D	89 02 52		
	R	270 56 57	+ 00 57 02	
Δ Tula Tank	D	90 00 02		Top of Finial
	R	269 59 47	− 00 00 08	1:30 P.M.

FIGURE 72.—Notes of vertical angles.

ments are graduated only to 30 seconds. (Ten seconds amounts to 0.5 foot at 2 miles.)

g. The *notes of vertical angles* (fig. 72) must give in the heading the height above the station mark of any parts of the signal on which vertical angles from other stations might be read, and the height of the instrument above the station mark whenever vertical observations were made. In the "Remarks" column should be noted, in words or by sketch, the part of each object pointed upon, in order that the observations may be properly referred to the station mark. For other types of instrument than the Wild, the *B* and mean vernier columns would have been filled out.

111. Special instructions.—*a. Observers.*—(1) A good observer is one who can consistently secure results commensurate with the possibilities of the instrument which he is using. Proficiency can be attained only by careful study of the instrument, by constant exercise of good judgment, and by making a careful study of all the factors affecting the accuracy of theodolite observations. Due regard must be had for the relative importance of the different classes of errors. The actual pointing of an instrument on an object is a simple operation. It is a mistake to try to perfect a pointing after the perceptions are once satisfied that the object has been centered on the wires. The most satisfactory observations are usually those which are made rapidly and methodically but not carelessly. Speed can be attained by the observer by his training himself in deftness of movement in manipulating the instrument and in studying how he can perform the manipulation with the fewest movements. Aside from blunders, such as reading a vernier incorrectly, and the allowable inaccuracies of pointing upon an object, there are four principal causes of error in the measurement of horizontal angles: instability of instrument support, instrumental errors, phase, and eccentricity, and horizontal refraction. The relative importance of these four factors will vary with the field conditions encountered.

(2) For any kind of triangulation, the theodolite should be kept in good adjustment by the methods described in section XI, without excessive use of the adjusting screws, which might cause them to become worn and loose. The programs of observation are designed to balance out small errors of most of the adjustments, but those due to parallax and lack of horizontality of the graduated circle cannot be so eliminated. The striding level and the horizontal axis adjustment should be tested frequently. For the best results the theodolite and tripod should be protected from direct sunlight and wind, which cause small changes and strains in the instrument. Since all of these are changing in amount from moment to moment, no system of observation will eradicate them. This is one of the principal reasons why rapid pointings upon the object give more accurate results than slow pointings. Another element which is often not appreciated is the manner of manipulation of the instrument. In an endeavor to make rapid pointings, instrumental errors are sometimes introduced which are larger than those the observer is seeking to avoid. The hand should not rest heavily upon the instrument at any time. Slow-motion screws and the graduated heads on the screws of reading microscopes should be turned with a true rotary motion without lateral thrust. The final half-turn of a tangent screw must always be against the spring.

(3) In order to obtain the best results, the following precautions should be observed:

(*a*) Adjust and level the instrument carefully before proceeding with measurements.

(*b*) Keep away from tripod when making measurements.

(*c*) Turn plates gently, taking hold of plates or limbs and not of telescope.

(*d*) Work steadily and carefully, do not speed.

(*e*) Do not screw tripod legs too tightly.

(*f*) Do not turn clamps or leveling screws too tightly.

(*g*) Leave horizontal axis lightly clamped while observing horizontal angles.

(*h*) The lower motion should be unclamped during vertical angle observations.

(*i*) Never carry the theodolite on its tripod.

(*j*) Refrain from walking around a repeating instrument to read the verniers, as the lower motion is unclamped between angles.

(*k*) Drill the recorder to record all readings in the notebook immediately, at the same time calling back the figures so promptly that observing is not retarded.

(*l*) Above all, the observer must not try to force the observations by sighting upon a different part of the object from that which his judgment says is the proper point, for a poor triangle closure may be due to an error at any one of the three stations involved. He must cultivate an impersonal attitude toward his results and read the angles without bias, for an angle forced to give a good triangle closure will often result in large angle and side corrections when the adjustment is made.

b. Recorders.—(1) *General.*—The recorder should be able to work quickly and accurately. He should be a man thoroughly familiar with all phases of reading angles and recording. He should have sufficient mathematical knowledge to make field computations. Finally, he should be a man who can be considered as an understudy to the triangulator

(2) Under favorable conditions for observing, the readings are made so rapidly that the recorder has to be methodical in order not to hinder the observations. His mental arithmetic must be infallible, as there is no time for setting down readings and meaning by long hand on scratch paper during all the steps of recording, computing, and transcribing results. All of the records and blank forms should be neat and clear, not only for appearance, but because they form a valuable part of the permanent record of the survey, and may have to be referred to by many men other than the observing party. Particular

attention must be paid to recording the correct name or number of every station mentioned, preceding the name of each by the sign of the type, as △ for third order, *I* for intersection, *R* for resection, and ▽ for grid triangulation station.

(3) As the work progresses, the recorder under direction of the observer prepares and maintains the following:

(*a*) Complete records of all observations.

(*b*) Title page and index on the first three pages of the notebook.

(*c*) Cross referencing wherever necessary

(*d*) Descriptions of all stations (par. 98*n* and 108).

(*e*) Transcript of results (par. 114*b*).

(*f*) Record of triangle closures.

c. Methods.—(1) Before taking the field, the recorder should become thoroughly familiar with all of the notes and forms of this section, not only in general, but in the rapid, clear, accurate execution of every step in the preparation of the completed form from the observer's readings. In the notebook, enough headings and column rulings should be prepared beforehand that observing may proceed without delay. (See also par. 33.)

(2) All observations should be recorded clearly and distinctly on the proper forms. The numbers must be written so plainly that there is no chance of misunderstanding them. Erasures must never be made in the original records; if it is necessary to change a figure, it should be lightly crossed out and the correct figure placed above it or to one side, leaving the original entry still legible. The recorder must be ever alert to record each reading on its appropriate line so that no changes will have to be made in the records. More uncertainty in the results is caused by hurried and ill-considered changes in the recorded figures to correct supposed discrepancies than from any other one source. The doubt is greatly increased if the original figures are illegible. The observer must make certain before beginning work that the recorder understands the importance of observing the requirements of this section.

(3) In the repetition method (par. 107*c*) the recorder obtains the approximate value of the sixth reading for the information of the observer. By subtracting any even multiples of 60° from the first angle before the multiplication, the actual degrees of the sixth repetition are obtained. In multiplying an angle by 6, the tens of seconds are not actually multiplied but carried over as units.

112. Equipment.—Before going into the field to occupy a station, the triangulator should—

a. Study the reconnaissance plat and with a protractor measure the angles between the stations of the main and secondary nets which

have not been previously obtained, scale the distances between stations, and prepare a tentative program for everything to be done at the station.

b. Assemble necessary maps, descriptions of stations to be occupied and read in, a tracing from the reconnaissance plat, and a list of angles already observed from adjacent stations. The above articles, together with prepared notebook, TM 5–236, and blank forms should be kept in a case to be carried by himself or the recorder.

c. Know that the theodolite is in adjustment, and that all accessories are in place. In the field, the theodolite is constantly under the care of the observer and the recorder.

d. If night reading is contemplated, check over the illumination system of the instrument and make certain that any lights to be shown from a station are complete with spare bulbs and batteries. See that suitable arrangements have been made for any other lights to be shown.

e. Check to see that the following articles are carried by the observing party:

(1) Theodolite and tripod with accessories.
(2) Map and notebook case with contents (*b* above).
(3) Field glasses.
(4) Prismatic compass.
(5) Spare batteries and bulbs for instrument and flashlight, and signal lamps.
(6) Flashlights.
(7) Signal lamps, etc.
(8) Plumb bob and cord.
(9) Steel tape.
(10) Vertical collimator (if available).
(11) Pencils and eraser.
(12) Celluloid protractor.
(13) Engineer scale.
(14) Watch.
(15) Canvas for wind and sun shield if required.
(16) Ax.
(17) Hammer.
(18) Nails, wire, and signal cloth.
(19) Station mark, sledge, and signal (for grid triangulation).

113. Method of observing.—*a. Plumbing signal.*—Upon reaching the station the observer should first examine the signal to see that the part pointed upon is plumb over the station mark. If not, and the station has been observed upon from any other station, the eccentric distance and angle must be measured and recorded (par. 109*c*). The

vertical collimator should be used in determining the vertical line to the station mark. If no collimator is carried, this line may be determined by setting the theodolite on two positions at right angles to each other and dropping the signal center to the ground mark, or vice versa. If a tower has a spar structure upon which the signal is mounted, a point should be established on the superstructure directly over the station mark, and directly below the center of the signal.

b. Plumbing instrument.—After the signal has been plumbed, the theodolite is set directly under the mark on the superstructure of the tower by hanging a plumb bob from the mark and centering the theodolite by shifting the instrument until the point of the plumb bob rests directly over the small countersunk mark in the top center of the telescope. The instument should be secured to prevent shifting. If the tripod is used, plumb the instrument by a plumb bob hung from the bottom of the instrument.

c. Observations.—The method and number of observations are predetermined by the type of instrument and the order of accuracy, as already described. Except in grid triangulation, vertical angle observation should be confined to the 3 hours after noon (par. 110*d*), and the horizontal angles of the main scheme and distant azimuth marks should be observed whenever conditions are favorable. As time is available, the following observations should be made:

(1) Measure and record any eccentricity of signal or instrument.

(2) Supplementary stations.

(3) Intersection stations.

(4) Reference and azimuth marks.

(5) Heights of instrument and various parts of signal above station mark.

(6) Check measurements and complete the description of station.

d. The triangulator should then—

(1) Check to make sure that all of above have been completed satisfactorily.

(2) Check all the notes and mark each page checked with initials.

(3) Check triangle closures, if any have been completed.

(4) Compute reductions to center if any triangle closures are involved.

(5) Check that signal is secured in position plumb over the mark.

(6) Check that no Government property is left behind.

e. In *grid triangulation* definite orders should be given the observers as soon as the reconnaissance is finished, covering the following:

(1) Name of station occupied.

(2) Names of stations to be observed, with description and approximate azimuths.

(3) How Form 6, "Transcript of results," will be collected.

(4) When to leave the station and where to proceed.

114. Disposition of records.—*a. General.*—Throughout the triangulation, all concerned must bear in mind that any maps, notes, records, etc., which come into their hands are very valuable and that they are to be kept safe and in good condition and returned promptly to headquarters when no longer needed in the field. Before taking the field, all notebooks should be permanently numbered on the cover, the prescribed title page filled out, the index started, and all double pages numbered in the upper right-hand corner. The triangulator and recorder must take constant care that the notebooks are kept up to date and checked in every respect and that all ambiguities are avoided. Should the record on any page or part page be rejected, two neatly ruled diagonals should be drawn over that part of the record, the word "void" and the reason marked beside it, with reference to the page replacing it.

b. Transcript of results, Form 6 (fig. 82).—(1) The chief difficulty in making out this form lies in deciding what observations to reject. For observations with the type of direction theodolite usually used on third-order triangulation, the rejection limit for the angular value of a direction on any one position of the circle may ordinarily be taken as 5 seconds from the mean. Specific ways in which the rejection limit is to be applied are indicated below:

(*a*) No reading should be rejected if it falls within the limit of retention unless rejected at the time of taking the observation. The observer's reason for rejection should then appear in the original record. This rule will not apply to the case where one set of observations of a direction is rejected in favor of another set of an equal number of positions.

(*b*) If two or more readings have been taken for a single position, the mean should be used if all readings come within the limit of retention.

(*c*) If one reading falls without the limit and one within the limit, do not use a mean even though the mean be within the limit. Use instead the single reading within the limit.

(*d*) If both readings fall without the limit, one being abnormally high and the other abnormally low, and the mean falls within the limit, reject both readings and try the position again.

(*e*) Before computing a trial mean, any observation so far from the approximate mean as to be very evidently the result of blunders should be rejected. After a trial mean is obtained and the rejection limit applied, none of the observations rejected should be again in-

cluded even though the new mean would bring them within the limit of rejection.

(*f*) The results obtained by applying rigorously the limit of rejection, even though the quantities rejected are just outside the limit, will probably be but little different from those derived after long consideration, and much time can be saved the field party by a strict application of the rules given above.

(2) Similar rules may be devised for vertical angles and horizontal angles by repetition. It must always be remembered that the first sure criterion of the quality of observation is the average error of triangle closure. Any method of selecting values in such manner as to arbitrarily obtain favorable closures is not to be thought of, as the final criterion is to close on a check line. Provided the conditions outlined in paragraph 111*a* are maintained, the instrument may be relied upon to furnish suitable readings to meet both criteria, if only it is skillfully handled and the readings made impartially with the idea of recording exactly what the circles show at each stage of the work.

(3) Form 6, "Transcript of results," is not made out until the observer has checked every figure of the notes. The observing party fills in the date, the station occupied and a diagram of its signal heights, and the *H.I.*; the stations observed and the observed angles or directions; and the last four lines showing the vertical angles, the parts of the signals observed, and the notebook references. As some preliminary computations of third-order triangulation and the complete computations of grid triangulation will be made before the notebook reaches the computing section, above copying must be double-checked. The use of Form 6 in the computing section is described in paragraph 118*f*(2)(*c*).

c. The party progress chart may be kept on a tracing from the map or plat of the system. The triangle closures are checked as soon as completed, and any additional observations are made before the party moves on. The maps should be frequently consulted with a view of making intersections wherever useful, and of planning the successive occupation of the stations with the smallest expenditure of time and effort.

d. Exercise XVII. Grid triangulation.—Observe all the pertinent horizontal and vertical angles of a two-triangle system and at least one intersection, similar to the work computed in paragraph 118. Fill out Forms 5 and 6 with the field data on which the computations of exercises XVIII to XXII will be based.

SECTION XIX

ADJUSTMENT AND COMPUTATION OF TRIANGULATION

	Paragraph
General	115
Procedure of computations	116
Computation of third-order triangulation	117
Solution of grid triangulation	118
Miscellaneous computations	119

115. General.—*a. Necessity for careful planning.*—For many purposes, the data from field observations have to be computed or converted to other forms. In order to maintain the standard of accuracy secured in the field work, the computations have to be performed in the proper order and by approved methods, and to be checked throughout. The orderly procedure of work through the office, the publication of results, and the indexing and filing of records must have careful attention. Special consideration must be given the selection of personnel and the assignment of tasks. While the more complicated work, particularly the self-checking variety, is done by the more experienced men, there is much of a routine nature, requiring frequent comparisons which can be accomplished by beginners.

b. Printed forms.—These have been provided to save time, to insure consistent methods, and to assist in checking. Such forms are often self-explanatory, but notes have been placed at the top of those less commonly employed in order to avoid too frequent reference to the manual. Until they are familiar, each form should be studied as to its purpose and general arrangement, and the filled out specimens in the manual should be inspected to see how the light vertical dotted lines are intended to keep digits vertically in column, and the light vertical dashed lines to take the place of decimal points. Whenever a plus or minus sign (±) is placed opposite a space, the correct sign should there be written as a guide to the positive or negative value of the quantity. Following common practice, Greek letters appear in some of the forms. (See sec. XLVII, TM 5–236 for the Greek alphabet.) Form 1A (substitute) is designed for graphs, special computations, and as a substitute if the other forms are not available.

c. Dropping extra decimal places.—No question arises if the figures dropped represent either more or less than one-half a unit of the last decimal place retained. When the figures in the dropped decimal places represent exactly one-half a unit of the last decimal place retained, then the number adopted may have two values both of which are equally correct. To avoid confusion the engineers have arbi-

trarily adopted the plan of using the nearest *even* figure (divisible by 2) for the last decimal place retained. For example, in rounding off to three decimal places the numbers 0.4215 and 0.6245, the adopted numbers should be 0.422 and 0.624, respectively.

d. Advice to beginners.—(1) The headings of every form should always be filled in as soon as the form is started.

(2) All work should be done carefully and legibly. Do not write one figure over another. Erase entirely the one which is superseded.

(3) Consideration should be given to the order in which the lines of a form are to be filled in. Where data have to be copied from other sources, all of the data should be copied at once, including the station names, always preceded by the symbol showing type of station. If several functions of an angle have to be looked up, all should be done at one opening of the book.

(4) Computers should form their habits so as to economize on effort, which may be saved by a convenient and habitual placing of the blank form, tables, eraser, and so on. Each computer should develop his own system after consultation with those having more experience. For example, the looking up of a log sine to the nearest second in the Vega tables should follow a definite routine. Carrying the degrees, minutes, and tens of seconds in the head, find the correct page and column in the tables; place the forefinger of the left hand below the four figures corresponding to the required 10 seconds, but do not read them; read the integral part of the log and first three decimals and record them on the form. Then read the units of seconds required, look up the corresponding proportional part, apply it to the four figures marked by the finger, and record.

(5) An inexperienced computer should feel free at all times to consult the noncommissioned officer in charge or any of the senior computers, though knowledge obtained by study and hard work is more easily retained.

(6) The computer or checker should sign and date the form as soon as his part of the work is completed.

e. Use of log tables.—The Vega seven-place tables will be utilized only where the five-place tables in TM 5–236 will not serve. The special Vega 1-second tables of log sines and log tangents from 0° to 5° and of log cosines and log cotangents from 85° to 90° will be employed wherever they are applicable. Beside other tables in TM 5–236 the computer should become familiar with any others required for his work, which might include the following Special Publications of the United States Coast and Geodetic Survey:

No. 5. Tables for a Polyconic Projection of Maps.

No. 8. Formulae and Tables for the Computation of Geodetic Positions.
No. 59. Grid System for Progressive Maps in the United States.
No. 71. Relation between Plane Coordinates and Geographic Positions.

f. Mechanical aids.—These include squared paper for making graphs (form 1A), the slide rule, and the calculating machine. The slide rule is extremely useful in checking work for gross errors and in small computations of all sorts. An instruction book is or should be issued with each rule. The slide rule is easier to use than logarithms and is just as accurate, within its capacity. The calculating machine is in general use in many offices. Many models have automatic multiplication and division; some models are made to run either by electric power or by a hand crank. For most calculations involving more than three significant figures, the result may be obtained more quickly by the calculating machine than by any other method. Its use relieves the computer of the mental fatigue accompanying arithmetical or logarithmic calculations. The chances of error are greatly decreased. The forms in this manual are designed for logarithms, but they may be readily adapted to the use of the machine and Ives' or Benson's tables of natural trigonometric functions. If machines were in general use in the Army, many of the forms could be rearranged to shorten the work still more.

g. Aids to accuracy.—Self-checking forms are those in which identical results have to be obtained by means of two completely different computations. Form 9, "Computation of position," is an example. Such a check is an almost certain proof of the correctness of all stages of the computation, provided the check is obtained without too much searching for mistakes in the preceding work. Other forms, like Form 7, "Quadrilateral adjustment," and Form 8, "Solution of triangles," are partially self-checking and the other portions should be otherwise checked. Computation in duplicate is not a complete check on accuracy, since both computers might make the same error. This is the more likely to be true with beginners whose errors are so frequent as to necessitate comparisons at each stage of the work, in order to avoid much waste of time on account of undetected errors. All should understand that both computer and checker are responsible for any error in the final computation.

116. Procedure of computations.—*a. General.*—The steps involved in the computation of third-order triangulation (par. 117) are as follows:

(1) Assembling data, Forms 6 and 13, and notebooks.

(2) Eccentric corrections, Form 5.
(3) Quadrilateral adjustment, Form 7.
(4) Solution of triangles, Form 8.
(5) Computation of position, Form 9.
(6) Geographic to grid coordinates, Form 10.
(7) Azimuth and distance from grid coordinates, Form 11.
(8) Trigonometric elevations, Form 12.
(9) Record of results, Forms 13 and 15.
(10) Disposition of completed computations.

Though numbers (6) and (7) usually are not considered as parts of third-order computations, they are included here because a majority of positions thus obtained are intended for control points on which grid triangulation will be based.

b. Spherical excess.—The observed angles, after the horizon adjustment and reduction to center, are not plane angles, but are really angles of a spheroidal or spherical triangle on a sphere of the mean radius of curvature of the spheroid at the mean latitude of the vertices of the triangle. The sum of the three angles should be 180° plus the spherical excess, which depends upon the radius of curvature and the area of the triangles, being approximately 1 second for each 75.6 square miles. To compute the spherical excess of a triangle it is necessary to make a preliminary computation of the triangle to obtain the logarithms of the sines of the angles and the logarithms of the lengths. In this preliminary computation, the logarithms need be carried out to only 4 places of decimals. The excess is then computed in the following manner:

$$\begin{aligned} \log a &= 4.7176 \\ \log b &= 4.7362 \\ \log \sin C &= 9.9099-10 \\ \log m &= 1.4041-10 \\ \hline \log \epsilon &= 0.7678 \end{aligned}$$

Spherical excess (ϵ)=5.859 seconds. a and b are two sides of the triangle in meters, C is the included angle, and log m (in meters) is taken from table XVII, TM 5–236. When the spherical excess is applied one-third of the excess is subtracted from each of the three angles of the triangle, thus reducing the spherical angles to plane angles whose sum should be 180°. The areas in third-order work are so small that the spherical excess will seldom exceed 1 second. To avoid the preliminary computation of sides, the figures are accordingly adjusted on the plane angles, the sum of the three angles of each triangle being adjusted to 180°. To secure the desired checks in the computation of position, the spherical angles must be used after being

computed as shown on Form 8, "Solution of triangles," figure 75. The plane angles have been used elsewhere.

c. Azimuth and back azimuth.—On Form 9, "Computation of position" (fig. 76), it will be noticed that the differences of the azimuths and back azimuths of the lines differ from 180° by an amount denoted as $\Delta\alpha$, which is caused by the convergency of the meridians toward the poles.

$\Delta\alpha$ is computed on the form. In figure 73, the line NS represents the meridian passing through station A, $N'S'$ the meridian passing through station B. The dotted line yY_1 is parallel to $N'S'$. From the figure it is evident that the angle α_1 (or the azimuth from A to B)

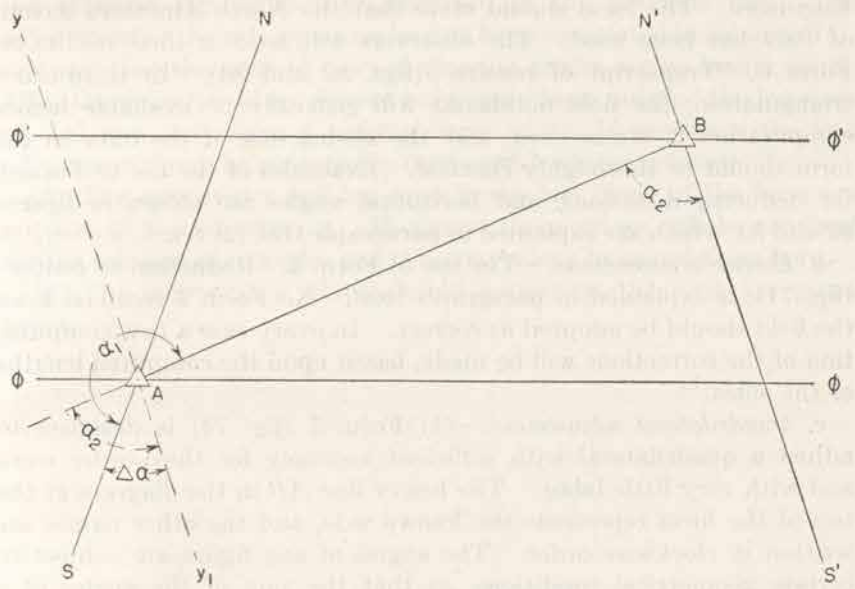


FIGURE 73.—Azimuth and back azimuth.

is not exactly 180° different from the angle α_2 (or the azimuth from B to A, which is also called the back azimuth of A to B). This figure shows that the azimuth BA is equal to

$$\text{azimuth } AB - 180° + \text{angle } \Delta\alpha,$$

$\Delta\alpha$ being the correction in seconds computed from the formula

$$\Delta\alpha = (\lambda - \lambda^1) \sin \frac{1}{2}(\phi + \phi^1)$$

in which $(\lambda - \lambda^1)$ is the difference of the longitude (in seconds) between A and B, and $\frac{1}{2}(\phi + \phi^1)$ is the middle latitude between A and B.

$\Delta\alpha$ must be applied so as always to make westerly azimuths greater than easterly azimuths, not counting the 180°. In figure 73, the azimuth of BA (or the back azimuth of AB) is equal to

$$\alpha_1 - 180° + \Delta\alpha$$

The azimuth of AB (or the back azimuth of BA) is equal to

$$\alpha_2 + 180° - \Delta\alpha$$

117. Computation of third-order triangulation.—*a. Assembling data.*—The basic data of the survey will generally be obtained from "Geographic Positions" furnished by the United States Coast and Geodetic Survey, or Form 13 (same title, fig. 80) by the Corps of Engineers. The form should state that the North American datum of 1927 has been used. The observers will send in their results on Form 6, "Transcript of results" (figs. 82 and 83). In third-order triangulation, the field notebooks will generally be available before computation is commenced, and the abstracting of the data to the form should be thoroughly checked. Examples of the use of Form 6 for deducing directions, and horizontal angles are shown in figures 82 and 83, which are explained in paragraph 118f (2) (*c*).

b. Eccentric corrections.—The use of Form 5, "Reduction to center" (fig. 71), is explained in paragraph 109*d*. No Form 5 received from the field should be adopted as correct. In every case a new computation of the corrections will be made, based upon the computed lengths of the sides.

c. Quadrilateral adjustment.—(1) Form 7 (fig. 74) is designed to adjust a quadrilateral with sufficient accuracy for third-order work and with very little labor. The heavy line AB in the diagram at the top of the form represents the known side, and the other names are written in clockwise order. The angles of any figure are subject to certain geometrical conditions, as that the sum of the angles of a plane triangle is 180°. Since no observations are perfect, the observed angles will not exactly satisfy the geometrical conditions, but it is possible to obtain improved values of the angles by distributing among them such minimum corrections as will cause them to satisfy the conditions.

(2) The geometrical conditions which the angles must fulfill are as follows:

$$1+2+3+4+5+6+7+8=360°$$
$$1+2=5+6$$
$$3+4=7+8$$

These adjustments are made in columns (3) to (7) in such manner that the sums of the angles of each triangle are 180°.

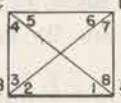
(3) In order that the computation of any one of the unknown sides may give the same result, no matter which triangles are employed in the computation, the following side equation must be satisfied: log sin 1+log sin 3+log sin 5+log sin 7—log sin 2—log sin 4—log sin 6—log sin 8=0

(4) The amount of the error (l) is calculated in columns (8) and (9). The correction in seconds for each angle is then $\pm d_n \frac{l}{\sum d^2}$. These corrections have to be given such signs for the odd angles and for the even angles that l will be brought to zero. The log sine of an angle over 90° decreases as the angle increases, and so such corrections take the opposite sign from that described in the previous rule. The adjustment of the side equation having been completed in column (14), it is seen that the sums of pairs of opposite angles are no longer equal. After the geometrical conditions have again been satisfied the log sines of (18) and (19) show a residual error of 1. If necessary the two adjustments may be repeated in turn until both are satisfied.

(5) The sum angles and log sines in the last block of the form are no part of the adjustment. However, these values will be required for the solution of triangles and it is convenient to record them here.

(6) The instructions which should appear on the form are given below.

French C D Lee



Herkness B A Proctor

(1) No. of Angle	(2) Observed angles (Ecc. corr.)			(3) Corr. to 360°	(4) Adj. to 360°	(5) Sums of Pairs			(6) Corr. opp. ang.	(7) Opp. ang. adj.	(8) Log sines odd numbered angles			(9) Log sines even numbered angles		
1	19	30	36.8	- 0.9	35.9				—	35.9	9	523	7086			
2	35	22	34.0	- 0.9	33.1	54	53	09.0	+ 0.1	33.2				9	762	6321
3	54	40	22.8	- 0.9	21.9				+ 0.5	22.4	9	911	6178			
4	70	26	29.0	- 1.0	28.0	125	06	49.9	+ 0.5	28.5				9	974	1886
5	27	40	03.1	- 0.9	02.2				- 0.1	02.1	9	666	8322			
6	27	13	08.0	- 0.9	07.1	54	53	09.3	- 0.1	07.0				9	660	2836
7	94	45	41.8	- 1.0	40.8				- 0.5	40.3	9	998	4987			
8	30	21	11.9	- 0.9	11.0	125	06	51.8	- 0.4	10.6				9	703	5710
	360	00	07.4		00.0			4)0.3		00.0	39.100		6573	39.100		6753
		8)07.4						0.0+0.3								6573
		0.9+0.2						4)1.9				l = +			l = -	180
								0.4+0.3								7

(10) No. of Angle	(11) d for 1 sec.	(12) d^2	(13) $d\frac{l}{\Sigma d^2}$	(14) Corr. angle	(15) Sums of pairs	(16) Corr. opp. angle	(17) Adjusted angles			(18) Log sines odd numbered angles			(19) Log sines even numbered angles		
1	59.4	3528.4	+ 1.2	37.1		-0.1	19	30	37.0	9	523	7152			
2	29.7	882.1	- 0.6	32.6	09.7	- 0.2	35	22	32.4				9	762	6297
3	14.9	222.0	+0.3	22.7		- 0.2	54	40	22.5	9	911	6179			
4	7.5	56.2	-0.2	28.3	51.0	-0.2	70	26	28.1				9	974	1883
5	40.2	1616.0	+0.8	02.9		+0.2	27	40	03.1	9	666	8363			
6	40.9	1672.8	-0.8	06.2	09.1	+0.1	27	13	06.3				9	660	2808
7	-1.7	2.9	—	40.3		+0.2	94	45	40.5	9	998	4987			
8	36.0	1296.0	-0.7	09.9	50.2	+0.2	30	21	10.1				9	703	5692
	Sum	9276.4		00.0			360	00	00			6681			6680
												6680			
												l = + 1			l = -

$\frac{l}{\Sigma d^2} = \frac{180}{9276.4} = 0.02$

59.4 X 0.02 = 1.2
29.7 X 0.02 = 0.6
Etc.

	Add	Sum angles			Log sines	
A	1 & 8	49	51	47.1	9 883	3810
B	2 & 3	90	02	54.9	9 999	9999
C	4 & 5	98	06	31.2	9 995	6362
D	6 & 7	121	58	46.8	9 928	5167

Date: March 25, 1940 Computed by: J. H. B. Checked by: S. S. H. FORM 7

FIGURE 74.—Quadrilateral adjustment (approximate).

QUADRILATERAL ADJUSTMENT (APPROXIMATE)

INSTRUCTIONS

(1) and (10) The angles are numbered as in diagram.

(2) Filled from form 6 or from notebooks.

(3) and (6) Apply the extra tenths to the larger angles.

(4) Spherical excess of less than 1 second per triangle may be disregarded at this point.

(8) and (9) Log sines of corrected angles shown in (7). l is algebraic sum of log sines in (8) and (9), regarding even numbers as negative.

(11) Tabular difference (log sin) for 1 second, negative if angle is over 90°.

(13) $\frac{l}{\Sigma d^2} \times d_n$ = correction applied to each angle, remembering that even numbers are considered negative.

(15) The sums of pairs unbalanced during the side adjustment are corrected in (16), keeping sum 360° and all triangles 180°.

(18) and (19) To detect residual errors.

Last block, sum angles and log sines are placed here for convenience in solving triangles.

d. Solution of triangles.—(1) Form 8 (fig. 75) is arranged for the computation of log sides by the well-known law of sines. The clockwise manner of listing the angles and triangles should be habitually followed as outlined at the top of the form. Misunderstandings are thus avoided, and at the same time the form is adapted to other purposes as described in paragraph 119*b* and *c*. The instructions which should appear on the form are given below.

SOLUTION OF TRIANGLES

INSTRUCTIONS

(1) Entered from the diagram in clockwise order, naming first the ends of the known side. The triangles are listed in clockwise succession, beginning with the one containing the known side.

(3) and (4) The extra tenths to larger angles. The seconds of (2) may now be marked out to avoid confusion.

(5) and (6) Subtracting log (sine of opposite angle) from log (known side) yields log (known side divided by sine of opposite angle) which is written in fourth line of (5). By adding the latter in turn to each of the other log sines, the log (side opposite each) is obtained and written in (6).

(7) Using same notation as in (1).

(8) For convenience in checking identical values.

(9) If spherical angles are required, the spherical excess is computed and entered in fourth line of (1). One-third of spherical excess is added to each of the adjusted plane angles of (4).

(1) Station and letters	(2) ~~Observed Sph. Exc. angles~~	Corr. Angles (3) ~~Corr.~~	(4) ~~Corr. Plane~~ Angle	(5) Log sine	(6) Log side	(7) Side	(8) See No.	(9) Sph. Angle
French C A D Lee						BC		
						AC		
Herkness B C A Proctor						AB		
No. 1 Sph. Exc.								
(1) Δ Proctor A	BC = 3.8281 AB = 4.2785	19° 30′	37.0	9 523 7152	3 828 0596	BC	3	37.1
Δ Herkness B	B = 0.0000	90 02	54.9	9 999 9999	4 304 3443	AC	4	55.0
(4) Δ French C	m = 1.4046	70 26	28.1	9 974 1883	4 278 5327	AB	5	28.2
No. 2 Sph. Exc. = 0.3	log 0.32 = 9.5112	180 00	00.0	4 304 3444				
(3) Δ Herkness B	BC = 3.8281 CD = 4.0794	54 40	22.5	9 911 6179	4 079 3967	CD	4	22.6
Δ French C	c = 9.9956	98 06	31.2	9 995 6362	4 163 4150	BD	5	31.3
(6) Δ Lee D	m = 1.4046	27 13	06.3	9 660 2808	3 828 0596	BC	2	06.3
No. 3 Sph. Exc. = 0.2	log 0.20 = 9.3077	180 00	00.0	4 167 7788				
(5) Δ French C	AD = 4.0427 CD = 4.0794	27 40	03.1	9 666 8363	4 042 6638	AD	5	03.2
Δ Lee D	D = 9.9285	121 58	46.8	9 928 5167	4 304 3443	AC	2	46.9
(8) Δ Proctor A	m = 1.4046	30 21	10.1	9 703 5692	4 079 3967	CD	3	10.2
No. 4 Sph. Exc. = 0.3	log 0.29 = 9.4552	180 00	00.0	4 375 8275	(Because of 1=1 on Form 7)			
(7) Δ Lee D	AB = 4.2785 AD = 4.0427	94 45	40.5	9 998 4987	4 278 5328	AB	2	40.7
Δ Proctor A	A = 9.8834	49 51	47.1	9 883 3810	4 163 4150	BD	3	47.2
(2) Δ Herkness B	m = 1.4046	35 22	32.4	9 762 6297	4 042 6638	AD	4	32.5
No. 5 Sph. Exc. = 0.4	log 0.41 = 9.6092	180 00	00.0	4 280 0341				
	38 36 12	Δ	ABC	Sph. Exc. 0.32				
	38 42 01	Δ	CDA	0.29				
	38 42 08		Sum	0.61				
No. 6 Sph. Exc. 3	116 00 21	Δ	BCD	0.20				
	38 40 07	Δ	DAB	0.41 Check				
	log m = 1.40465-10			0.61				
No. 7 Sph. Exc.								
No. 8 Sph. Exc.								

Date: March 25, 1940 Computed by: J.H.B. Checked by: S.E.H. FORM 8

FIGURE 75.—Solution of triangles.

(2) In figure 75, the figure diagram has been copied from Form 7, "Quadrilateral adjustment," as were the corrected angles and log sines. The headings of columns (2), (3), and (4) have been changed to leave space in column (2) for the computation of the spherical excess. The sum of the spherical excess for two triangles with one diagonal of the quadrilateral common to both should exactly equal the sum for the two triangles on the other diagonal, as shown in the lower part of the form. Solution of only two of the triangles would enable the computation to have been carried forward. Nevertheless, all of the triangles should invariably be solved, in order that the agreement of the values of common sides, derived by different routes, may independently check the accuracy of the competition.

e. Computation of position.—Form 9 (fig. 76) is in two separate halves, the new position being computed from station 2 in the left half, and from station 3 in the other. In this form the angles are still numbered clockwise but start with the unknown angle. The instructions which should appear on the form are given below.

COMPUTATION OF POSITION

INSTRUCTIONS

Latitudes and longitudes of stations 2 and 3, and the azimuth and back azimuth between them must be known or computed.

The logs *s* (side in meters) and the *spherical* angles are obtained from form 8, angles 1 and 2, being positive, and 3 negative.

The logarithmic factors *A'*, *B*, *C*, and *D* are found in Special Publication No. 8, United States Coast and Geodetic Survey or Bulletin 650, United States Geological Survey, *A'* for latitude of new station, the others for old station.

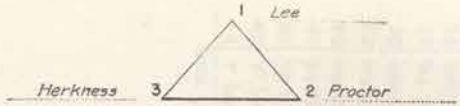
The first term is negative if new station is north, positive if south of the old station. Second and third terms are always positive.

$\Delta\lambda$ is negative if new station is east of old.

After transcribing all above (except *A'*) to the form, find $\alpha(2-1)$ in third line, solve for $\Delta\phi$ in lower left quarter, deduce ϕ' in the ninth line and solve for $\Delta\lambda$ and $\Delta\alpha$ in the lower right quarter of each half of the form and deduce λ' and α' in the upper portion.

If latitudes or longitudes fail to check by one unit in the last place of decimals, change one of the values by one unit in such manner as to make both this position and the next one check.

If azimuth (1—2) plus angle (1) fails to check azimuth (1—3) by one unit in the last decimal place, a correction must be applied to make the result consistent before proceeding with the next position.



α	2		to 3			124°	38'	59.5"	α	3		to 2			304°	32'	15.7"
L2			B			+49	51	47.2	L3			B			−35	22	32.5
α	2		to 1			174	30	46.7	α	3		to 1			269	09	43.2
Δα ±						−	00	27.3	Δα ±						+	06	16.9
					±	180	00	00.0						±	180	00	00.0
α'	1 Δ Lee		to 2 Δ Proctor			354	30	19.4	α'	1 Δ Lee		to 3 Δ Herkness			89	16	00.1
					Angle 1 +	94	45	40.7									
						89	16	00.1									

Check

Φ	38°	35'	11.707"	2	λ	76°	58'	34.834"	Φ	38°	42'	01.368"	3	λ	77°	09'	21.326"
ΔΦ ±	+	05	56.140		Δλ ±	+	00	43.656	ΔΦ ±	+	00	06.478		Δλ ±	−	10	02.835
Φ'	38	42	07.847	1 Δ Lee	λ'	76	59	18.490	Φ'	38	42	07.846	1 Δ Lee	λ'	76	59	18.491

	Logarithms			Value in seconds
s	4	042	6638	
Cos α	9	998	0054	
B	8	510	9574	±
h	2	551	6266	1st term −356".145
s²	8	085	33—	
Sin² α	7	961	10—	
C	1	306	87—	
	7	353	30—	2d term + 0.002
h²	5	103	3—	
D	2	381	4—	
	7	484	7—	3d term + 0.003
				−ΔΦ −356.140

½(Φ + Φ') 38° 39' 09".777

	Logarithms			Value in seconds
s	4	042	6638	
Sin α	8	980	5504	
A'	8	509	1511	
Sec Φ'	0	107	6791	
Δλ ±	1	640	0444	+ 43.656
Sin ½(Φ+Φ')	9	795	6008	
−Δα ±	1	435	6452	+ 27.3

	Logarithms			Value in seconds
s	4	163	4150	
Cos α	8	165	1059	
B	8	510	9501	±
h	0	839	4710	1st term − 6".910
s²	8	326	83—	
Sin² α	9	999	91—	
C	1	308	37—	
	9	635	11—	2d term + 0.432
h²	1	678	9—	
D	2	381	7—	
	4	060	6—	3d term + 0.000
				−ΔΦ − 6.478

½(Φ + Φ') 38° 42' 04".608

	Logarithms			Value in seconds
s	4	163	4150	
Sin α	9	999	9535	
A'	8	509	1511	
Sec Φ'	0	107	6791	
Δλ ±	2	780	1987	− 602.835
Sin ½(Φ+Φ')	9	796	0607	
−Δα ±	2	576	2594	− 376.9

Date: March 26, 1940 Computed by: J. H. B. Checked by: G. J. H. FORM 9

FIGURE 76.—Computation of position.

f. Geographic to grid coordinates.—(1) If the control points are furnished on Form 13, "Geographic positions," the need for this conversion before grid computation is obvious. United States Coast and Geodetic Survey Special Publication No. 59 shows a method of converting by interpolation, and a more precise method of computing by formula. Form 10 (fig. 77①) follows the latter with some modifications. Form 10 also differs from the coast artillery form in that the meridional distance on the central meridian is interpolated from the coordinate tables instead of from Special Publication No. 8. The results check within a few tenths of those obtained by the usual "computation of artillery grid coordinates from geographic coordinates." Computers accustomed to using seven-place tables will find form 10 less laborious.

(2) The logarithm of K is obtained in the first third of the form, followed by X', log Y'. The last quarter of the form is devoted to obtaining the Y value on the central meridian for the latitude of the station, and the Y coordinate results. The formulas for each are

$$K = \frac{\alpha \cot \phi}{\sqrt{1-e^2 \sin^2 \phi}}$$

$$X' = K \sin (\delta\lambda \sin \phi)$$

$$Y' = 2K \sin^2 (\delta\lambda \sin \phi \div 2)$$

X' is the distance of the station grid-east or grid-west of the central meridian in yards; Y' is the distance in yards of the station grid-north of the intersection of the central meridian of the zone and the parallel of latitude passing through the station. The instructions which should appear on the form are given below.

GEOGRAPHIC TO GRID COORDINATES

INSTRUCTIONS

X and Y coordinates are on (25) and (44) respectively.
CM=central meridian, e=eccentricity, α=equatorial radius in yards.
(21) and (29) From table XII, TM 5-236.
(24) Be careful to get all significant figures correct. X' is positive east, negative west of CM.
(34) Y' is due to curvature of parallels.
(35) $Y5$ is CM value at first 5-minute intersection above or below station latitude.
(39) $\delta\phi$ is difference in seconds between ϕ of station and first 5-minute grid line to south.
Use seven-place tables with careful interpolation.

1	Station	Δ Herkness	Δ Proctor		
2	Zone Letter	B	B		
3	Latitude ϕ	38 42 01 368	38 36 11 707		
4	Longitude λ	77 09 21 326	76 58 34 834		
5	$\delta\lambda$ (sec. to CM)	13 838 674	14 485 166		
6	log sin ϕ (see 16)	9 796 0522	9 795 1317		
7	log $\sin^2\phi$ (X 2)	9 592 1044	9 590 2634		
8	log e^2	7 830 5026	7 830 5026	7 830 5026	7 830 5026
9	log $e^2 \sin^2\phi$ (sum)	7 422 6070	7 420 7660		
10	$e^2 \sin^2\phi$	0 002 6461	0 002 6349		
11	$1-e^2 \sin^2\phi$	0 997 3539	0 997 3651		
12	log $(1-e^2\sin^2\phi)$	9 998 8493	9 998 8541		
13	$\frac{1}{2}$ log $(1-e^2\sin^2\phi)$	9 999 4246	9 999 4270		
14	colog $(1-e^2\sin^2\phi)^{\frac{1}{2}}$	0 000 5754	0 000 5730		
15	log α	6 843 5615	6 843 5615	6 843 5615	6 843 5615
16	log cot ϕ	0 096 2797	0 097 7890		
17	log K (sum)	6 940 4166	6 941 9235		
18	log $\delta\lambda$ (see 5)	4 141 0945	4 160 9235		
19	log sin ϕ (from 6)	9 796 0522	9 795 1317		
20	log $(\delta\lambda \sin\phi)$ (sum)	3 937 1467	3 956 0552		
21	Σ for log $(\delta\lambda \sin\phi)$	4 685 4475	4 685 4359		
22	log K (from 17)	6 940 4166	6 941 9235		
23	log X^1 (sum)	5 563 0108	5 583 4146		
24	X^1	365 603.9	383 190.4		
25	$X = 1{,}000{,}000 \pm X^1$	1 365 603 9	1 383 190 4		
26	log $(\delta\lambda \sin\phi)$ (from 20)	3 937 1467	3 956 0552		
27	colog 2	9 698 9700	9 698 9700	9 698 9700	9 698 9700
28	log $(\delta\lambda \sin\phi \div 2)$ (sum)	3 636 1167	3 655 0252		
29	Σ for log $(\delta\lambda \sin\phi \div 2)$	4 685 5430	4 685 5401		
30	log sin $(\delta\lambda \sin\phi \div 2)$ (sum)	8 321 6597	8 340 5653		
31	log $\sin^2(\delta\lambda \sin\phi \div 2)$ (X2)	6 643 3194	6 681 1306		
32	log 2	0 301 0300	0 301 0300	0 301 0300	0 301 0300
33	log K (from 17)	6 940 4166	6 941 9235		
34	log Y^1 (sum)	3 884 7660	3 924 0841		
35	Y5 N. of Sta. (see 42)	1 787 518 5	1 777 401 9		
36	diff. Y5's (35–42)	10 116 6	10 116 5		
37	log (diff. Y5's)	4 005 0345	4 005 0302		
38	colog 300	7 522 8787	7 522 8787	7 522 8787	7 522 8787
39	log $\delta\phi$ (sec. over 5 S.)	2 084 1042	1 855 5616		
40	log $\delta Y(\phi-5)$	3 612 0174	3 383 4705		
41	$\delta Y(\phi-5)$	+ 4 092 8	2 418 1		
42	Y5 S. of Sta.	1 777 401 9	1 767 285 4		
43	Y^1 (antilog 34)	+ 7 669 5	8 396 2		
44	Y (sum)	1 789 164 2	1 778 099 7		

Date: March 31, 1940 Computed by: J.N.B. Checked by: B.H. FORM 10

① Geographic to grid coordinates.

FIGURE 77.—Interpolations and conversions.

INTERPOLATION OF GRID COORDINATES FROM GEOGRAPHIC COORDINATES

INSTRUCTIONS

X is on line 19 Y is on line 32

(2) and (5): Seconds of ϕ and λ above next lower 5-minute line for 5-minute intersections, above next lower minute line for 1-minute intersections.

(3) and (6): Divide by 300 (seconds) for the 5-minute intersections, and by 60 (seconds) for the 1-minute intersections, 4 and 3 decimal places respectively.

No.	Symbol	Operation	Herkness B	Herkness B		
1	ϕ		38° 42' 01.368"	38° 42' 01.368"		
2	Sec. ϕ over	5 or 1 min.	21.368	1.368		
3	Cϕ	(2)/300 or 60	.4046	.023		
4	λ		77 09 21.326	77 09 21.326		
5	Sec. λ over	5 or 1 min.	261.326	21.326		
6	Cλ	(5)÷300 or 60	.8711	.355		
7	X1	SW	1364 753.7	1364 584.6		
8		(13)X(3) ±	- 171.3	- 2.0		
9	X2	NW	1364 330.2	1364 499.8		
10	X3	NE	1372 245.5	1366 083.6		
11	X4	SE	1372 678.4	1366 168.8		
12		(14)X(3) ±	- 175.2	- 2.0		
13	X2-X1	(9)-(7) ±	- 423.5	- 84.8		
14	X3-X4	(10)-(11) ±	- 432.9	- 85.2		
15	XA	(7)+(8)	1364 582.4	1364 582.6		
16	XB	(11)+(12)	1372 503.2	1366 166.8		
17		(18)X(6) ±	- 6 899.8	- 562.4		
18	XA-XB	(15)-(16) ±	- 7 920.8	- 1584.2		
19	X	(16)+(17)	1365 603.4	1365 604.4		
20	Y1	SW	1785 025.7	1789 074.4		
21		(26)X(3)	4 095.2	46.6		
22	Y2	NW	1795 147.3	1791 098.6		
23	Y3	NE	1795 482.6	1791 165.1		
24	Y4	SE	1785 360.8	1789 140.6		
25		(27)X(3)	4 095.3	46.6		
26	Y2-Y1	(22)-(20)	10 121.6	2 024.2		
27	Y3-Y4	(23)-(24)	10 121.8	2 024.5		
28	YA	(20)+(21)	1789 120.9	1789 121.0		
29	YB	(24)+(25)	1789 456.1	1789 187.2		
30		(31)X(6) ±	- 292.0	- 23.5		
31	YA-YB	(28)-(29) ±	- 335.2	- 66.2		
32	Y	(29)+(30)	1789 164.1	1789 163.7		

Date: March 23, 1940 Computed by: JKB Checked by: BBH FORM 10A

② Interpolation of grid coordinates from geographic coordinates.

FIGURE 77.—Interpolations and conversions—Continued.

X is on line 19; Y is on line 32.

(2) and (5) Seconds of ϕ and λ above next lower 5-minute line for 5-minute intersections, above next lower minute line for 1-minute intersections.

(3) and (6) Divide by 300 (seconds) for the 5-minute intersections, and by 60 (seconds) for the 1-minute intersections, 4 and 3 decimal places, respectively.

CORRECTIONS TO CONVERT GEOGRAPHIC AZIMUTHS TO GRID AZIMUTHS

CSW, CNW, CNE and CSE = Azimuth corrections at the degree intersection SW, NW, NE and SE of point, from Special Publication No. 59.

$\delta\phi$ and $\delta\lambda$ = minutes and seconds of latitude and longitude expressed as 3-place decimal.

(15) = difference between geographic and grid azimuths, (−) in east half, (+) in west half of zone.

	Station	Herkness B	Herkness A	See page 19 Sp. Pub. No. 59		
	Zone					
	ϕ	38 42 01	38 42 01	42 50 00		
	λ	77 09 21	77 09 21	70 18 00		
1	CNW	1 53 15	3 08 39	1 21 50		
2	CSW	1 50 47	3 04 33	1 20 17		
3	(1)−(2)	2 28	4 06	1 33		
4	$\delta\phi$	0.700	0.700	0.833		
5	(4)X(3)	1 44	2 52	1 17		
6	(2)+(5)	1 52 31	3 07 25	1 21 34		
7	CNE	2 30 58	2 30 58	2 02 44		
8	CSE	2 27 41	2 27 41	2 00 25		
9	(7)−(8)	3 17	3 17	2 19		
10	(4)X(9)	2 18	2 18	1 56		
11	(8)+(10)	2 29 59	2 29 59	2 02 21		
12	(6)−(11) ±	− 37 28	+ 37 26	− 40 47		
13	$\delta\lambda$	0.156	0.156	0.300		
14	(13)X(12) ±	− 5 51	+ 5 50	− 12 14		
15	(11)+(14) ±	−2 24 08	+2 35 49	−1 50 07		

	Station					
	Zone					
	ϕ					
	λ					
1	CNW					
2	CSW					
3	(1)−(2)					
4	$\delta\phi$					
5	(4)X(3)					
6	(2)+(5)					
7	CNE					
8	CSE					
9	(7)−(8)					
10	(4)X(9)					
11	(8)+(10)					
12	(6)−(11) ±					
13	$\delta\lambda$					
14	(13)X(12) ±					
15	(11)+(14) ±					

Date: March 23, 1940 Computed by: J.H.B Checked by: G.G.H FORM 108

② Corrections to convert geographic azimuths to grid azimuths.

FIGURE 77.—Interpolations and conversions—Continued.

CSW, *CNW*, *CNE*, and *CSE* = azimuth corrections at the degree intersection *SW*, *NW*, *NE*, and *SE* of point, from Special Publication No. 59.

$\delta\phi$ and $\delta\lambda$ = minutes and seconds of latitude and longitude expressed as 3-place decimal.

(15) Difference between geographic and grid azimuths, (—) in east half, (+) in west half.

(3) While five-place logarithms are generally employed in grid computations, they are not useful, on Form 10, as X', for example, often has seven significant figures, which require complete interpolation.

(4) Form 10A (fig. 77②) is a rearrangement of the "Computation of artillery grid coordinates from geographic coordinates," shown in Special Publication No. 59. Either 5-minute or 1-minute intersections may be used, and logarithms are not needed. The coordinates should not differ more than a yard from those computed on Form 10. The instructions which should appear on the form are given below.

(5) Form 10B, "Corrections to convert geographic azimuths to grid azimuths" (fig. 77③), is an application of the "Transformation of geographic azimuths to grid azimuths" in United States Coast and Geodetic Survey Special Publication No. 59, a method of interpolating in the table of "Corrections for the reduction of geographic azimuths to grid azimuths" in the same publication.

g. Azimuth and distance from grid coordinates.—(1) In Form 11 (fig. 78), before the X and Y coordinate differences are obtained, the magnification of scale (see par. 118*c*) is deducted from the Y component, and the grid-azimuth is found through the tangent of its bearing. The direction diagram should always be plotted, as the quadrant of the azimuth is thus indicated with certainty. The form is partly self-checking as the log distance is found by both sine and cosine of the bearing. The two values should check closely, and the mean is taken as the log distance.

(2) In figure 78, one of the lines of the figure computed on the third-order forms is used as an example. This is an extreme case, since grid computation is not intended for such large figures. While the distance always checks within a yard or so, the geodetic line being the shorter, the azimuth may differ 10 seconds or more. This variation may be partly due to the use of five-place tables and the coarseness of the interpolation of the scale magnification. However, the differences between grid and geographic azimuths should be remembered because geographic coordinates derived from grid triangulation cannot be used to carry third-order work forward. In using the grid azimuth correc-

AZIMUTH AND DISTANCE FROM GRID COORDINATES

The diagram shows the quadrant of the azimuth.
As the magnification of scale will not be carried further, it is deducted from δY.

Sta. 1 △ Herkness	X	1 365 603.9	Y	1 789 164.2	Magnification of scale	
Sta. 2 △ Proctor	X	1 383 190.4	Y	1 778 099.7	at △ Proctor	
	δX	17 586.5	δY	11 064.5	1000 yds. (10ths.)	11.1
Grid ↓ North	log δX	4 245 18	MnS	− 16.6	yds. per 1000	1.5
	−logδ(Y-M)	4 043 28	δ(Y-M)	11 047.9	MNS	16.6
	log tan α.	0 201 90			Bearing	57 51 47
	log δX	4 245 18	logδ(Y-M)	4 043 28	Az. 1-2	302 08 13
Mean	−log sin α.	9 927 77	−log cos α.	9 725 86		
logs 4.31742	log S	4 317 41	log S	4 317 42	Yards	20769.0
Sta. 1	X		Y		Magnification of scale	
Sta. 2	X		Y		at	
	δX		δY		1000 yds. (10ths.)	
Grid ↓ North	log δX		MnS	−	yds. per 1000	
	−logδ(Y-M)		δ(Y-M)		MNS	
	log tan α.				Bearing	
	log δX		logδ(Y-M)		Az. 1-2	
Mean	−log sin α.		−log cos α.			
logs	log S		log S		Yards	
Sta. 1	X		Y		Magnification of scale	
Sta. 2	X		Y		at	
	δX		δY		1000 yds. (10ths.)	
Grid ↓ North	log δX		MnS	−	yds. per 1000	
	−logδ(Y-M)		δ(Y-M)		MNS	
	log tan α.				Bearing	
	log δX		logδ(Y-M)		Az. 1-2	
Mean	−log sin α.		−log cos α.			
logs	log S		log S		Yards	
Sta. 1	X		Y		Magnification of scale	
Sta. 2	X		Y		at	
	δX		δY		1000 yds. (10ths.)	
Grid ↓ North	log δX		MnS	−	yds. per 1000	
	−logδ(Y-M)		δ(Y-M)		MNS	
	log tan α.				Bearing	
	log δX		logδ(Y-M)		Az. 1-2	
Mean	−log sin α.		−log cos α.			
logs	log S		log S		Yards	
Sta. 1	X		Y		Magnification of scale	
Sta. 2	X		Y		at	
	δX		δY		1000 yds. (10ths.)	
Grid ↓ North	log δX		MnS	−	yds. per 1000	
	−logδ(Y-M)		δ(Y-M)		MNS	
	log tan α.				Bearing	
	log δX		logδ(Y-M)		Az. 1-2	
Mean	−log sin α.		−log cos α.			
logs	log S		log S		Yards	

Date: April 3, 1940 Computed by: J.W.B. Checked by: J.J.K. FORM II

FIGURE 78.—Azimuth and distance from grid coordinates.

tion table in Special Publication No. 59 (duplicated in table L, TM 5–236), the interpolation must be as close as possible (fig. 77③). In using some of the tables in the front of Special Publication No. 59, close attention must be given the grid zone letters before entering the longitude columns. In the tables of grid coordinates for 5-minute intersections, the values for the east half of each zone are given on the left hand pages.

h. Trigonometric elevations.—The determination of elevations from vertical angles to the ocean horizon, the effect of curvature, the opposite but varying effect of refraction, and the use of the correction angle K (table XVI; TM 5–236) have been explained in paragraph 110*a* to *d*, inclusive. Examples of the computations on Form 12, "Trigonometric Elevations," are shown in figures 79 and 86. The instructions which should appear on the form are given below.

TRIGONOMETRIC ELEVATIONS

INSTRUCTIONS

(1) and (5) Station 2, elevation unknown, station 1, elevation known.

(7) and (10) Cross out inapplicable unit.

(8) and (9) Cross out one or both conversion logs.

If reciprocal angles have been observed.—Use block II, and omit block III.

Block I and line (22) The correction for heights of signals and instruments is $\frac{1}{2}(S1+I1-S2-I2)$.

(11), (12), and (14) Signs must be shown; (11) same sign as Form 6; (12) sign opposite to Form 6; (14) same sign as the greater of (11) and (12).

If the vertical angle has been observed at only one station.—Use block III, and omit block II.

Block I and line (22) The correction for heights of signal and instrument is—

$+I1-S2$ if the observation was made at station 1 (known elevation).

$+I1$ if the station mark at station 2 has been observed.

$-I2+S1$ if the observation was made at station 2 (unknown elevation).

(16) K is interpolated from table XVI, TM 5-236. Note that tabular distances are in feet.

(17) Algebraic sum of (15) and (16).

If observations have been made only at station 2 (unknown elevation).—(21) Δh has sign opposite that of V, line (17).

(In all other cases Δh has same sign as V.)

Use five-place logarithms, except that log tan V should be taken from single-second table in Vega (rounded off to five places). Angles to nearest second only.

| | | |

GEOGRAPHIC POSITIONS

Organization: 64th. Engineer Co(Topo) No. 1001 Date: March 30, 1940

MAP INDEX ..... DATUM North American 1927 Vicinity: Thomasville ..... Thomas County Virginia State

Line No.	Station	Latitude and Longitude	Elevation Feet	Azimuth	Back Azimuth	To Station	Distance Logarithm(meters)	Distance Meters	Distance Feet
1	△Herkness	38 42 01.368	357.3	269 09 43.2	89 16 00.1	△Lee	4.163 4150	14 568.51	47 796.8
2		77 09 21.326		304 32 15.7	124 38 59.5	△Proctor	4.278 5327	18 990.34	62 304.1
3									
4	△Proctor	38 36 11.707	294.6	174 30 46.7	354 30 19.4	△Lee	4.042 6638	11 032.24	36 195.0
5		76 58 34.834							
6									
7	△Lee	38 42 07.847	489.2						
8		76 59 18.491							
9									
10	△French		303.1						
11									
12									
13									
14									
15									
16									
17									
18									
19									
20									
21									
22									
23									
24									

Computed by: J.M.B. Checked by: B.A.L. FORM 13

FIGURE 80.—Geographic positions.

i. Geographic positions.—Form 13 (fig. 80), copied from a similar United States Coast and Geodetic Survey form of the same title, has been filled out so far as the computations have been carried in this paragraph. The data are obtained from Forms 13, 8, 9, and 12. The azimuth and back azimuth of a line are given only once. The compiler and checker initial only the office copy.

j. Disposition of completed computations.—On completion of a project in time of peace, all notebooks, with general index, all computations, and a set of the aerial photographs used are sent to the headquarters of the corps area or department in which the work was done. In time of war, the above records are retained in the organization as long as they may be of further use, and are then disposed of as directed by higher authority.

118. Solution of grid triangulation.—*a. General.*—The chief purposes of the military grid are to—

(1) Aid in the designation of map positions.

(2) Provide a reference for azimuths on parallel lines, independent of the convergency of the meridians.

(3) Facilitate the accurate calculation of the azimuth and distance between points in the field.

All map and control data furnished the combat troops are referred to standard military grid coordinates, which are also adapted for the determination of such control data. All basic geographic data have to be converted before being used. Grid azimuths are clockwise from grid or *Y* north.

b. Computing on military grid coordinates.—As grid triangulation is intended for the most rapid determination of control data, the computations have been simplified. Since the figures are generally simple triangles, adjustment is reduced to correcting the sum of the angles to 180°, though a method of obtaining consistent results through more complicated figures by small adjustments during solution of the component triangles is explained in paragraph 119*b* and *c*. Lengthy position computations are avoided on the rectangular grid where small discrepancies of coordinates and azimuths may be adjusted by adopting the means of discordant results, provided no errors have been made in the calculations. As the notebooks may never be accessible during computation, Form 6, "Transcript of results," from which the computing section works, must be most thoroughly checked in the field to insure freedom from mistakes, omissions, and ambiguities. Five-place logarithms are used throughout, except that it is convenient to get the log tangents of the vertical angles needed for Form 12, "Trigonometric elevations," from the single-second tables in the

seven-place Vega, rounding them off to five places for the computation.

c. Magnification of scale.—The polyconic projection enlarges north and south distances, the amount of enlargement varying from zero on the central meridian of the grid zone to 2 yards per thousand yards at 4° east or west of the central meridian. The rectangular grid of course remains constant in scale and the error appears in the coordinates as magnification of scale. A table of "Corrections to y coordinates for magnification of scale" is given in United States Coast and Geodetic Survey Special Publication No. 59, and is reprinted as table XLIX, TM 5–236. In the computations of grid triangulation, one of the known stations is designated as the *base station* for the computation. The one farthest north or south is preferred, in order to avoid confusing positive and negative corrections. The differences in the y coordinates between the chosen base station and each of the other known stations has to be corrected before computation by applying the correction for that vicinity to the grid y differences. After the computing is otherwise completed, the magnification of scale in the correct amount for the y differences between the base station and each of the others is so applied to the y coordinate of the latter as to increase the distances, thus restoring all the stations to the correct military or map grid coordinates.

d. Map and true grid azimuths.—Because of the magnification of scale, no map grid azimuth (except one exactly grid north-south or east-west) computed from map grid coordinates is the true grid azimuth. The table "Corrections for the Reduction of Map Grid Azimuths to True Grid Azimuths," Special Publication No. 59, shows these errors, which may amount to several minutes, for various azimuths and parts of the grid zone. By the procedure described in *c* above, the magnification of scale is eliminated and all azimuths obtained during the computation must be the desired true grid azimuths. The latter should agree closely with any obtained by applying the "Corrections for the reduction of geographic azimuths to grid azimuths" (see table in Special Publication No. 59; also table L, TM 5–236), to the geographic azimuths, allowing 180° for the change of reference from south to north (par. 117*f*(5)).

e. Procedure of computation.—(1) The computation of a grid triangulation should be commenced as soon as sufficient data are received from the field, as this may be the slowest part of the work unless several computers are available and the work is well organized. By careful consideration in the assignment of tasks, time can be saved in two ways:

(*a*) If the work can be so organized that errors are detected at once, less time will be spent in searching for mistakes.

(*b*) The work to be done by each computer should be reduced to a minimum and kept to computations of the same nature, so far as possible.

(2) Detection of errors is best insured by carrying out all computations in the same form, and by having each computation done by two men working concurrently, but separately and independently. To do this throughout the work would mean that each portion of it is done twice, but it should be generally sufficient to duplicate the work in this way only so far as the computation of the base and the log sides. Coordinates of each point must be computed from two known points. If these two computations are done by separate computers, the fact that the coordinates agree is a check on both computations.

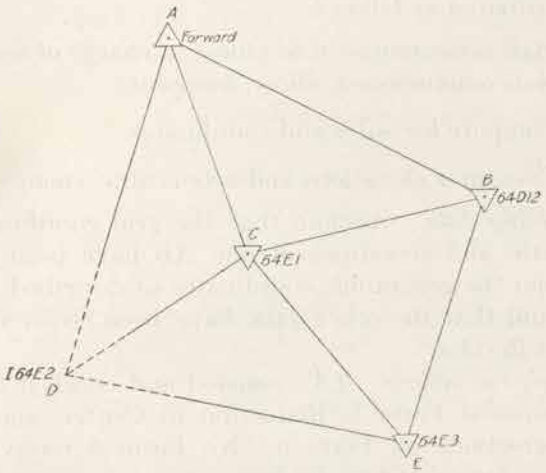


FIGURE 81.—Diagram of grid triangulation.

Intersected points must be fixed by at least two triangles; thus they have their own internal check.

(3) The order in which the main scheme triangles are to be solved must first be decided, it being remembered that a triangle cannot be solved until one side is known. Coordinates of the point forming the apex of a triangle should be computed as soon as the triangle is solved, a check thus being secured on the log sides. If this were not done, an undetected error in a log side might be carried through the entire computation, and much time wasted.

f. Organization of computing section.—(1) The diagram of a small grid triangulation, traced from the reconnaissance plot (par. 98*o*), is shown in figure 81. A rough copy should be given each computer.

An inspection of the diagram shows the order in which the computations should be accomplished, as follows:

Triangle	Base	Log sides	Coordinates
ABC	*AB*	*AC*, *BC*	*C* from *A* *C* from *B*
CBE	*CB*	*CE*, *BE*	*E* from *C* *E* from *B*
ACD	*AC*	*AD*, (*CD*)	*D* from *A*
CED	*CE*	(*CD*), *ED*	*D* from *E*

(*CD*) is check side for an intersected point. All of these computations are shown in figures 82 to 87, inclusive.

(2) In a computing section of five experienced men the work might be distributed as follows:

(*A*) Staff noncommissioned officer in charge of section.
(*B*) Noncommissioned officer, assistant.
(*C*), (*D*) Compute log sides and coordinates.
(*E*) Computes elevations and acts as draftsman.

(*a*) *Assembling data.*—Assume that the grid coordinates, log distance, azimuth, and elevations of line *AB* have been furnished or computed from the geographic coordinates as described in paragraph 117*f* and *g*, and that the other data have been received on Form 6, Transcript of Results.

(*b*) *Eccentric corrections.*—(*A*), assisted and checked by (*B*), computes any required Form 5, Reduction to Center, and makes the necessary corrections on Form 6. No Form 5 received from the field will be adopted. Any preliminary log computations needed in connection with the eccentric corrections are made by (*C*) and (*D*) as outlined in (*d*) below.

(*c*) *Calculation of azimuths.*—Assisted and checked by (*B*), (*A*) completes the upper part of Form 6 and works out the azimuths from each point to all points observed. This must be done in order for all occupied stations, beginning with *A* and ending with *E*. As deduced on Form 6, the azimuths are entered on Form 15, Control Data. Slight differences between azimuths and back azimuths will not be meaned, but will be entered as found.

(*d*) *Solution of triangles.*—(*A*), assisted by (*B*), prepares two copies of Form 8, Solution of Triangles, for the triangle *ABC*, filling in the observed angles at *A*, *B*, and *C*, and log base *AB*. These forms are then handed to (*C*) and (*D*), who correct the observed angles and compute logs *AC* and *BC*.

(*e*) *Computation of coordinates.*—Assisted by (*B*), (*A*) prepares two copies of Form 14, Coordinates From Azimuth and Distance, filling in the following data:

(1)	(2)
Station *C*	Station *C*
From station *A*	From station *B*
X for *A*	*X* for *B*
Y for *A*	*Y* for *B*
Azimuth *AC*	Azimuth *BC*
log *AC*	log *BC*

Form (1) above is handed to (*C*) and form (2) to (*D*) who compute the coordinates of *C* from *A* and *B*, respectively. Stages (*d*) and (*e*) are then repeated alternately until the coordinates of all points have been computed. (*A*) enters the log sides and coordinates on Form 15 as soon as he is satisfied that they are correct.

(*f*) *Trigonometric elevations.*—Obtaining the data required from Forms 6 and 15, (*E*) computes all elevations on Form 12. The elevation of every point is checked by being independently computed from two other points of known elevation. This stage is carried on concurrently with stages (*d*) and (*e*), as the necessary data become available.

(*g*) *Record of results.*—(*E*) may also be usefully employed in plotting the complete system on a gridded sheet at the largest convenient scale, thus providing a graphic check which may be useful in finding errors. (*A*), assisted and checked by (*B*), now finishes and prepares Form 15 for publication.

(*h*) All of the computations and other data are finally collected, indexed, and filed in a large envelope. They are preserved as long as they may be of any possible use, and then disposed of as directed by higher authority.

g. Assembling data.—Precautions must be taken that all needed data are at hand, that the figures are the latest adjusted values and based upon the North American datum of 1927. All copies of Form 6 (figs. 82 and 83) should be carefully examined on receipt for any obvious mistakes, omissions, or ambiguities. The instructions which should appear on the form are given below.

(1) Transcript of results at △ Forward Date: April 2, 1940 H.I. 4.7 feet

Signal heights 24.8, 23.0, 20.4, 14.3

(2) With ▽64D12 With

(3) Azimuth from ditto 298° 06' 12" ditto ° ' "

(4) ±180 00 00 ±180 00 00

(5) Azimuth to ditto 118 06 12 ditto

(6) Angle from initial sta. − 00 00 00 −

(7) Correction +118 06 12 +

Computed by: J.N.B. Checked by: J.B.N. (8) Mean correction +

(9) Stations Observed	▽64D12	▽64E1	I64E2			This angle at left side.
Horizontal Angles						
(10) ~~Observed~~ Calculated		36 26 53				
(11) Eccentricity corrected						
(12) Adjusted angle						
Directions				° ' "	° ' "	
(13) Observed ~~Calculated~~	00° 00' 00"	43° 15' 26"	79° 42' 19"			
(14) Eccentric reduction	—	—	—			
(15) Eccentricity corrected	—	—	—			
(16) Azimuth	118 06 12	161 21 38	197 48 31			
(17) Adjusted azimuth						
(18) Sketch of signal	X		Top of grey tank			
Note Book 10 Pages 9, 10						
(19) Copied by L.Y.						
(20) Checked by T.R.						
(21) Vertical angle ±	—	+01 18 36	-00 28 46			

(1) Transcript of results at ▽ 64D12 Date: April 2, 1940 H.I. 5.0 feet

Signal heights 16.1, 15.0, 12.1, 10.0

(2) With △ Forward With

(3) Azimuth from ditto 118° 06' 12" ditto ° ' "

(4) ±180 00 00 ±180 00 00

(5) Azimuth to ditto 298 06 12 ditto

(6) Angle from initial sta. − 00 00 00 −

(7) Correction +298 06 12 +

Computed by: J.N.B. Checked by: J.B.N. (8) Mean correction +

(9) Stations Observed	△Forward	▽64E3	▽64E1			This angle at left side.
Horizontal Angles						
(10) Observed ~~Calculated~~	40 29 54	261 12 15	158 17 51			
(11) Eccentricity corrected						
(12) Adjusted angle						
Directions				° ' "	° ' "	
(13) ~~Observed~~ Calculated	00° 00' 00"	261° 12' 15"	319° 30' 06"			
(14) Eccentric reduction	—	—	—			
(15) Eccentricity corrected	—	—	—			
(16) Azimuth	298 06 12	199 18 27	257 36 18			
(17) Adjusted azimuth						
(18) Sketch of signal	X	Instrument				
Note Book 12 Pages 22, 23						
(19) Copied by B.D.						
(20) Checked by C.S.						
(21) Vertical angle ±	—	+01 18 15	+02 46 17			

FORM 6

FIGURE 82.—Transcript of results at △ Forward.

TRANSCRIPT OF RESULTS

INSTRUCTIONS

Every figure of the notebook must be checked by the observer before this form is made out. The observing party fills in only the date, the station occupied, a diagram of its signal heights, and the *H.I.*; the stations observed and the observed angles or directions; the last four lines showing the vertical angles, and the parts of the signals to which vertical angles were read; the notebook references, etc. Every figure of the transcript must be checked by the observer to insure freedom from mistakes, omissions, and ambiguities, since computations will usually be started, and often completed, before the notebook reaches the computing section.

No corrections for eccentricity will be made on Form 6 in the field, but a copy of computed Form 5 should be attached for the guidance of the computing section.

(1) Transcript of results at ▽64E1 Date: April 2, 1940 H.I. 5.1 feet

Signal heights 9.8, 9.2, 7.0

(2)	With ▽64D12	With △Forward
(3) Azimuth from	ditto 257° 36' 18"	ditto 161° 21' 38"
(4)	±180 00 00	±180 00 00
(5) Azimuth to	ditto 77 36 18	ditto 341 21 38
(6) Angle from initial sta.	− 96 14 31	− 00 00 00
(7) Correction	+341 21 47	+341 21 38

Computed by: J.H.B. Checked by: E.E.H. (8) Mean correction + 341 21 42

(9) Stations Observed	△Forward	▽64D12	▽64E3	▽64E2	
Horizontal Angles					
(10) ~~Observed~~ Calculated	104 00 30	96 14 31	64 29 48	95 15 11	This angle at left side
(11) Eccentricity corrected					
(12) Adjusted angle					
Directions					
(13) ~~Observed~~ Calculated	00° 00' 00"	96° 14' 31"	160° 44' 19"	255° 59' 30"	° ' "
(14) Eccentric reduction	—	—	—	—	
(15) Eccentricity corrected	—	—	—	—	
(16) Azimuth	341 21 42	77 36 13	142 06 01	237 21 12	
(17) Adjusted azimuth					
(18) Sketch of signal			Sta. mark	Grey tank (Top)	
Note Book 10 Pages 11, 12					
(19) Copied by L.V.					
(20) Checked by C.R.					
(21) Vertical angle	± -01 15 06	-02 42 57	-01 22 17	-02 09 05	

(1) Transcript of results at ▽64E3 Date: April 2, 1940 H.I. 5.2 feet

Signal heights 9.5, 9.0, 7.0

(2)	With ▽64D12	With ▽64E1
(3) Azimuth from	ditto 199° 18' 27"	ditto 142° 06' 01"
(4)	±180 00 00	±180 00 00
(5) Azimuth to	ditto 19 18 27	ditto 322 06 01
(6) Angle from initial sta.	−00 00 00	−302 47 19
(7) Correction	+19 18 27	+19 18 42

Computed by: J.H.B. Checked by: E.E.H. (8) Mean correction + 19 18 35

(9) Stations Observed	▽64D12	I64E2	▽64E1		
Horizontal Angles					
(10) Observed ~~Calculated~~	57 12 41	263 56 06	38 51 13		This angle at left side
(11) Eccentricity corrected					
(12) Adjusted angle					
Directions					
(13) ~~Observed~~ Calculated	00° 00' 00"	263° 56' 06"	302° 47' 19"	° ' "	° ' "
(14) Eccentric reduction	—	—	—		
(15) Eccentricity corrected	—	—	—		
(16) Azimuth	19.18 35	283 14 41	322 05 54		
(17) Adjusted azimuth					
(18) Sketch of signal		Top of grey tank	Instrument		
Note Book 12 Pages 24, 25					
(19) Copied by C.S.					
(20) Checked by C.J.					
(21) Vertical angle	± -01 16 47	-00 24 24	+01 19 06		

FORM 6

FIGURE 83.—Transcript of results at ▽64E1.

TRANSCRIPT OF RESULTS

INSTRUCTIONS

Every figure of the notebook must be checked by the observer before this form is made out. The observing party fills in only the date, the station occupied, a diagram of its signal heights, and the *H. I.*; the stations observed and the observed angles or directions; the last four lines showing the vertical angles, and the parts of the signals to which vertical angles were read; the notebook references, etc. Every figure of the transcript must be checked by the observer to insure freedom from mistakes, omissions, and ambiguities, since computations will usually be started, and often completed, before the notebook reaches the computing section.

No corrections for eccentricity will be made on Form 6 in the field, but a copy of computed Form 5 should be attached for the guidance of the computing section.

(1) Station and letters		(2) Observed angles			(3) Corr. ±	(4) Corr. Plane Angle	(5) Log sine	(6) Log side	(7) Side	(8) See No.	(9) Sph. Angle
Forward	A	43	15	26	+ 3	29	9 835 87	3 545 53	BC	2	
D12	B	40	29	54	+ 3	57	9 812 53	3 522 19	AC	3	
E1	C	96	14	31	+ 3	34	9 997 41	3 707 07	AB		
No. 1 Sph. Exc.		179	59	51			3 709 66				
E1	C	64	29	48	- 7	41	9 955 47	3 576 38	BE		
D12	B	58	17	54	- 7	44	9 929 81	3 550 72	CE	4	
E3	E	57	12	41	- 6	35	9 924 62	3 545 53	BC	1	
No. 2 Sph. Exc.		180	00	20			3 620 91				
Forward	A	36	26	53	- 1	52	9 773 85	3 492 18 (10)	CD	4	
E1	C	104	00	30	- 2	28	9 986 89	3 705 16	AD		
TE2	D	39	32	41	- 1	40	9 803 92	3 522 19	AC	1	
No. 3 Sph. Exc		180	00	04			3 718 27	(3105.4 yd 3105.1 yd		Error 3 yd	
E1	C	95	15	11	+ 3	14	9 998 17	3 692 75	DE		
E3	E	38	51	13	+ 2	15	9 797 50	3 492 08 (10)	CD	3	
TE2	D	45	53	29	+ 2	31	9 856 14	3 550 72	CE	2	
No. 4 Sph. Exc.		179	59	53			3 694 58				
No. 5 Sph. Exc.											
No. 6 Sph. Exc.											
No. 7 Sph. Exc.											
No. 8 Sph. Exc.											

Date: April 2, 1940 Computed by: J.W.B. Checked by: A.B.H. FORM 8

FIGURE 84.—Solution of triangles.

h. Eccentric corrections.—The use of Form 5, "Reduction to center" (fig. 71), has been explained in paragraph 109.

i. Calculation of azimuths is accomplished on Form 6 as shown in figures 82 and 83.

j. Exercise XVIII. Calculation of azimuths.—Commencing the office computation of the data resulting from exercise XVII, and pertinent basic data furnished by the instructor, the student will now complete Form 6, incidentally making any required reductions to center on Form 5, as outlined in *f*(2)(*b*) and (*c*) above.

k. Solution of triangles.—Form 8 has been explained in paragraph 117*d*. Figure 84 shows the usual employment of this form with the triangles listed clockwise, giving first the ends of the known side. In those with one station unoccupied, the third angle has been deduced from the azimuths of the sides, which gives a more probable value than would be obtained by subtracting the sum of the other two angles from 180°. Neatly lining out discarded values, as the seconds in column (2), is permissible in any Form where it is believed necessary to prevent abstracting an uncorrected value, provided the crossed figures are left quite legible. In this example, the antilogs for the side *CD* have been set down in order to show that their difference is negligible.

l. Exercise XIX. Solution of triangles.—Utilizing the completed Form 6 and the given data, the student will fill out Form 8 and compute all logs sides from the corrected angles. This exercise and the next should be worked concurrently, thus securing an independent check on the log sides before continuing with the solution of the next triangle.

Sta. 2 E1	log(1-2) 3 522 19	log(1-2) 3 522 19	Magnification of scale	
Sta. 1 Δ Forward	log sin α 9 504 63	log cos α 9 976 60	From Δ Forward	
(1-2)α 161° 21' 36"	log ΔX 3 026 82	log Δy 3 498 79	Thousand yds.(to tenths)	3.2
Graph	ΔX ± + 1063 7	Δy ± − 3153 5	Yds. per thous.(to tenths)	1.5
	X1 1 378 830 1	y1 1 826 191 7	Magnification ±	-4.8
No. 1	X2 1 379 893 8	y2 1 823 038 2	Map y2 =	
Adopted coordinates	X2 = 1 379 893 8	y2 =	Map y2 =	1,823,033.4
Sta. 2 E1	log (1-2) 3 545 53	log (1-2) 3 545 53	Magnification of scale	
Sta. 1 D12	log sin α 9 989 76	log cos α 9 331 73	From	
(1-2)α 257 36 18	log ΔX 3 535 29	log Δy 2 877 26	Thousand yds.(to tenths)	
Graph	ΔX ± − 3430 0	Δy ± − 753 8	Yds. per thous.(to tenths)	
	X1 1 383 323 7	y1 1 823 792 0	Magnification ±	
No. 2	X2 1 379 893 7	y2 1 823 038 2	Map y2	
Adopted coordinates	X2 = —	y2 =	Map y2 =	See No. 1
Sta. 2 E3	log(1-2) 3 576 38	log(1-2) 3 576 38	Magnification of scale	
Sta. 1 D12	log sin α 9 519 35	log cos α 9 974 86	From Δ Forward	
(1-2)α 199 18 27	log ΔX 3 095 73	log Δy 3 551 24	Thousand yds.(to tenths)	6.0
Graph	ΔX ± − 1246 6	Δy ± − 3 558 2	Yds. per thous.(to tenths)	1.5
	X1 1 383 323 7	y1 1 823 792 0	Magnification ±	-9.0
No. 3	X2 1 382 077 1	y2 1 820 233 8	Map y2	
Adopted coordinates	X2 = 1 382 077 0	y2 = ~~do~~	Map y2 =	1,820,224.8
Sta. 2 E3	log (1-2) 3 550 72	log(1-2) 3 550 72	Magnification of scale	
Sta. 1 E1	log sin α 9 788 37	log cos α 9 897 12	From	
(1-2)α 142 06 01	log ΔX 3 339 09	log Δy 3 447 84	Thousand yds.(to tenths)	
Graph	ΔX ± + 2183 2	Δy ± − 2 804 4	Yds. per thous.(to tenths)	
	X1 1 379 893 8	y1 1 823 038 2	Magnification ±	
No. 4	X2 1 382 077 0	y2 1 820 233 8	Map y2	
Adopted coordinates	X2 = —	y2 = —	Map y2 =	See No. 3
Sta. 2 IE2	log (1-2) 3 705 16	log(1-2) 3 705 16	Magnification of scale	
Sta. 1 Δ Forward	log sin α 9 485 49	log cos α 9 978 68	From Δ Forward	
(1-2)α 197 48 31	log ΔX 3 190 65	log Δy 3 683 84	Thousand yds.(to tenths)	4.8
Graph	ΔX ± − 1551 1	Δy ± − 4 828 8	Yds. per thous.(to tenths)	1.5
	X1 1 378 830 1	y1 1 826 191 7	Magnification ±	-7.2
No. 5	X2 1 377 279 0	y2 1 821 362 9	Map y2	
Adopted coordinates	X2 = 1 377 279 1	y2 = ~~1 821 363 0~~	Map y2 =	1,821,355.8
Sta. 2 IE2	log(1-2) 3 692 75	log(1-2) 3 692 75	Magnification of scale	
Sta. 1 E3	log sin α 9 988 29	log cos α 9 360 05	From	
(1-2)α 283 14 41	log ΔX 3 681 04	log Δy 3 052 80	Thousand yds.(to tenths)	
Graph	ΔX ± − 4 797 8	Δy ± + 1129 3	Yds. per thous.(to tenths)	
	X1 1 382 077 0	y1 1 820 233 8	Magnification ±	
No. 6	X2 1 377 279 2	y2 1 821 363 1	Map y2	
Adopted coordinates	X2 —	y2 —	Map y2	See No. 5

Date: April 2, 1940 Computed by: J.W.B. Checked by: J.J.M. FORM 14

FIGURE 85.—Coordinates from azimuth and distance.

COORDINATES FROM AZIMUTHS AND DISTANCE

INSTRUCTIONS

In a system of grid triangulation, this form should be computed after the solution of each triangle, thus securing an independent check on the log sides.

Every point is computed independently from two points of known coordinates; station 1, known; station 2, unknown.

The azimuth (α) is obtained from Form 6; the graph indicates the correct signs for ΔX and ΔY.

The log sides are obtained from Form 8.

For the magnification of scale, the thousands of yards are obtained by subtracting Y of the base station selected for the computation from Y of the new station. The yards per thousand are interpolated from the table, "Corrections to y coordinates for magnification of scale" in Special Publication No. 59 or the identical table *XLIX*, TM 5–236. The sign of the product must be such as to increase the true grid distance to the map grid distance.

The adopted coordinates are the means of the independently determined coordinates. Adopted Y_2 may be crossed out after map grid Y_2 is obtained.

I	From Forms 6 and 8	(1) Station 2	▽64E1			▽64E1			▽64E3			▽64E3			I 64E2			I 64E2			I 64E2		
		(2)	Heights	S2	9.8	Heights	S2	9.8	Heights	S2	5.2	Heights	S2	0.0	Heights	S2		Heights	S2		Heights	S2	
		(3)		I2	5.1		I2	5.1		I2	5.2		I2	5.2		I2			I2			I2	
		(4)		2\	14.9		2\	14.9		2\	10.4		2\	5.2		2\			2\			2\	
		(5) Elevation of 1	385.6 ft	−	7.4	103.5 ft	−	7.4	103.5 ft	−	5.2	611.0 ft	−	2.6	385.6 ft			611.0 ft			363.3 ft		
		(6) Station 1	△ Forward			▽64D12			▽64D12			▽64E1			△ Forward			▽64E1			▽64E3		
		(7) log (1-2) yds. m.	3.52219	S1	14.3	3.54553	S1	15.0	3.57638	S1	15.0	3.55072	S1	5.1	3.70516	S1		3.49210	S1		3.69275	S1	
		(8) log yds.-ft	0.47712	I1	4.7	0.47712	I1	5.0	0.47712	I1	5.0	0.47712	I1	5.1	0.47712	I1	4.7	0.47712	I1	5.1	0.47712	I1	5.2
		(9) log m. to ft.	~~0.51598~~	2\	19.0	~~0.51598~~	2\	20.0	~~0.51598~~	2\	20.0	~~0.51598~~	2\	10.2	~~0.51598~~	2\		~~0.51598~~	2\		~~0.51598~~	2\	
		(10) log (1-2) ft. m.	3.99931	+	9.5	4.02265	+	10.0	4.05350	+	10.0	4.02784	+	5.1	4.18228	+	4.7	3.96922	+	5.1	4.16987	+	5.2
II		(11) +VA1 ±	+ 01	18	36	+ 02	46	17	+ 01	18	15	− 01	22	17									
		(12) −VA2 ±	+ 01	15	06	+ 02	42	57	+ 01	16	47	− 01	19	06									
		(13) Divide sum by	2\ 02	33	42	2\ 05	29	14	2\ 02	35	02	2\ 02	41	23	2\			2\			2\		
		(14) V ±	+ 01	16	51	+ 02	44	37	+ 01	17	31	− 01	20	42									
III		(15) VA1 ±													− 00	28	46	− 02	09	05	− 00	24	24
		(16) K	+			+			+			+			+	01	04	+	00	39	+	01	02
		(17) V ±													− 00	27	42	− 02	08	26	− 00	23	22
IV		(18) log tan V	8.34944			8.68053			8.35319			8.37068			7.90622			8.57261			7.83233		
		(19) log (1-2)	3.99931			4.02265			4.05350			4.02784			4.18228			3.96922			4.16987		
		(20) log Δh (sum)	2.34875			2.70318			2.40669			2.39852			2.08850			2.54183			2.00220		
		(21) Δh ±	+ 223.2 ft			+ 504.9 ft			+ 255.1 ft			− 250.3 ft			− 122.6 ft			− 348.2 ft			− 100.5 ft		
		(22) HI's & HS's ±	+ 2.1			+ 2.6			+ 4.8			+ 2.5			+ 4.7			+ 5.1			+ 5.2		
		(23) Elevation of 1	385.6			103.5			103.5			611.0			385.6			611.0			363.3		
		(24) Elevation of 2	610.9			611.0			363.4			363.2			267.7			267.9			268.0		
		(25) Adopted El. of 2	611.0						363.3						267.9								

Date: April 2, 1940 Computed by: J.H.B. Checked by: E.A.H. FORM 12

FIGURE 86.—Trigonometric elevations.

CONTROL DATA

Organization: 64 th Engr Co (Topo) No. 1 Date: April 2, 1940

Map Index ____ Zone Letter B

Basic datum: North American 1927

Vicinity: Carlton Carlton County Maryland State

Line No.	Station and Description	Grid Coordinates (Yards) X	Grid Coordinates (Yards) Y	Elevation Feet	True Grid Azimuth	To Station	Log (Yds.) Distance	Distance Yards	Line No.	Diagrams
1	△ Forward	1378830.1	1826191.7	385.6	118 06 12	▽64D12	3707.07	5094.1	1	24.8 23.0 20.4 14.3 ; Y ; △ 53 ft from intersection
2	Concrete mon.				161 21 38	▽64E1	3522.19	3328.1	2	
3					197 48 31	I64E2	3705.16	5071.8	3	
4									4	
5	▽64D12	1383323.7	1823792.0	1035	298 06 12	△ Forward	3707.07	5094.1	5	18.1 15.0 12.1 10.0 ; 83 ft from corner stonewall
6	2½ Pipe				199 18 27	▽64E3	3576.38	3770.3	6	
7					257 36 18	▽64E1	3545.53	3511.8	7	
8									8	
9	▽64E1	1379893.8	1823033.4	611.0	341 21 42	△ Forward	3522.19	3328.1	9	9.8 9.2 7.0 ; 5 ft S.W. of high stump
10	2" Pipe				77 36 13	▽64D12	3545.53	3511.8	10	
11					142 06 01	▽64E3	3550.72	3554.0	11	
12					237 21 12	I64E2	3492.10	3105.3	12	
13									13	
14	▽64E3	1382077.0	1820224.8	363.3	19 18 35	▽64D12	3576.38	3770.3	14	9.5 9.0 7.0 ; 47 ft E of ℄ rd. ; 96 ft NE of ruin
15	2" Pipe				283 14 41	I64E2	3692.75	4928.9	15	
16					322 05 54	▽64E1	3550.72	3554.0	16	
17									17	
18	I64E2	1377279.1	1821355.8	267.9					18	Top of grey tank.
19	Grey Tank			(Top)					19	
20									20	
21									21	
22									22	
23									23	
24									24	

Computed by J.H.B. Checked by A.G.H. FORM 15

FIGURE 87.—Control data.

m. Exercise XX. Coordinates from azimuth and distance.—In conjunction with exercise XIX the coordinates of the unknown point in each triangle should be computed on Form 14 (fig. 85) as soon as each triangle is solved. The instructions which should appear on the form are given on page 263.

n. Trigonometric elevations.—Form 12 (fig. 86) has been explained in paragraph 117*h*, where the instructions have also been reprinted. General information on curvature, refraction, and the *K* correction angles is given in paragraph 110*a* to *d*, inclusive.

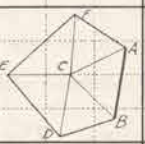
o. Exercise XXI. Trigonometric elevations.—From the data furnished and derived on Forms 6 and 8 compute on Form 12 the elevations of all stations of the grid triangulation scheme, showing the adopted elevation for each of them.

p. Control data.—All data in military grid coordinates should be published on Form 15 (fig. 87). Only the office copy is signed by the compiler and checker. The given data in this sample have been underlined.

q. Exercise XXII. Control data.—Fill out Form 15 for the triangulation and computations which have been performed in exercises XVII to XXI, inclusive.

119. Miscellaneous computations.—*a. General.*—Several examples of computations which are less frequently needed have been collected in this paragraph. The list is not exhaustive, but familiarity with all of the procedures in this section should enable a computer to undertake any problem that may arise short of least-square adjustments and such specialities as the adjustment of large level nets and the final adjustment of positions in extensive triangulation systems.

b. Adjustment of central point figure.—(1) This figure may occasionally come up in grid triangulation as the result of the single point circuit described in paragraph 128. A diagram is shown in figure 88, where Form 8, "Solution of triangles," is utilized to make the adjustment during the solution of the triangles. The column headings have been changed to suit the case. Having *AB* as the known side, listing the triangles in the usual clockwise order and naming first the ends of the known side in each triangle, the solution is carried out until it is discovered that the log side *AC* in block No. 5, disagrees with the common side in triangle *ABC* by +8 in the last decimal place. In order to leave space on the form for the adjustment, the sums of each triangle have been brought to 180° before listing. Anticipating the need of adjustment the tabular differences in each log sine for a 10-second change, in the peripheral angles only, have been entered in (4) as the sines were looked up, with the negative sign for angles exceed-

(1) Station and letters		(2) Observed (△ closed) angles		Corr ±	(3) Corr. angle ±	(4) ~~Corr~~ ~~Plane~~ ~~Angle~~ Tab. Diff. 10 sec.	(5) ± See corr. × 1/10 Tab. Diff. Log sine	(6) Corr ± Log side	(7) Side	(8) ~~See~~ Adj. log ~~No.~~	(9) ~~Sph.~~ side ~~Angle~~
△ Pool	A	54	41	-5 52	47	1	9 911 75 0	3 736 49 +1	BC	3.73	649
△ Pelican	B	54	27	+5 37	42	2	9 910 48 +1	3 735 22 +1	AC	3.73	523
▽ Cow	C	70	50	31	31		9 975 26	3 800 00	AB	3.80	000
No. 1 Sph. Exc.							3 824 74				
	C	54	16	32	32		9 909 47	3 654 72	BD	3.65	472
	B	47	11	-5 20	15	2	9 865 46 -1	3 610 71 -1	CD	3.61	070
▽ Wolf	D	78	32	+5 08	13	0	9 991 24 0	3 736 49	BC	3.73	649
No. 2 Sph. Exc.							3 745 25				
	C	81	28	28	28		9 995 17	3 742 88 -2	DE	3.74	286
	D	51	41	-5 03	40 58	2	9 894 65 -1	3 642 36 -3	CE	3.64	233
▽ Bramble	E	46	50	+5 29	34	2	9 863 00 +1	3 610 71 -1	CD	3.61	070
No. 3 Sph. Exc.							3 747 71 -2				
	C	93	40	41	41		9 999 10	3 758 83 -4	EF	3.75	879
	E	36	34	-5 37	32	3	9 775 17 -2	3 534 90 -6	CF	3.53	484
▽ Seacow	F	49	44	+5 42	47	2	9 882 63 +1	3 642 36 -3	CE	3.64	233
No. 4 Sph. Exc.							3 759 73 -4				
	C	59	43	48	48		9 936 34	3 676 23 -8	AF	3.67	615
	F	81	40	-5 46	41	0	9 995 41 0	3 735 30 -7 -8	AC	3.73	523
	A	38	35	+5 26	31	3	9 795 01 +2	3 534 90 -6	CF	3.53	484
No. 5 Sph. Exc.							3 739 89 -8				
						17	8 17 X 10 = ± 5	log AC error was -8 in 5th place			
No. 6 Sph. Exc.											
No. 7 Sph. Exc.											
No. 8 Sph. Exc.											

Date: January 17, 1940 Computed by: J.H.B Checked by: E.E.H FORM 8

FIGURE 88.—Adjustment of central point figure.

ing 90°. The spaces in (4) opposite the central angles are left blank as no change in these is contemplated. The sum of the tabular differences is shown at the foot of (4).

(2) The change in each angle is seen to be 8 (the difference between logs AC) $\times 10$ seconds, divided by 17 (the sum of the tabular differences). At the foot of (5) this works out as ± 5, the sign for the successive angles alternating. To shorten AC the opposite angle must be diminished and so the correction for the upper angle of each pair is negative as the signs are shown on the form. The corresponding changes in the log sines are computed, $+5 \times 2 \div 10 = +1$ for log sin B,

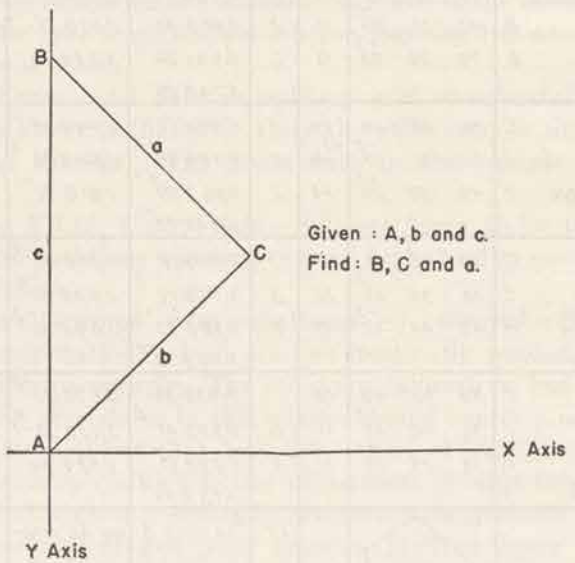


FIGURE 89.—Triangle computation by coordinates method.

for example, and written above the log sines. The changes are then marked in for each of the other quantities as they would change if the computation were repeated with the altered log sines. The desired change of -8 is obtained for log AC in block 5, but it is made -7 to take care of one unit increase which has occurred in the first log AC.

(3) The object of the adjustment was to obtain this agreement so that subsequent work would be free from discrepancies which would nullify any internal checks in the computations. It should not be supposed that the adjustment increases the accuracy of the positions.

c. Adjustment of quadrilateral.—In the rare case where a quadrilateral might occur in grid triangulation, consistency in the log sides may be obtained by the same method as explained in *b* above. In

the quadrilateral no changes should be expected in the sum angles at the lettered corners of the figure, so the positive and negative corrections are applied to the odd- and even-numbered angles of the diagram, similar to that at the head of Form 7 (fig. 74) in such sense as to obtain the desired agreement of the log sides.

d. Triangle computation using two sides and included angles.—(1) Any organization employing the angle method of adjusting large systems of triangulation will meet this problem frequently. They will doubtless secure a supply of the United States Coast and Geodetic Survey Form 665 for this purpose, which contains its own check computations.

(2) The solution is easiest on coordinates. If the coordinates of the apex of the known angle are known, the solution is obvious. If not, suppose A is at the origin and AB coincides with either axis of a system of local coordinates (fig. 89). The coordinates of B are determined by the length of the side AB, those of C are computed, and the azimuth and length of BC can then be found. The calculations may be checked by solution on Form 8.

e. Inverse solution.—It sometimes becomes necessary in the adjustment of triangulation to compute the azimuths and length of a line joining two stations which are fixed in position, but which have not been directly connected by the observations. In order to compute this line an inverse or back computation must be made. This computation can be made on Form 9, "Computation of Position" (fig. 76), but it can be made more easily and simply on Form 16. In figure 90, triangulation stations Herkness and Proctor are fixed in position (latitude and longitude) and it is desired to determine the azimuth, back azimuth, and length of the line Herkness to Proctor. The instructions which should appear on the form are given below.

NAME OF STATION

1. ϕ	38°	42'	01.368"	△ Herkness		λ	77°	09'	21.326"
2. ϕ'	38	36	11.707	△ Proctor		λ'	76	58	34.834
$\Delta\phi(=\phi'-\phi)$	—	05	49.661			$\Delta\lambda(=\lambda'-\lambda)$	-	10	46.492
$\frac{\Delta\phi}{2}$	-	02	54.830			$\frac{\Delta\lambda}{2}$	-	05	23.246
$\phi m\ (=\phi+\frac{\Delta\phi}{2})$	38	39	06.538						
$\Delta\phi$(secs.)	—		349.661"			$\Delta\lambda$(secs.)			646.492"
log $\Delta\phi$	2.543 647 2m					log $\Delta\lambda$	2.810 563 1m		
cor. arc–sin *	—					cor. arc–sin *	—		2
log $\Delta\phi_1$ *						log $\Delta\lambda_1$ *	2.810 562 9m		
log cos $\frac{\Delta\lambda}{2}$ *	9.999 999 5					log cos ϕm	9.892 626 5		
colog B_m	1.489 046 2					colog A_m	1.490 847 5		
log $\{s_1 \cos(\alpha+\frac{\Delta\alpha}{2})\}$	4.032 692 9			(opposite in sign to $\Delta\phi$)		log $\{s_1 \sin(\alpha+\frac{\Delta\alpha}{2})\}$	4.194 036 9m		
						log $\{s_1 \cos(\alpha+\frac{\Delta\alpha}{2})\}$	4.032 692 9		
log $\Delta\lambda$	2.810 563 1m			3 log $\Delta\lambda$		log tan $(\alpha+\frac{\Delta\alpha}{2})$	0.161 344 0		
log sin ϕm	9.795 592 3			log F		Bearing	5 55° 24' 22.4" E		
log sec $\frac{\Delta\phi}{2}$ *	0.000 000 1			log b		$\alpha+\frac{\Delta\alpha}{2}$	304° 35' 37.6"		
log a	2.606 155 5m					log sin $(\alpha+\frac{\Delta\alpha}{2})$	9.915 504 4		
a	—	403.8				log cos $(\alpha+\frac{\Delta\alpha}{2})$	9.754 160 4		
b *						log s_1	4.278 532 5		
$-\Delta\alpha$ (secs.)	—	403.8"				cor. arc–sin *	+		2
$-\frac{\Delta\alpha}{2}$	—	201.9"				log s *	4.278 532 7		
	-	03'	21.9						
$\alpha+\frac{\Delta\alpha}{2}$	304	35	37.6						
α(1 to 2)	304	32	15.7						
$\Delta\alpha$	+	06	43.8						
	180								
α'(2 to 1)	124	38	59.5						

Date: March 25, 1940 Computed by: J.N.B. Checked by: J.J.H. FORM 16

FIGURE 90.—Inverse solution.

INVERSE SOLUTION

INSTRUCTIONS

The nine spaces marked with asterisks may be left blank if not required.

"Arc-sin corrections for inverse position computation," is given in table XXV, TM 5-236. This table is not to be confused with the table given on page 17 of United States Coast and Geodetic Survey Special Publication No. 8, which is entirely different.

The letter *n* should be written after the logarithms of negative quantities. The quadrant in which the angle $\left(\alpha+\frac{\Delta\alpha}{2}\right)$ occurs depends upon the algebraic signs of the quantities $\sin\left(\alpha+\frac{\Delta\alpha}{2}\right)$ and $\cos\left(\alpha+\frac{\Delta\alpha}{2}\right)$.

Log *s* is obtained in two ways, first by subtracting log $\sin\left(\alpha+\frac{\Delta\alpha}{2}\right)$ from log $\left[s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right]$ and second by subtracting log $\cos\left(\alpha+\frac{\Delta\alpha}{2}\right)$ from log $\left[s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right]$.

This form has no internal check. However, the values of α and log *s* determined by this inverse solution may be checked in the following manner. Starting with the azimuth, the logarithm of the length, and the fixed latitude and longitude of one of the stations, the latitude and longitude of the other are computed on Form 9, "Computation of position." The values thus obtained should check the fixed values of the latitude and longitude within one in the last place of decimals.

f. Alternate solution.—Sometimes the desired information need not be so accurate as that obtained by the inverse solution. Table XI, TM 5-236, may be used to reduce the differences in geographic coordinates to feet. Solving the right triangle will then furnish a rough approximation of the azimuth and distance between the two points (par. 136*c*). United States Coast and Geodetic Survey Special Publication No. 71 gives tables and shows a short method of com-

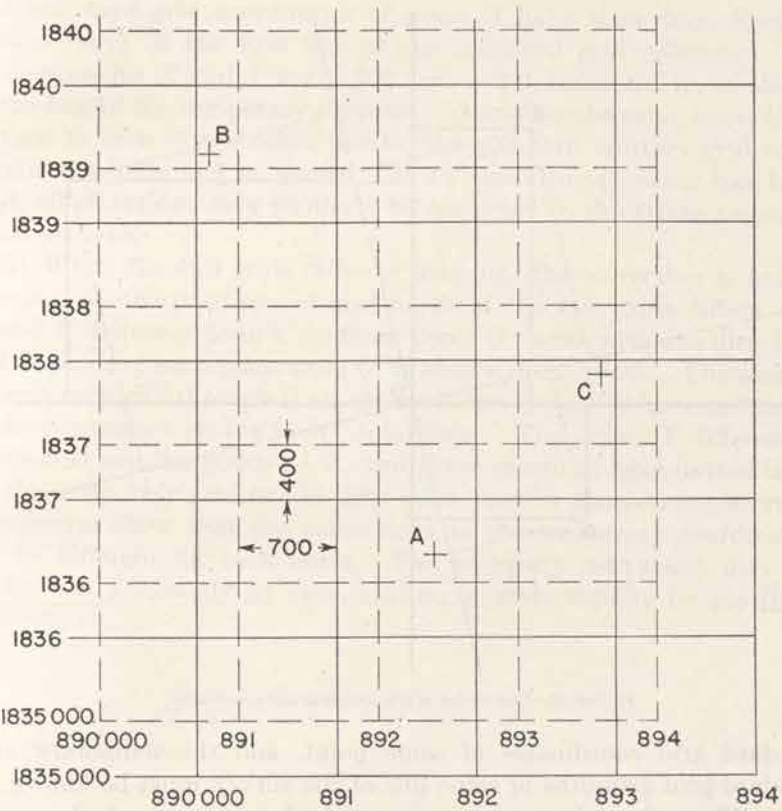


FIGURE 91.—Arithmetical conversion of coordinates.

puting the azimuth and distance on local coordinates with the origin at one of the stations. Both distance and azimuth are quite close, though the latter is not geodetic.

g. Grid to geographic coordinates.—As this computation is so seldom made, no form is provided. The "Computation of geographic coordinates from grid coordinates" form in United States Coast and Geodetic Survey Special Publication No. 59 should be followed.

h. Assumed coordinates to military grid coordinates.—(1) The necessity and methods of assuming a temporary grid is considered in paragraph 137. Here the conversion of coordinates on an assumed grid to coordinates on the standard military grid will be explained. The two grids may differ in the locations of their origins, in the azimuths of their axes, or in both. In order to convert the temporary coordinates to their equivalents on the standard grid, the temporary and

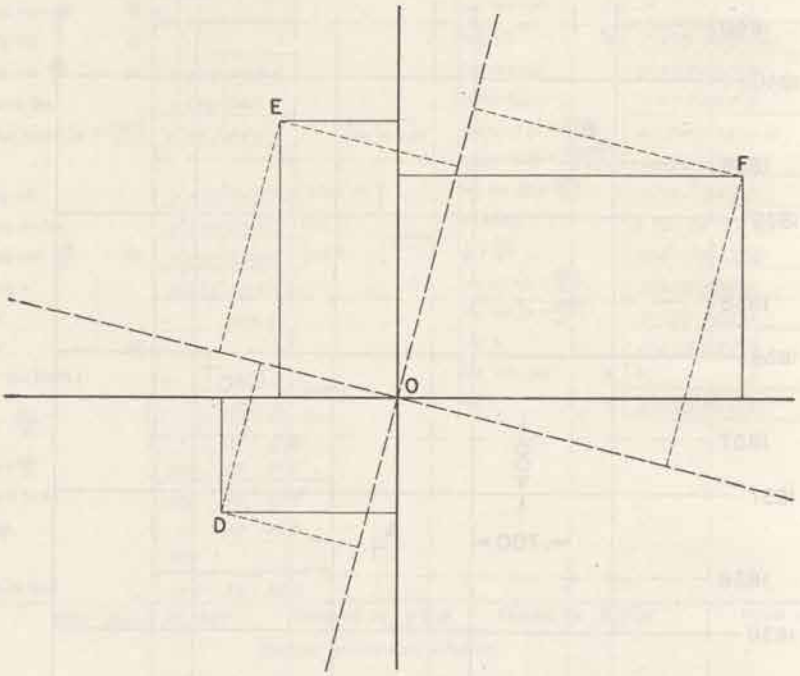


FIGURE 92.—Conversion of grid azimuths and coordinates.

standard grid coordinates of some point, and the temporary and standard grid azimuths of some line of the survey must be known.

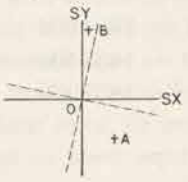
(2) When the temporary and standard grids are of the same azimuth but the coordinates differ, the temporary X and Y coordinates of each point are changed by the same amounts as are required to correct the X and Y coordinates, respectively, of any single point for which the coordinates on both grids have been determined. The dashed lines in figure 91 represent a temporary grid on which the positions have been determined as shown below in the first two columns:

	Temporary		Standard	
	X	*Y*	*X*	*Y*
A	892400	1836200	891700	1836600
B	890800	1839100	890100	1839500
C	893600	1837500	892900	1837900
Correction				
	−700	+400		

The standard grid coordinates of point *A* have since been fixed as shown above in the first line of the standard grid columns. The corrections for *X* and *Y* are −700 and +400, respectively, as shown at the foot of the temporary columns. Applying the same corrections in turn to each of the other points, the standard military grid coordinates are obtained as shown. If an assumed elevation has been used, all elevations may similarly be corrected to elevations based on mean sea level.

(3) When the two grids differ in azimuth, the correction is not so simple. As the positions of grid north of the two grids differs, the *X* and *Y* distances from a common point *O* varies with the direction and distance of each point from *O*, as shown in figure 92. The axes of a temporary grid through *O* are shown in heavy dashed lines and those of the permanent grid in heavy solid lines. The *X* and *Y* differences between *O* and the points *D*, *E*, and *F* are shown in light dotted lines for the temporary grid and in light solid lines for the permanent grid. The figures show that the corrections to the temporary coordinates will be different for each point. The necessary conversion may be carried out accurately by computation or more rapidly by plotting.

TEMPORARY TO STANDARD GRID COORDINATES



In this form initials are substituted as follows
S = standard grid. T = temporary grid.
Z = azimuth from grid north.
B = bearing (cardinal points must be shown).
MS = magnification of scale (same sign as δSY)
Z Corr. ± = correction to make TZ = SZ (same sign as T north error).
The diagram should show that corrections have been made in the proper sense. Show temporary grid in dashed lines

Sta. O = ▽ 64 E 10 Sta. A = ▽ 64 F 2 T north error ± = + 1°53'25"

Symbols	Values	Logs	Symbols	Values	Elevations – feet
O – TX	810 538 2		O – TY	1 819 948 8	O – T 213 2
A – TX	812 038 0		A – TY	1 818 528 1	O – S 205 6
δ TX	+ 1 499 8	3 176 03	δ TY	– 1 420 7	Corr. ± – 7 6
δ TY		3 152 50	TZ	133 26 55	A – T 199 8
(tan) TB	S 46 33 05 E	0 02353	Z Corr. ±	+ 1 53 25	A – S 192 2
SB	S 44 39 40 E		SZ	135 20 20	Logs
δ TX		3 176 03	δ TY		3 152 50
– sin TB		9 860 93	– cos TB		9 837 40
dist. O – A		3 315 10	← check →		3 315 10
sin SB		9 846 90	cos SB		9 852 04
δ SX	+ 1 452 1	3 162 00	δ SY	– 1 469 4	3 167 14
O – SX	810 588 9		O – SY	1 819 937 6	MS per 1000 0.4
A – SX	812 041 0		A – SY	1 818 467 6 / 468 2	–0.6 ± MN δ SY

Sta. O = ▽ 64 E 10 Sta B = ▽ 64 F 5 T north error ± = above

Symbols	Values	Logs	Symbols	Values	Elevations – feet
O – TX	810 538 2		O – TY	1 819 948 8	O – T above
B – TX	810 499 0		B – TY	1 822 324 6	O – S
δ TX	– 39 2	1 593 29	δ TY	+ 2 375 8	Corr. ± – 7 6
δ TY		3 375 81	TZ	359 03 17	B – T 229 1
(tan) TB	N 00 56 43 W	8 217 48	Z Corr. ±	+ 1 53 25	B – S 221 5
SB	N 00 56 42 E		SZ	00 56 42	Logs
δ TX		1 593 29	δ TY		3 375 81
– sin TB		8 217 41	– cos TB		9 999 94
dist. O – B		3 375 88	← check →		3 375 87
sin SB		8 217 27	cos SB		9 999 94
δ SX	+ 39 2	1 593 15	δ SY	+ 2 375 8	3 375 81
O – SX	810 588 9		O – SY	1 819 937 6	MS per 1000 0.4
B – SX	810 628 1		B – SY	1 822 314 4 / 313 4	+1.0 ± MN δ SY

Sta. O = Sta. = T north error ± =

Symbols	Values	Logs	Symbols	Values	Elevations – feet
O – TX			O – TY		O – T
– TX			– TY		O – S
δ TX			δ TY		Corr. ±
δ TY			TZ		– T
(tan) TB			Z Corr. ±		– S
SB			SZ		Logs
δ TX			δ TY		
– sin TB			– cos TB		
dist. O –			← check →		
sin SB			cos SB		
δ SX			δ SY		
O – SX			O – SY		MS per 1000
– SX			– SY		± MN δ SY

Date: May 6, 1940 Computed by: J.H.B Checked by: B.B.H FORM 17

FIGURE 93.—Temporary to standard grid coordinates.

(4) The heading of Form 17 (fig. 93) is as follows:

(TEMPORARY TO STANDARD GRID COORDINATES)

In this form initials are substituted as follows:
S=standard grid; T=temporary grid.
Z=azimuth from grid north.
B=bearing (cardinal points must be shown).
MS=magnification of scale (same sign as δSY).
Z cor.±=correction to make $TZ=SZ$ (same sign as T north error).
The diagram should show that corrections have been made in the proper sense. Show temporary grid in dashed lines.

The computation is carried out on the form in four steps:

(*a*) Compute the temporary grid azimuth and log distance O to A from the temporary coordinates of O and A (any point whose temporary grid coordinates are known).

(*b*) Correct the temporary azimuth to the standard grid azimuth by applying the azimuth correction.

(*c*) Compute the standard grid difference of X and Y between O and A.

(*d*) Secure the standard grid coordinates of A by applying these X and Y differences and the magnification of scale to the standard coordinates of O.

(5) The plotting method is less reliable, but it is probably the quickest when a large number of points have to be converted in a short time and computers are scarce. It is sufficiently accurate to afford a rough check on the computations, if desired. This method requires three steps:

(*a*) All of the points to be converted are plotted on a gridded board by their temporary coordinates, also the temporary azimuth from the common point O to some other preferably distant point P. If the temporary coordinates of P are known, this point will be plotted and the azimuth line drawn by connecting O and P. If only the azimuth of P is known, the azimuth is drawn in by calculating cutting points as described in paragraph 122.

(*b*) The point A and the azimuth line AP are plotted on a second board, cutting points being used, unless the permanent grid coordinates of P are known.

(*c*) A tracing from the "temporary" board is now placed over the second board in such manner that the points O and the two azimuth lines OP coincide throughout. All points on the tracing are now pricked through and their standard grid coordinates read off the board.

i. Triangulation between single points.—This problem may come up wherever the sides of the main triangulation range from 10 to 20 miles in length, especially if the original observations have been made

from towers, when such line has to be broken down by grid triangulation with sides of 1 to 5 miles for topographical or other purposes. △West and △North in figure 94 are two main scheme stations. Beginning at △West, the stations in a chain of small triangles extending to △North have been marked and occupied. The standard grid coordinates, the true grid azimuth and log (true grid distance in yards between the two main scheme stations) having been given or computed, the grid triangulation can be solved in the following steps:

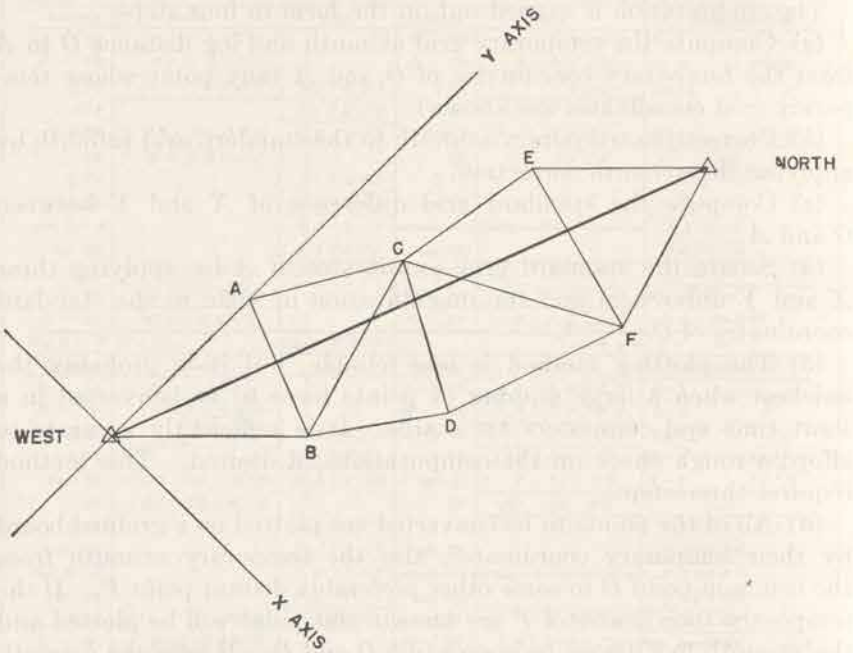


FIGURE 94.—Triangulation between single points.

(1) Assuming either of the short sides from △West, in the present example △West–A, as unity, compute the grid triangles through to △North on Form 8.

(2) In order to determine the log distance △West–△North at the same scale, the line of stations on either side of the system (in the present case *A*, *C*, and *E* have been selected) may be computed as a traverse. For convenience △West is assumed as the origin, and the line △West–A as the positive *Y* axis of a system of local coordinates. Deducting the sums of the corrected triangle angles at *A*, *C*, and *E* (obtained from form 8) from 360°, the exterior angles, which in this case are the traverse angles, are obtained. Beginning with △West–*A* as zero azimuth, the successive azimuths to include *E*–

△North are calculated on Form 1. Obtaining the log distances from Form 8, the traverse is computed on Form 2.

(3) The azimuth and log distance △West–△North are computed on Form 11 from the local coordinates of △North as found in (2) in terms of log distance △West–A.

(4) The relation between the true and assumed scales is now found by subtracting the log distance △West–△North as computed in (3) on the local coordinates from the log (true grid distance between the same stations) as derived from the basic data. By adding the resulting log factor to all the log sides of the grid triangulation as computed in (1) above, the true log sides are found.

(5) The standard grid azimuth of the line △West–*A* can be found by subtracting the local grid azimuth of △West–△North, as determined in (3), from the known standard grid azimuth of the latter line.

(6) All the other azimuths may be calculated by the adjusted triangle angles from Form 8 in (1).

(7) From the true grid azimuths and log sides, the computation of the coordinates of all points of the grid triangulation should close the grid triangulation on △North, after correcting all the *Y* coordinates for magnification of scale, using △West as the base station.

SECTION XX

INTERSECTIONS

Paragraph
General 120
Graphic method 121
Intersection by semigraphic method 122

120. General.—*a. Types.*—The position of an unknown point is established by means of the intersection of direction rays sighted from two or more occupied points of known position. The known points may be plotted on a plane table board and the unknown point located at the intersection of the rays drawn on the board. The known points may be occupied with transit or theodolite, and angles or directions observed. From this the azimuths of the lines and the position of their intersection may be computed to an accuracy limited only by the accuracy of the known positions, the definition of the intersected point, the quality of the instrument, and the skill of the observer.

b. Advantages.—The desirability of obtaining the position and elevation of points without occupying them needs no demonstration. The utilization of the plane table in triangulation reconnaissance (sec. XV) is founded on graphic intersections. The importance of

securing additional points by intersection, often in remote and difficult terrain, incidental to the observations of the main scheme station, has been explained in section XVIII. The ease and advantage of intersecting additional points, thus increasing the controlled area, during the running of control traverses is less commonly realized. An adjacent traverse station, preferably the one to the rear, should be used as an initial on these observations. Not only can such points be determined during the traversing, but the intersected points may later be occupied with an instrument, and more distant points intersected with an accuracy suitable for many purposes. The necessity in war of advancing the control as far as practicable into inaccessible territory is so important as to justify some modification of the requirements mentioned below. For extra long sights which might run as high as 20 miles, observation from both flanks will require special arrangements with other units.

c. Requirements.—No intersection of less than three rays should be accepted. While two rays will give a location, there is no check against a faulty identification of the object sighted upon, or any other error. The directions from at least two of the stations should, if practicable, form a good angle of intersection at the object to be located.

121. Graphic method.—*a. Drawing rays.*—When the table is set up and properly oriented, a line prolonged through the plotted position of the table and the plotted position of any other object will pass through the object. Thus, if a line is drawn on the sheet so that it passes through the plotted position of the table and, if prolonged, through an unlocated object, the latter must lie somewhere on the drawn line. The method of obtaining such lines is as follows: Assuming in figure 95 that the table is set up and oriented at station *A* and that an object exists at *S*, the position of which is unknown, the alidade is placed so that the edge of the base passes through *a*, and the object at *S*, seen through the telescope, is bisected by the vertical line of the telescope. The position of *S* on the sheet is then on a line drawn along the edge of the base—that is, on the line marked "spire" in the figure. Such a line of position is called a "cut" or "ray." The position of the object on the line is unknown, of course, but if the table is set up and oriented at some other suitable point, as at station *B*, a second cut from the new station will locate the position of *S* at the intersection of the two cuts. The position should be verified by a third cut from another station, such as *C*. At each station occupied, the surrounding territory should be scrutinized to select all objects to which cuts are desirable. The first cut to an object is usually marked

in a distinctive manner, and the first intersection by ringing it in pencil. If numerous cuts are taken, it may be desirable to number and record them in a notebook. A chisel-pointed 5H or 6H pencil is best for drawing lines; a sharp-pointed 4H pencil is generally used for lettering. Care must be taken to keep the flat side of the chisel-pointed pencil against the edge of the base and to hold the pencil at a constant angle to the plane of the paper, so that there will be no deviation of the line from the edge of the base. To avoid confusion and unnecessary marking of the sheet, each cut should be drawn only in the general location of the object but must be drawn so that, if extended, it will pass through the center of the plotted point representing the position of the station from which the cut is obtained. If practicable, the same edge of the base should be used for all cuts as a precaution in case the two edges are not exactly parallel.

b. Orientation lines.—In order that the second orientation may be as accurate as possible, it is desirable that the alidade be pointed as closely as practicable in exactly the opposite direction from the first pointing. For this purpose, advantage should be taken of the full length of the alidade base, when the first cut is obtained, by drawing short orientation lines along the edge of the base at both ends. When the second station is occupied it is obvious that the alidade will be reversed more exactly by placing the edge of the base against the orientation lines than by depending on two points, or a point and a short line, relatively close together. Orientation lines are always drawn when a cut is taken to a station where the table is to be set up and oriented by a backsight on the station from which the cut is taken; otherwise they are not required.

c. Adjustment of position.—When three cuts to an object from a series of stations fail to intersect at one point they form a triangle called the "triangle of error." The usual causes of such discrepancies and remedies therefor are given below:

(1) *Erroneous plotting of control stations.*—This is unlikely if the plotting has been carefully checked, but it may be desirable in some cases to recheck the positions.

(2) *Failure to identify the object from one or more of the stations or if the object is indefinite, to sight on the same point of the object from all stations.*—This should be eliminated by careful pointings from each station; if unavoidable, it can often be corrected by additional cuts from other stations.

(3) *Small angle of intersection between two cuts.*—This should be corrected by a fourth cut from a suitably located station; adjustment of position, in case a fourth cut cannot be obtained, is discussed below.

(4) *Distortion of sheet.*—Large errors due to abnormal distortion should be eliminated by allowances at the stations from which the cuts are obtained. Errors caused by slight distortion may be considered as a part of the discrepancy due to small errors in orientation and adjusted as described below.

(5) *Errors in orienting table at stations.*—Large errors should be avoided by checking the orientation by sighting to a second known point.

(6) *Adjustment of small errors.*—If there is a triangle of error and the topographer is reasonably certain that the discrepancy is not due

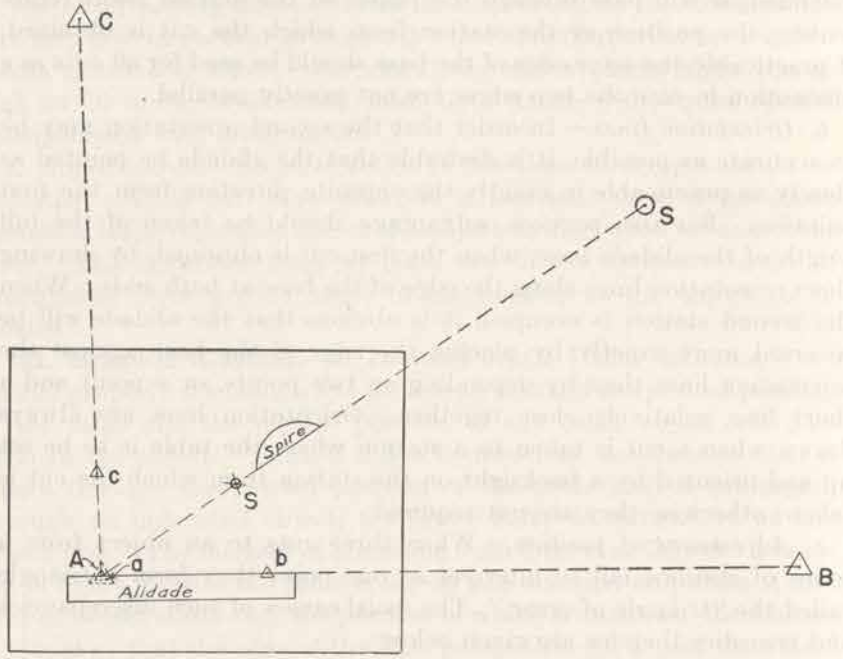


FIGURE 95.—Graphic intersection.

to the causes mentioned in (1) and (2) above (large errors of orientation or distortion), it is probably caused by small deflections in azimuth due to the limitations of the instrumental equipment. Such triangles of error should be quite small and may be adjusted by selecting the most probable position of the object.

d. Graphic triangulation.—This is also called "plane table triangulation" and is useful in reconnaissances. It is mostly intersection, with a few graphic resections (par. 124). A typical illustration is given in figure 96. In this example, there are triangulation stations, Ash and Bay, at the entrance points of a small bay but no

stations inside, and it is desired to provide the latter by extending graphic triangulation from the base Ash–Bay. It is desirable that the table be set up with its plotted position plumbed over the station; if this is impracticable at any station, the table should be set in line with two stations, and its exact position, after work at the station is completed, should be marked by a flag for use in orienting by a back-

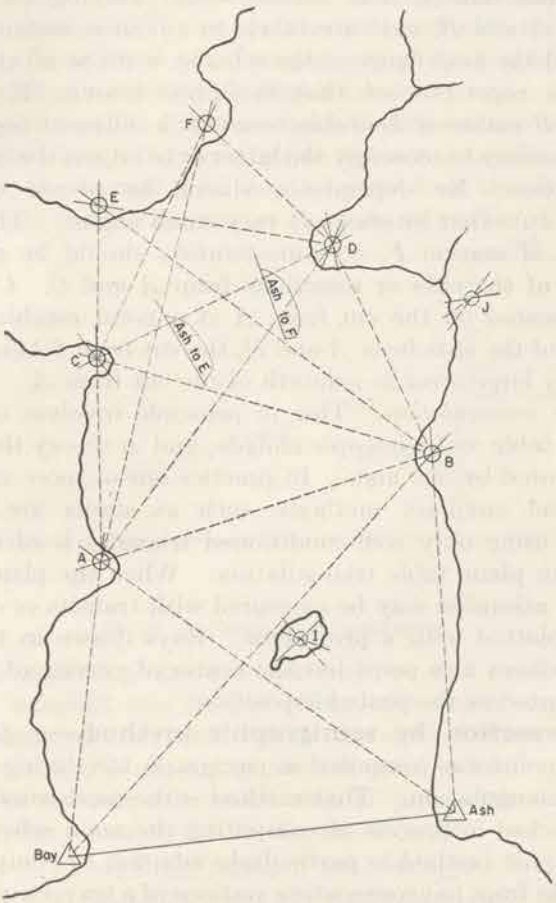


FIGURE 96.—Graphic triangulation.

sight. Great care must be taken in drawing cuts to observe the precautions previously mentioned. Orientation lines should be drawn for each cut unless it is certain that they will not be required for orienting. Plane table operations may be started either at Ash or Bay. If the former, the station routine should be about as follows: Set up table at Ash; orient on Bay; draw magnetic meridian; cut to

A and *B* and to any supplemental objects, such as *I*, that it is desired to locate by intersections. Theoretically, the observations at Ash and Bay are for the purpose of locating *A* and *B*, and no additional cuts are necessary. Actually, the table is located and oriented more accurately at Ash and Bay than will be the case at any other station, and it is therefore advisable to cut from these stations to all other stations of the scheme that can be seen. Having established the positions of *A* and *B*, cuts are taken to advance stations, including those beyond the next figure of the scheme, work at all stations being essentially a repetition of that described above. If information obtained at *B* makes it desirable to adopt a different position for *A*, it will be necessary to reoccupy the latter or to adjust the cuts observed at that station. No dependence should be placed on positions obtained by cuts that intersect at very small angles. Thus, in fixing the position of station *F*, no consideration should be given to the intersection of the cuts or resections from *A* and *C*. Ordinarily, *F* should be located on the cut from *A* at a point established by the intersection of the cuts from *A* and *D*, the cut from *C* being used only to detect any large error in azimuth of the cut from *A*.

e. Graphic triangulation.—This in principle involves only the use of the plane table and telescopic alidade, and in theory the operations may be executed by one man. In practice one or more assistants are employed and auxiliary methods, such as stadia are used. The principle of using only well-conditioned triangles is of the greatest importance in plane table triangulation. When the plane table cannot be used, azimuths may be measured with transits or other instruments and plotted with a protractor. Rays drawn in this way are not likely to meet at a point but the center of gravity of all positions may be accepted as the probable position.

122. Intersection by semigraphic method.—*a. General.*—An intersection point was computed in paragraph 118 during the solution of the grid triangulation. That method is the most convenient when it can be worked in process of computing the main scheme stations. The semigraphic method is particularly adapted to computing intersections made from nonconsecutive stations of a traverse or from other stations for which the interstation distances are unknown. In this method the position is accurately fixed from the coordinates and azimuths at stations selected almost at random, and by very simple means which do not involve the computation of any starting bases, or final computation of coordinates, which are read directly from the graph. Another advantage is that extra rays may be included in the determination with practically no added labor. The graph immediately exposes an incorrect observation, a faulty identification of the

observed object, or erroneous coordinates and indicates the correct point from the correct data, without any waste of time, which is advantageous in grid triangulation with its duplication of observers and lack of detailed reconnaissance.

b. Trial points.—The first trial point (T) coordinates are read off a gridded map, plane table sheet, or a rough small-scale graph plotted on a separate piece of squared paper. A 100-yard square, bounded by grid lines numbered in even 10's or 50's and with the trial point near its center, is selected and the numbers of the bounding lines are marked in position on the graph (figs. 98 and 99).

c. Cutting points.—(1) Cuts are used to draw a line or portion of a line on a given azimuth from a point of known coordinates. If the coordinates of the second point can be plotted on the sheet, the line connecting them is thus plotted at the true azimuth. If the coordi-

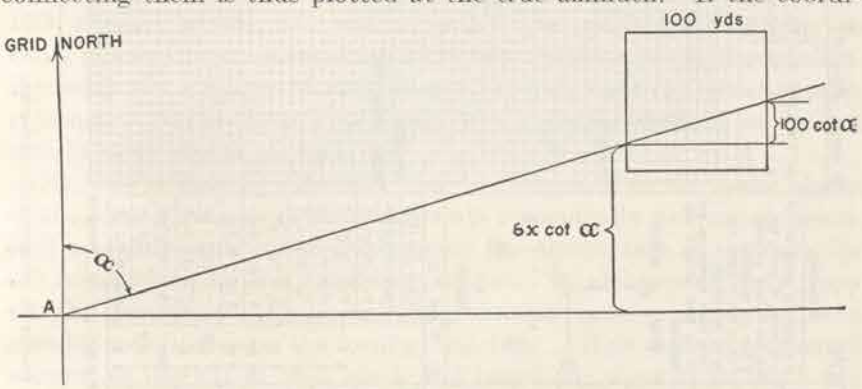


FIGURE 97.—Cutting points on grid square.

nates of the second point are unknown or not available for the purpose, the azimuth may be laid off by selecting a convenient grid line and computing the point where the desired line will intersect the grid line, which is chosen by the following rules:

Rule I. For an azimuth within 45° of grid-north or grid-south, work out the cut on west-east line by the formula

$$\delta X = \delta Y \text{ tan azimuth (or bearing)}.$$

Rule II. For an azimuth over 45° from grid-north or grid-south, work out the cut on a south-north line by the formula

$$\delta Y = \delta X \text{ cot azimuth (or bearing)}.$$

Incidentally, this is an excellent method of plotting a line of given azimuth on any gridded sheet, as the result is more accurate than can be obtained with a protractor.

(2) The *first cut* on the square laid out on the graph is computed by one of the above rules. In figure 97, as the azimuth is nearly

SEMIGRAPHIC INTERSECTION

From	① ▽64 F8		② ▽64 F5	
Azimuth	45 00 00		300 00 00	
Map Y (cross)	~~490 000 0~~		~~492 108 0~~	
± MnS Y	490 000 0		-38 492 104 2	
Station	X Y 490 000 0		X Y 570 000 0	
T1 Square	X Y 494 900 0		X Y 565 100 0	
δ	X Y 4 900 0		X Y 4 900 0	
log δ	X Y 3 690 20		X Y 3 690 20	
α log	cot tan 0 000 00		cot tan 9 761 44	
(Sum) log δ	Y X 3 690 20		Y X 3 451 64	
δ	Y X* + 4 900 0		Y X* + 2 829 1	
Station	Y X 560 000 0		Y X 492 104 2	
Cut A	Sum 564 900 0		Sum 494 933 3	
100 cot tan δ	Y X† + 100 0		Y X† + 57 7	
Cut B	Sum 565 000 0		Sum 494 991 0	
C=A± /10 δ	Y X† 564 990 0		Y X† 494 991 0	
D=C± 1/10 δ	Y X† 565 000 0		Y X† 494 996 8	

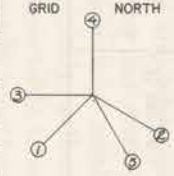
From	③ ▽64 H3		④ ▽64 H4		⑤ ▽64 F6	
Azimuth	90 00 00		180 00 00		330 00 00	
Map Y (cross)	~~495 000 0~~		~~500 000 0~~		~~490 000 0~~	
± MnS Y	-9.0 494 991 0		-180 499 982 0		490 000 0	
Station	X Y 560 000 0		X Y 499 982 0		X Y 490 000 0	
T2 Square	X Y 565 000 0		X Y 495 000 0		X Y 494 990 0	
δ	X Y 4 980 0		X Y 4 982 0		X Y 4 990 0	
log δ	X Y 3 697 40		X Y 3 697 40		X Y 3 698 10	
α log	cot tan		cot tan		cot tan 9 761 44	
(Sum) log δ	Y X		Y X		Y X 3 459 54	
δ	Y X*		Y X*		Y X* -2 881 0	
Station	Y X 494 991 0		Y X 565 000 0		Y X 567 881 6	
Cut C	Sum 494 991 0		Sum 565 000 0		Sum 565 000 6	
10 cot tan δ	Y X†		Y X†		Y X† -5 8	
Cut D	Y X 494 991 0		Y X 565 000 0		Y X 564 994 8	

Date: April 16, 1940 Computed by: J.H.B. Checked by: H.H.H. FORM 18

Instructions

If azimuth is within 45° of grid north or south, use log tan (second column).
*Sign by inspection.
†δX or δY is same sign as at asterisk (*).
Place block number on end of ray from which it was observed.

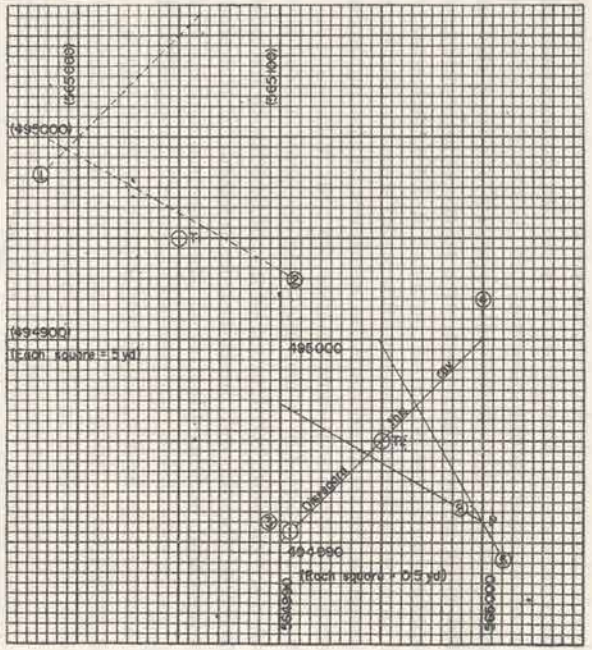
GRID NORTH



Diagram

At station R64 H7 Grid Zone B

	X	Y	Base station ①
1st trial point T1	565 050 0	494 950 0	MnS 1.8 yd per 1000
2 " " T2	564 995 0	494 995 0	P Map Y below
Final " P	565 000 0	~~494 991 0~~	495 000 0


(494900)
(1 each square = 1 yd)
495000
(495000)
(495000)
(494900)
494950
494990
495000
495000
(495000)
(495000)
(495000)

west-east, the Y of the cut on the first line of the square will be the Y of $A+\delta X \cot \alpha$. Since the second line of the square is 100 yards farther east, the Y of the second cut will be greater by 100 cot α than the y of the first cut. By plotting the two cuts on the corresponding sides of the square on the graph, a portion of the ray can be drawn.

(3) Form 18, "Semigraphic intersection" (fig. 98), shows the solution of a hypothetical problem to illustrate the method. The small diagram is invaluable with traverses, but should always be drawn as a guide in determining the signs at the asterisks. Usually only two rays, selected to give an intersection as near a right angle as possible, are computed with the first trial point in blocks (1) and (2) of the form. Plot these two rays in dashed lines on the graph and at the intersection obtain the coordinates of the second trial point ($T2$). Select the 10-yard square bounded by grid lines numbered in even 10's which contains $T2$, and renumber the graph lines forming its sides, placing parentheses about the other numbers to avoid confusion. Knowing the changes of coordinates for cuts on a 100-yard square, it is simple to figure the cuts on the new square as is done in the last two lines of blocks (1) and (2).

(4) The remaining cuts may now be computed in the lower blocks of the form. As one of the trial points occasionally proves erroneous, it is probably economical to postpone the abstraction of the data for the lower block until $T2$ has been tested. In abstracting data, copy all the necessary items, including station names, azimuths, and x and y coordinates on the form at one time. Rule out the unwanted column at the left of each block and insert the signs after the * and † marks. The calculations are simple, and by working the columns concurrently all of the logs and antilogs can be looked up at one opening of the book.

(5) As the graph lines are drawn, the end toward the station from which each was observed should be marked with the number of the block in which the cuts were obtained. The final position of P is selected by inspecting and weighing up the various rays. Rays which do not "plot" in fair agreement with the others should be disregarded. The final points in the triangle of error should be chosen so as to be nearest the shortest ray and farthest from the longest ray. The magnification of scale correction has to be added to the y coordinate in order to obtain the standard grid coordinates of the final position, as shown above the graph on the form.

(6) If the graphs are to too small a scale the intersection will be difficult to determine accurately. The size of the graph on Form 18 is limited. If the issue blocks of 8- by 10½-inch squared paper with the small $\frac{1}{10}$-inch squares are available, one or both of the graphs should

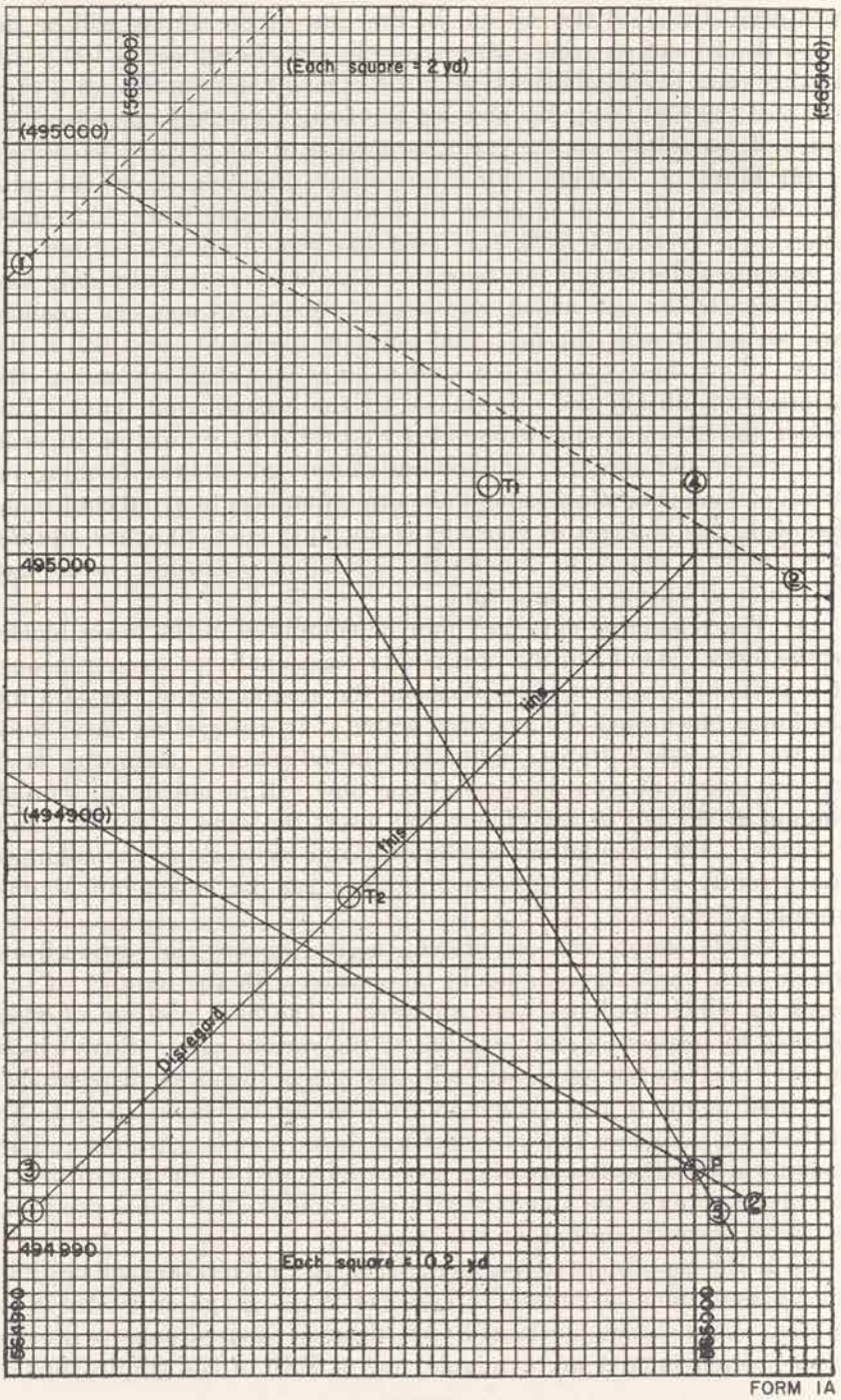


FIGURE 99.—Graphs on squared paper.

be drawn on that (fig. 99) instead of on the form. The small squares might well be taken as 5 yards each for the 100-yard square, and 0.2 yard for the 10-yard square.

d. The printed instructions, which space limited, are reprinted below, in case they are illegible on the form.

SEMIGRAPHIC INTERSECTION

If azimuth is within 45° of grid north or south, use log tan (second column).
*Sign by inspection. † δX or δY is same sign as at asterisk (*).
Place block number on end of ray from which it was observed.

e. Exercise XXIII. Semigraphic intersection.—From data furnished by the instructor, obtain the coordinates of an intersection point on Form 18, drawing the graph for T2 on both the form and a separate piece of squared paper, at a larger scale.

SECTION XXI

RESECTIONS

	Paragraph
General	123
Graphic method	124
Resection by semigraphic method	125
Computation of three-point problem	126

123. General.—*a. Types.*—Resection is the determination of the observer's position by means of observations taken to previously fixed points. The several methods of resection include:

(1) Observing horizontal angles **from** the unknown point to three known points.

(2) Observing horizontal angles from two unknown points to two known points. (See par. 130 for a mathematical solution of this problem.)

(3) Observing horizontal angles from one unknown point to two known points when the azimuth of one of them is known. These observations may be taken graphically with the plane table (par. 124*b*), or the angles may be measured with a transit or theodolite.

b. Trigonometric resection.—This is founded upon the same principles as plane table resection and the same rules are applicable in the selection of known points. However, the angles and positions are determined so much more accurately than by purely graphic methods that a position can be satisfactorily fixed somewhat closer to the "danger circle" (par. 124*c*(4)(*c*)) than with the plane table. In order to get a check on the result, the angles should be measured between four points instead of three. In grid triangulation the round should include every visible station, so that the position can be obtained, even though a line should fail to "plot." Vertical angles are measured so that the elevation as well as position may be computed.

c. Relative advantages.—(1) The semigraphic method, in which an approximate position is determined graphically, and the distance and direction of the true position from the approximate one are calculated (par. 125), permits the easy inclusion of extra points.

(2) Mathematical methods, in which the position is deduced directly from the angles (par. 126), permit only three rays to be used at a time and each trio may give a different result.

(3) Resection is useful for many purposes besides purely topographical work. While intersection is generally employed in recon-

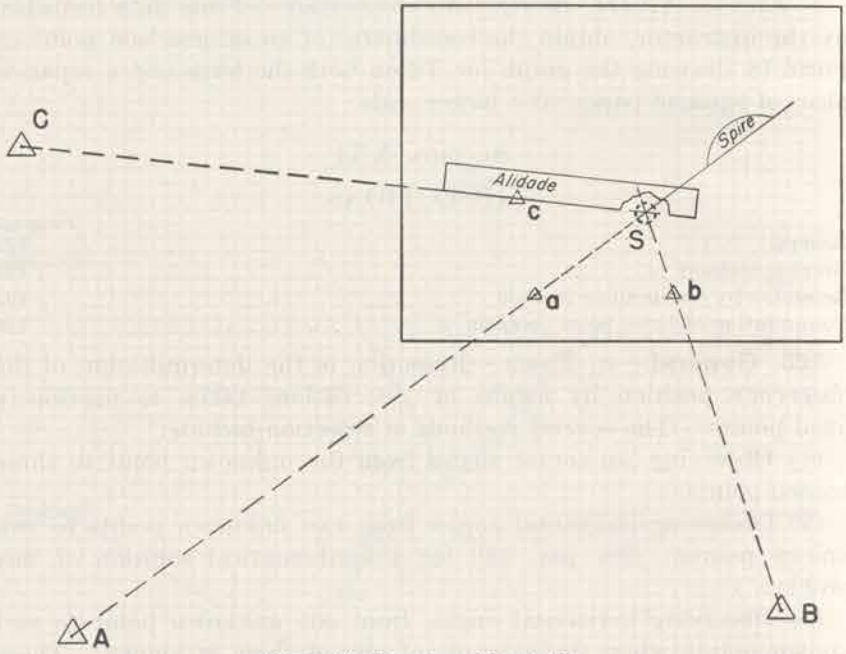


FIGURE 100.—Resection with backsight.

naissance for triangulation, resection is of occasional value in order to save retracing the route when the intersected points are found impracticable. Resection is quicker for establishing an auxiliary point than intersection which requires the occupation of three stations. The special use of resection for carrying forward control in the case of a rapid advance has been explained in paragraph 98*o*.

124. Graphic method.—*a. General.*—In resecting a position the board must first be approximately oriented in one of the following ways:

(1) By a ray previously "cut" through the position from a known point so marked that a backsight can be made.

(2) By means of the magnetic needle.

(3) By selecting a trial point and laying on a distant point.

b. Backsight method (fig. 100).—After the table has been set up and oriented at a located station and one cut has been obtained to an unlocated station, there is another method by which the latter can be located. Instead of obtaining cuts from other stations, the topographer may proceed directly to the unlocated station (point *S* in figs. 95 and 100) and determine its position in the following manner:

(1) Referring to figure 100, set up the table over *S* and place the alidade so that the edge of the base is against the cut line and passes through the plotted position (*a*) of station *A* from which the cut was obtained. The telescope should point toward *a*, that is, exactly opposite to its pointing when the cut was obtained. Swing and clamp the board so that the signal at *A*, seen through the telescope, is bisected by the vertical line of the latter. This operation orients the table. Next place the alidade so that the edge of its base passes through the plotted position *c* of station *C*, or any other station suitable for use, and the signal at *C* is bisected by the vertical line of the telescope. Having in mind the conditions that obtain when the table is oriented, it is evident that the position on the sheet of *S* must lie somewhere along the edge of the alidade base. A short line drawn along the edge to intersect the first cut, therefore, locates *S* at the point of intersection of the two lines. A second intersecting line to check the position should be obtained by resection on station *B*. Previous remarks relative to pencils to be used and precautions to be observed in drawing cuts also apply to resection lines.

(2) If the table cannot be set directly over station *S*, it is entirely practicable to place it near *S* and in line with *S* and *A*. The sheet is then oriented as just described and its position located by resection, after which the position of *S* can be determined by measuring its distance from the table. Having located a station by resection, cuts can be taken to locate any other unlocated points.

c. Three-point resection.—(1) When three control stations or other previously determined points can be seen and are *suitably situated*, the plane table can be located and oriented simultaneously by applying the principle of resection to effect a graphic solution of the three-point problem. Having in mind the principles of orientation, it is evident that, when the table is properly oriented, resection lines from two or more stations will intersect at a point on the sheet corresponding to the position of the table on the ground. If the table is not oriented accurately, resection lines from three stations will form a triangle of error, the size of which increases with the error of orientation.

(2) Location and orientation of the table by this method involve the selection of suitable control stations, securing a triangle of error by resecting on the three stations, and interpreting the triangle of error

in order to determine the position of the table. The procedure is as follows: Set up and level the table and orient it approximately. If the approximate position of the table is not known, the magnetic needle may be used conveniently for the preliminary orientation. Sight on the control stations and draw the resection lines, as explained previously, to form the triangle of error (fig. 101). The position of the point sought can then be estimated very closely by observing the following rules:

Rule I. The point sought is on the same side of all resection lines; that is, either to the right or left (as the observer faces the station from which the line is obtained) of all lines.

Rule II. The perpendicular distance of the true position from each ray is directly proportional to its distance from the corresponding known point.

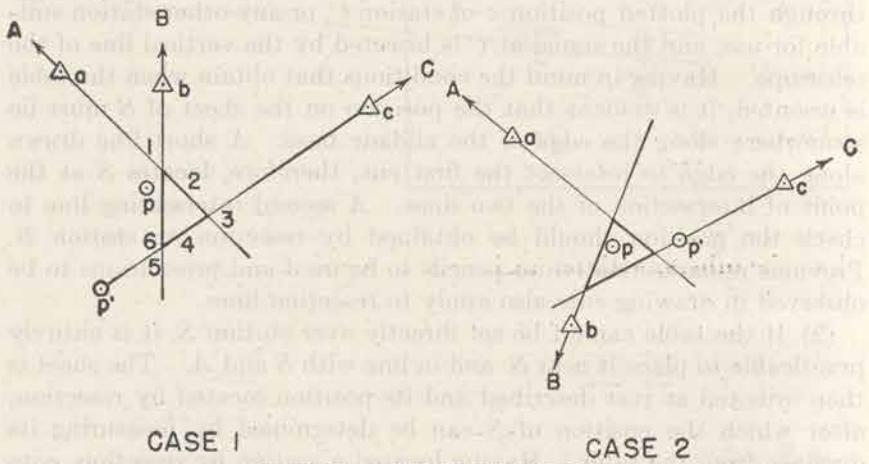


FIGURE 101.—Graphic three-point resection.

Having estimated and plotted the position of the point sought, place the alidade so that the edge of the base passes through the plotted point and the plotted position of the most distant station. (Always use the most distant station for orienting.) Orient on the latter and resect on the two nearer stations. If the position of the point sought was not estimated accurately, a second, but smaller, triangle of error will result, and the operation must be repeated until the correct position of the table is determined. The pencil lines should be fine and light and on the surface of the paper where they can be easily erased. The accuracy of the first estimate will increase with practice.

(3) This method may be understood by reference to figure 101, in which *a*, *b*, and *c* are the plotted positions of the stations *A*, *B*, and

C. The triangle of error, formed by the intersections of the resection lines passing through the plotted positions of the stations, is indicated by heavier lines. The position of the table assumed for the first orientation is shown at p'. In case 1, the plotted position (p) of the table must, in conformity with rule I, lie either in sector 3, to the right of all resection lines, or in sector 6, to the left of all lines. By rule II it must lie in sector 6 in order to be nearer to the line from the nearest station (B) than to the lines from the other stations. It is accordingly plotted in sector 6 at the proper proportionate distances from the three lines. In case 2, the point p must lie inside the triangle of error in order to satisfy the rules. Conditions illustrated in figure 101 hold good in all cases; that is, if the table is located outside the triangle formed by the three control stations, its position on the sheet will lie outside the triangle of error; if it is located inside the triangle, its position will lie inside the triangle of error. This is sometimes called rule III.

(4) The strength of the position and orientation depend chiefly on the relative positions of the three stations used. The principal considerations in selecting control stations are as follows:

(*a*) A strong position will be obtained when the position sought is inside the triangle formed by the three control stations (A, fig. 102) when the latter are nearly in line, or when the center station is nearer than the others, with no angle less than 30° (B).

(*b*) A weak position will generally result from using two stations so situated that the angle between them is small. An exception to this occurs when two stations a considerable distance apart are in range, or nearly in range, with the nearer station the center one (C), provided the angle to the third station is not less than 30°. When the more distant of the two stations is in the center, however, such a position may be very weak (D).

(*c*) A position is indeterminate when the point sought (E) lies on or near the circle passing through the three stations; the position weakens as the point approaches this circle (F and G, fig. 102).

d. Tracing paper solution.—Another solution is obtained by laying off the angles between three or more control stations on tracing cloth or paper and using this as a three-arm protractor to determine the position of the unknown point. Fasten the tracing sheet to the board and mark a point upon it to represent the unknown point. With the edge of the alidade on this point, sight on the known points and draw the several cuts. Release the tracing sheet and shift it over the plane table sheet until each of the cuts passes through the plotted position of the station to which the cut was taken. The intersection of the

cuts on the tracing sheet is now in the proper position over the plane table sheet to indicate the position of the point sought. Locations made by the tracing paper method are likely to be less accurate than those made by the graphic method described in *c* above. They should

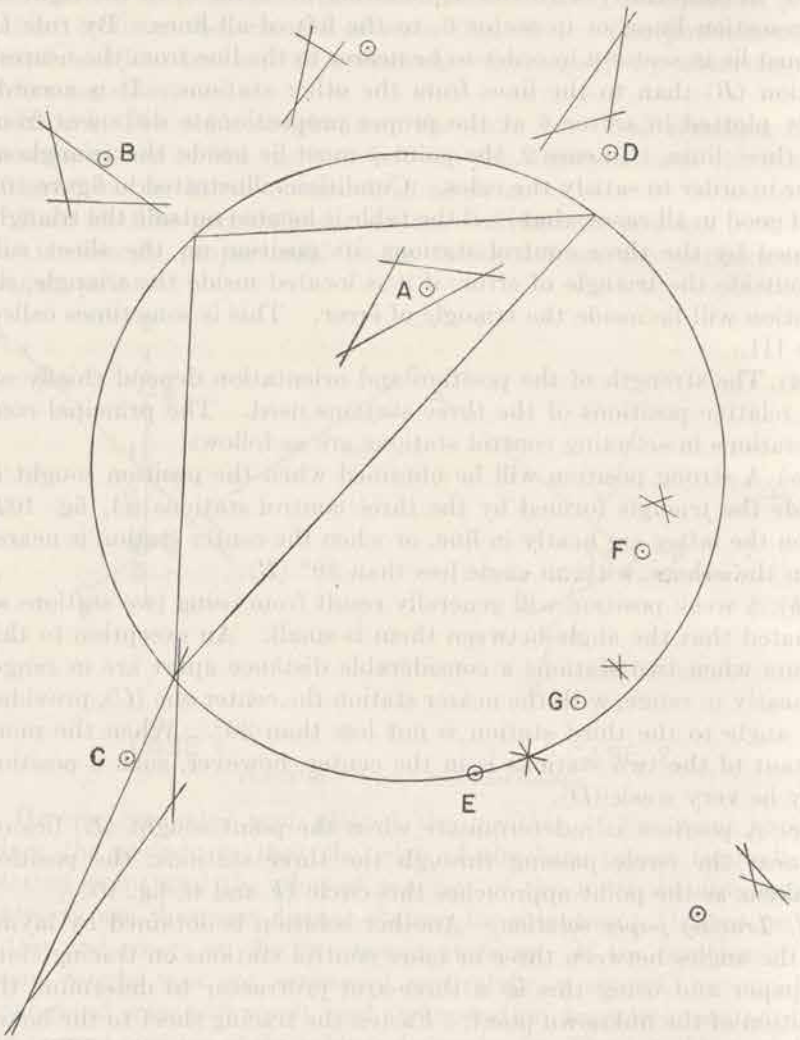


FIGURE 102.—Triangles of error and danger circle.

be checked after they are pricked through to the plane table sheet before using them in the location of other points. In general this method should be used only when special conditions make it necessary, or as a means for locating a trial point for the three-point method.

e. Two-point problem.—(1) It is sometimes desirable to locate and orient the table at a point from which only two control stations are visible; this may be done in several ways. When it is practicable to set up the table in range with the two stations, the position of an unknown point can easily be determined as follows: Set up the table anywhere on the range line and orient by the latter. Place the alidade at any convenient location on the sheet so that a flag or other object, erected over the unknown point, is bisected by the vertical line of the telesceope and draw orientation lines along the edge of the base. Leaving a flag at the first site of the table, proceed to the unknown point, set up the table, reverse the alidade, and orient by a backsight on the first point occupied. As the azimuth of the table is now the same as when on the range line and the table is, therefore, properly oriented, the position of the point sought is shown by the intersection of resection lines from the two control stations.

(2) The solution of the two-point problem, generally used when it is impracticable or undesirable to set up on range, is illustrated in figure 103. This method requires no linear measurements and demonstrates the effectiveness of the plane table as a surveying instrument. Two cases are illustrated in the figure; the description of the method given below is applicable to both cases as well as to other locations of the points. In the illustration, a and b are the plotted positions on the sheet of two control stations A and B, not conveniently accessible, from which it is desired to locate and orient the table at an unknown point (C). The procedure is as follows: Select a fourth point (D) where the resection lines from A and B will give a strong intersection. This point also must be so situated with respect to C that cuts on A and B from C and D will give good intersections. Orient as closely as possible by eye or with the magnetic needle, and resect on A and B. Intersection of the resection lines at d' gives a tentative position of D, differing from its true position because of the error in orientation. Sight on an object erected at C and draw orientation lines and a cut through the point d'. Plot c' on this cut at a distance from d' corresponding to the estimated distance from D to C. Next, set up the table at C, reverse the alidade on the orientation lines for c' d', and orient by a backsight on D. The error in orientation is now the same in extent and direction as at D. Sight on A and draw the cut through c' to intersect the line ad' at a'. Then sight on B and obtain the intersection at b'.

(3) The quadrilateral, $a'b'd'c'$ in case 1, or $a'c'b'd'$ in case 2, has the same shape as that formed by the four points on the ground, but is erroneous in size and position. In all cases, however, the line $a'b'$ is parallel to the line AB on the ground so that the error in

orientation is indicated by the angle between the line *ab* and the line *a'b'*. A convenient method of correcting the orientation is to place the alidade on the line *a'b'* and to note some distinctive and distant object that is bisected by the vertical line of the telescope. Then

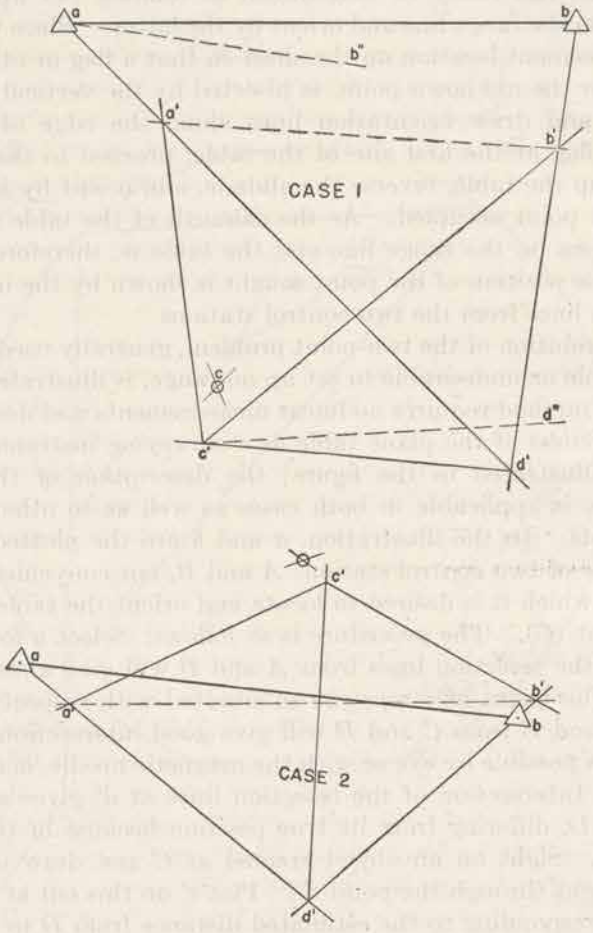


FIGURE 103.—Two-point resection.

place the alidade on the line *ab* and adjust the table until the same object is again bisected. The table is now oriented correctly, so that resection *A* and *B* establishes the true position of the point sought at *c*.

125. Resection by semigraphic method.—*a. General.*—This method is best for any case of grid computation in which a position is to be obtained for an unknown point at which angles between three or more known stations have been observed. In addition to the ad-

vantages ascribed to semigraphic intersection in paragraph 122*a*, it is more rapid and reliable than the usual mathematical methods, is partially self-checking, and extreme proximity to the danger circle is revealed by the loci of the positions falling on lines which are parallel or nearly so.

b. Comparison with semigraphic intersection.—(1) The selection of the first trial point T is made according to the same principles, but the advantages of a close determination are greater. The azimuths of the rays are unknown, but the approximate azimuth of the longest ray is computed and the other directions are corrected from this. The cutting point method is employed, but single cuts are made on either of the axes through $T1$, instead of on two lines of a square containing $T1$. The protractor is employed to plot the ray in azimuth with sufficient accuracy for the short portions of the rays shown on the large-scale graphs.

(2) As the rays plotted for $T1$ almost never meet at a point, the selection of a second trial point $T2$, the correction of the azimuths, recomputation of cuts, and a second plotting to a larger scale are necessary. The larger scale does not increase the relative accuracy, but makes possible the more accurate determination of position to the nearest tenth of a yard. The second trial point might be estimated by the rules in paragraph 124*c* (2) for graphic resection, but a more accurate determination is desirable.

c. Plotting loci.—The solid line rays numbered 1, 2, and 3 in figure 104 form a small triangle of error from which the probable position of the point P is desired. The relative distances of the triangle from the three known points may be determined from the same plat by which the coordinates of $T1$ were selected. Examine the graph to see in which direction all of the lines must be swung in order to bring them to a point. On the side to which this swing is required, draw parallel to each ray a line at a distance from the ray proportional to the distance to the station from which the observation was made. These parallels are the dashed lines of figure 104, numbered ①, ②, and ③. By connecting the intersection of any two of the solid rays and the intersection of the parallel dashed rays of the same numbers (dotted lines 1–2, 2–3, and 1–3), the locus of the point P is obtained for each pair of the solid rays. The three loci intersecting at a point show the final position of the point if the graph is at a scale from which the coordinates can be taken.

d. When more than three rays are observed, all the loci will not meet at a single point, but in several points, one for each trio for which all the loci are drawn. Each of the intersections of loci determines

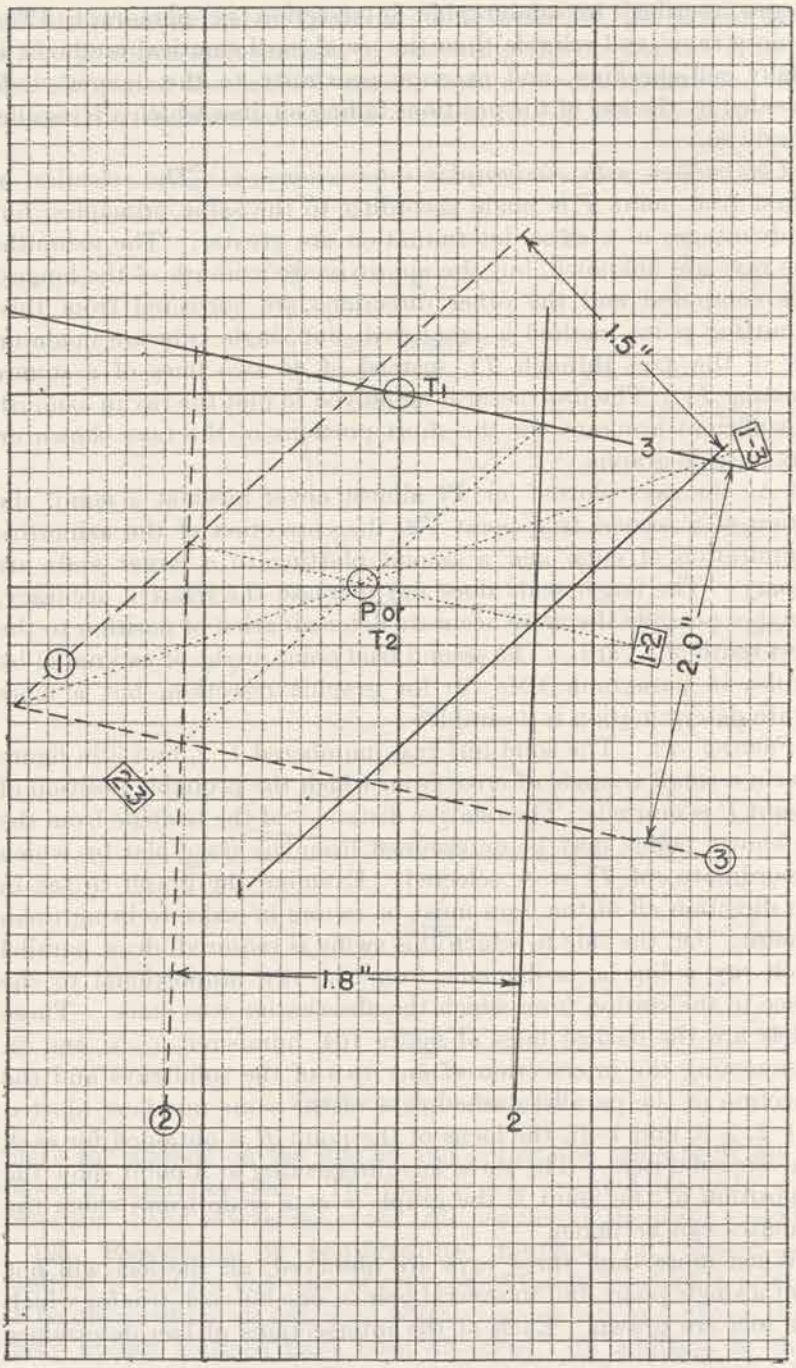


FIGURE 104.—Plotting resection on squared paper.

the position of the new station as fixed by the observations from the three stations represented by the rays and would agree with the position as determined from the same three points by ordinary computation. None of the rays should be discarded unless examination indicates that all the loci concerned with that point are doubtful, in which case the point should be disregarded. One great advantage is that the final position of the point may be fixed as the means of the coordinates determined by the separate trios.

e. Form 19, "Semigraphic resection" (fig. 105), is designed for this computation, which includes the following steps:

(1) Abstract the station names, coordinates of the known stations, and the directions to them to the spaces in the form. Give each station an initial letter to be used in the diagram and graph.

(2) Choose the base station (preferably the most southern), calculate the magnification of scale, and correct all Y coordinates to true distance.

(3) Obtain the coordinates of $T1$ from map or plat and measure distances from the observed stations. (The latter may be reduced to convenient proportions as shown beneath the coordinates' lines of the form.)

(4) Employing the most distant station as Z, compute the azimuth Z–$T1$ and obtain the correction to all the directions by subtracting the direction of Z from the reference line.

(5) Apply the correction thus obtained to the directions in the middle of the form to obtain the azimuths and bearings which are convenient in looking up logs tan and cot.

(6) Complete the diagram.

(7) Below the middle of the form, compute the cuts on $T1$ axes for all stations but Z. The signs of δX and δY are obtained by inspection of the diagram.

(8) On squared paper (fig. 106), plot the cuts and determine the coordinates of $T2$, as described in *c* and *d* above.

(9) Compute the azimuth Z–$T2$, and obtain the correction for the azimuths.

(10) Apply this correction to the azimuths.

(11) Compute new cuts on $T2$ axes and complete a second graph.

(12) Read off the final coordinates of P.

(13) Compute the azimuth Z–P and obtain final correction to azimuths.

(14) Correct all azimuths and convert them to azimuths from P to the known station.

(15) Record coordinates at top of form and correct Y for magnification of scale.

Grid ↓ North

SEMIGRAPHIC RESECTION

(Numbers in parentheses refer to paragraph 125e, TM 5–235)

(1) Sta. ▽ Gravel	X		Y		Base Sta. W			(2)
(3) 1st Trial Point, T1	1366	7500	1795	8500	MnS .15 yd per 1000			(2)
(8) 2nd " " , T2	1366	7920	1795	9000	MnS corr. to P ±.114 yd.			(15)
(6) Diagram (12) Final " P	1366	7911	~~1795~~	~~9000~~	17959114 P Map Y			(15)

Block I — Calculation of azimuth corrections for T1, T2 and P

Azimuths from Z (T) to		(4) T1		(9) T2		(14) P	
Station Z	X	1373	5700	1373	5700	1373	5700
" T1	X	1366	7500	1366	7920	1366	7911
(8) Plot	δX	6	8200	6	7780	6	7789
Station Z	Y	1802	5010	1802	5010	1802	5010
" T1	Y	1795	8500	1795	9000	1795	9000
T1 graph	δY	6	6510	6	6010	6	6010
(11) Plot	log δX	3	833 78	3	831 10	3	831 16
Plot	log δY −	3	822 89	− 3	819 61	− 3	819 61
T2 graph	log tan α	0	010 89	0	011 49	0	011 55
	(see diagram) α	225	43 05	225	45 29	225	45 43
	Z from reference line −	00	00 00	→ 225	43 05	→ 225	45 29
	Azimuth correction ±	225	43 05	± ~~r~~	02 24	± ~~r~~	00 14

Block II — Data, bearings and azimuths to T1, T2 and P

Stations observed	Temple T			Beacon B			Cupola C			Water Tower W		
(1) X	1373	5700		1372	1352		1372	1669		1366	9902	
(2) (± − MnS) Y	1802	5010		1798	2313		1790	3472		1788	3257	
(1) Map Y	~~1802~~	~~5223~~		~~1798~~	~~2461~~		~~1790~~	~~3502~~		~~1788~~	~~3257~~	
(3) Dist. from T & ratio	5"4		1"6	3"4		1"0	4"4		1"3	4"4		1"3
(1) Directions	00	00	00	20	40	19	90	09	58	132	44	02
(5) T1 Corrections ±	225	43	05	225	43	05	225	43	05	225	43	05
(5) Bearings Sta–T1	45	43	05	66	23	24	44	06	57	01	32	53
(5) Azimuths Do.	225	43	05	246	23	24	315	53	03	358	27	07
(10) T2 Az. corr. ±	~~r~~	02	24	~~r~~	02	24	~~r~~	02	24	~~r~~	02	24
(10) Bearings Sta–T2	45	45	29	66	25	48	44	04	33	01	30	29
(10) Azimuths Do.	225	45	29	246	25	48	315	55	27	358	29	31
(13) ± 180 ± P Az. Corr. ±	-179	59	46	-179	59	46	-179	59	46	-179	59	46
(13) Azimuth P to Sta.	45	45	43	66	26	02	135	55	41	178	29	45

Block III — Cutting points on T1 and T2 axes

(7) Station XY				x	1372	1352	y	1790	3472	y	1788	3257		
T1 XY				x	1366	7500	y	1795	8500	y	1795	8500		
δ XY				x		5 3852	y		5 5028	y		7 5243		
log δ XY				x	3	731 21	y	3	740 58	y	3	876 47		
Bearing log cot tan				cot	9	640 58	tan	9	986 60	tan	8	431 77		
(Sum) log δ YX				y	3	371 79	x	3	727 18	x	2	308 24		
± δ YX				y	−	2 3539	x	−	5 3356	x	−	2033		
Station YX				y	1798	2313	x	1372	1669	x	1366	9902		
Cut T1 axis YX				y	1795	8774	x	1366	8313	x	1366	7869		
(11) Station XY				x	1372	1352	y	1790	3472	y	1788	3257		
T2 XY				x	1366	7920	y	1795	9000	y	1795	9000		
δ XY				x		5 3432	y		5 5528	y		7 5743		
log δ XY				x	3	727 81	y	3	744 51	y	3	879 35		
Bearing log cot tan				cot	9	639 75	tan	9	985 99	tan	8	420 39		
(Sum) log δ YX				y	3	367 56	x	3	730 50	x	2	299 74		
± δ YX				y	−	2 3311	x	−	5 3765	x	−	1994		
Station YX				y	1798	2313	x	1372	1669	x	1366	9902		
Cut T2 axis YX				y	1795	9002	x	1366	7904	x	1366	7908		

Date: April 1, 1940 Computed by: J. M. B. Checked by: G. E. M. FORM 19

FIGURE 105.—Semigraphic resection.

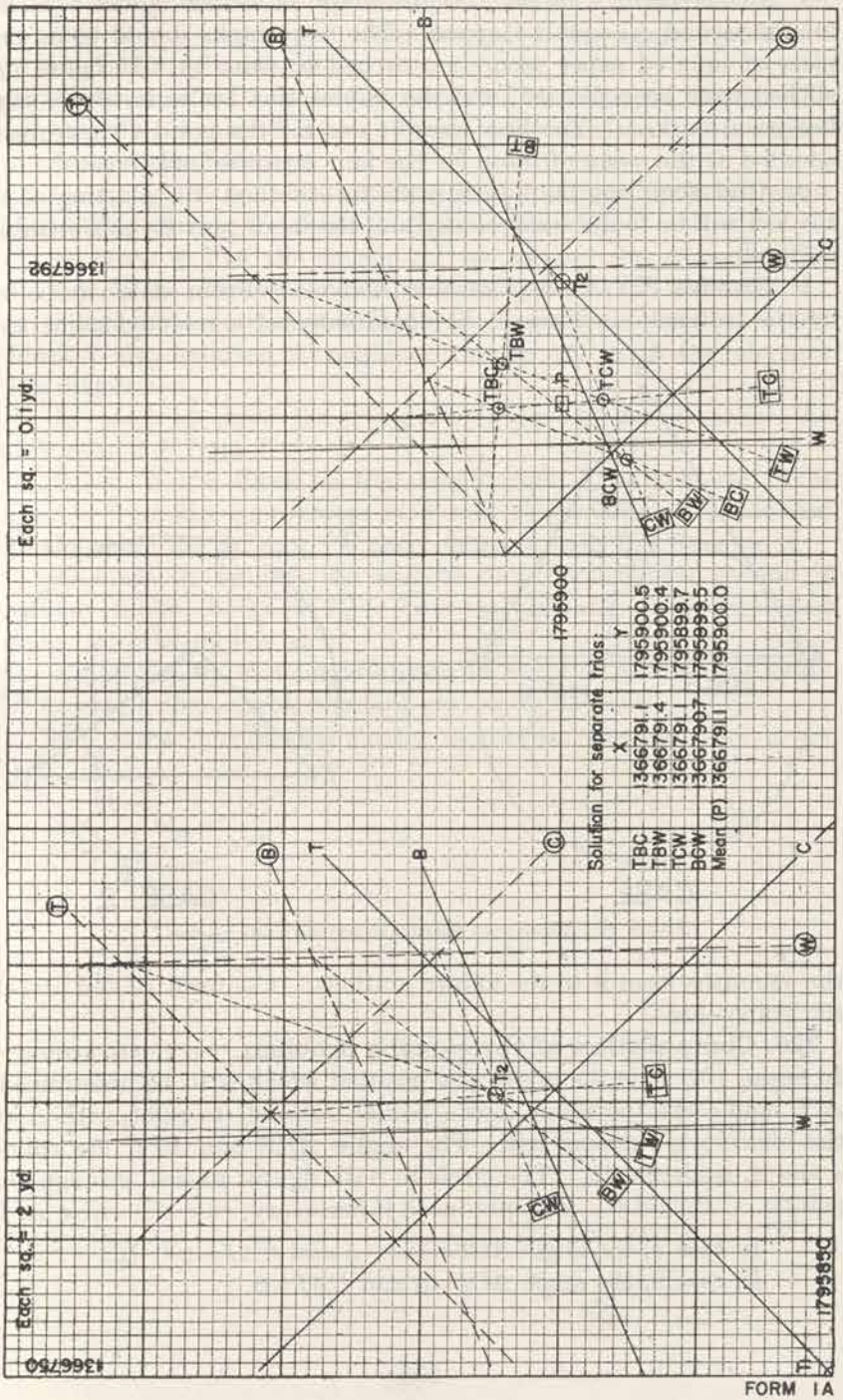


FIGURE 106.—Resection graphs on Form 1A.

f. A convenient scale for the first graph is $\frac{1}{10}$ inch equals 5 yards; for the second, $\frac{1}{10}$ equals $\frac{3}{10}$ yard. The two graphs should be on one piece of squared paper securely attached to form 19. In figure 106 all loci have been plotted, and the intersection for each trio of rays has been marked. The final coordinates for the point P have been adopted as the mean of those for the four separate locations.

g. After acquiring familiarity with this method the plotting of all the loci will be found unnecessary and the labor may be considerably reduced.

h. Exercise XXIV. Semigraphic resection.—From data furnished by the instructor, obtained the coordinates of a resected point and

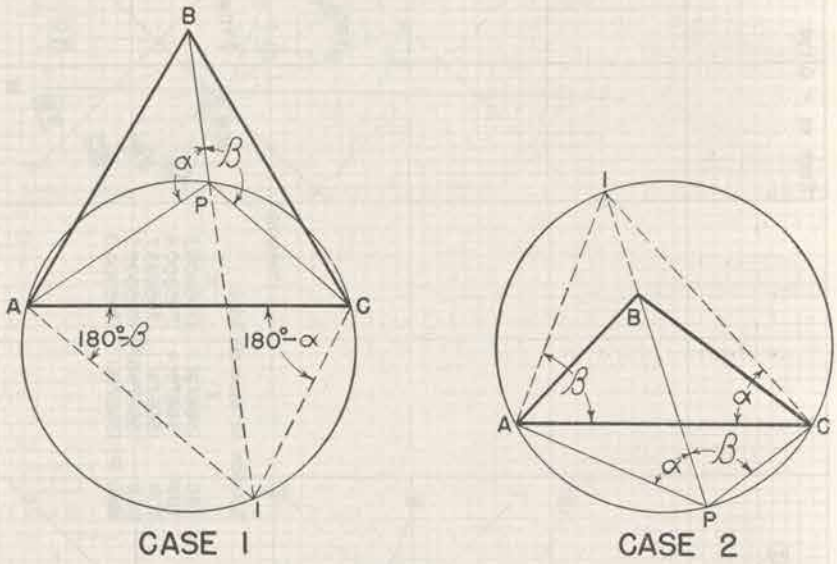


FIGURE 107.—Explanatory diagrams for grid coordinate resection.

the azimuths from it to the known stations. Use form 19 and attach the graph sheet.

126. Computation of three-point problem.—*a. Construction.*—In figure 107, P is the observer's position which is to be determined. A, B, and C are the three known points from left to right, with A on the observer's extreme left. The angles APB and BPC are called α and β, respectively. The circle in both figures passes through the points, A, P, and C. BP, produced if necessary, cuts the circle at I.

In case 1, the angle $ACI=180-\alpha$ and $CAI=180-\beta$.
In case 2, the angle $ACI=\alpha$ and $CAI=\beta$.

b. The computation for an example of case 2 is carried out on Form 20, "Grid coordinate computation of three-point problem" (fig. 108), in the following steps:

(1) Abstract all data, carefully construct the diagram, correct the Y coordinates for magnification of scale, and calculate the six angles at the head of the form.

(2) Compute the azimuth and length of AC from the coordinates of A and C, deducing the azimuth of AI in block ①.

(3) In block ② find the length AI by solving the triangle ACI, and compute the coordinates of I from the azimuth and distance.

(4) From the coordinates of B and I, compute the azimuth of BI in block ③ also deducing the angles of triangle AIP, with due reference to the diagram.

(5) In block ④ solve the triangle AIP for IP and compute the coordinates of P from I.

(6) Block ⑤ is a check on the computation. The azimuth PC is computed from the coordinates of P and C, the angle β is deduced from the azimuths PC and PB and compared with the observed β.

(7) In block ⑥ compute the azimuth to a fourth known station, if observed, deduce the angle BPD from the azimuths of its sides, and compare with the observed γ.

(8) Finally apply the magnification of scale to the Y coordinate of P to obtain the correct standard grid coordinates.

c. The instructions which should appear on the form are given below.

Final coordinates of P(▽*Gravel*)X=*1366791.3* Map Y=*1795911.4*

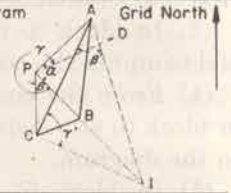
Pt.	Stations	X	True Y	Map Y	Directions
A	△*Temple*	1373 570 0	1802 501 0	~~1802 522 3~~	00 00 00
B	△*Cupola*	1372 166 9	1790 347 2	~~1790 350 2~~	90 09 58
C	△*Water Tower*	1366 990 2	1788 325 7	~~1788 325 7~~	132 44 02
D	△*Beacon*	1372 135 2	1798 231 3	~~1798 246 1~~	20 40 19
P	▽*Gravel*	MnS *1.5* yds. per 1000. Base Sta. △*Water Tower*			

α	90	09	58
β	42	34	04
γ	290	30	21
α'	90	09	58
β'	42	34	04
I	47	15	58

① Length AC Azimuths AC, AI, IA	
CX	1366 990 2
AX	1373 570 0
δX	6 579 8
CY	1788 325 7
AY	1802 501 0
δY	14 175 3
log δX	3 818 22
log δY −	4 151 53
log tan AC	9 666 69
AC *	204 54 00
CAI (β') ± *	− 42 34 04
AI *	162 19 56
IA	342 19 56
log δX	3 818 22
log sin AC −	9 624 32
log AC	4 193 90
log δY	4 151 53
log cos AC −	9 957 63
log AC	4 193 90
adopted log AC	4 193 90

③ Azimuths IP, PI	
IX ②	1380 027 4
BX	1372 166 9
δX	7 860 5
IY ②	1782 227 8
BY	1790 347 2
δY	8 119 4
log δX	3 895 45
log δY −	3 909 52
log tan BI	9 985 93
BI *	135 55 41
PI *	135 55 41
IP *	315 55 41
PB *	135 55 41
IP	315 55 41
IA ①	342 19 56
AIP (diff.)	26 24 15
API (α)	90 09 58
Sum	116 34 13
deduct from	180 00 00
PAI	63 25 47

Diagram



⑤ Check on computations (from point C)	
CX	1366 990 2
PX ④	1366 791 3
δX	198 9
CY	1788 325 7
PY ④	1795 900 0
δY	7 574 3
log δX	2 298 63
log δY −	3 879 35
log tan PC	8 419 28
PC *	178 29 45
PB ③	135 55 41
computed β (diff.)	42 34 04
observed β	42 34 04

② Coordinates of I	
log AC ①	4 193 90
log sin ACI (α)	0 000 00
log cosec AIC (1)	0 134 00
1) log AI (sum)	4 327 90
2) log sin AI	9 482 16
3) log cos AI	9 979 02
log δX (1+2)	3 810 06
log δY (1+3)	4 306 92
δX ± *	+ 6 457 4
AX	1373 570 0
IX (sum)	1380 027 4
δY ± *	− 20 273 2
AY	1802 501 0
IY (sum)	1782 227 8

④ Coordinates of P	
log AI ②	4 327 90
log sin PAI ③	9 951 53
log cosec API(α)	0 000 00
1) log IP (sum)	4 279 43
2) log sin IP ③	9 842 33
3) log cos IP ③	9 856 41
log δX (1+2)	4 121 76
log δY (1+3)	4 135 84
δX ± *	− 13 236 1
IX ②	1380 027 4
PX (sum)	1366 791 3
δY ± *	+ 13 672 2
IY ②	1782 227 8
PY (sum)	1795 900 0

⑥ Check on data (from point D)	
DX	1372 135 2
PX ④	1366 791 3
δX	5 343 9
DY	1798 231 3
PY ④	1795 900 0
δY	2 331 3
log δX	3 727 86
log δY −	3 367 60
log tan PD	0 360 26
PD *	66 25 50
PB ③	135 55 41
computed Y (diff.)	290 30 09
observed Y	290 30 21

Date: *April 2, 1940* Computed by: *J.Y.B.* Checked by: *B.B.H.* FORM 20

FIGURE 108.—Grid coordinate computation of three-point problem.

GRID COORDINATE COMPUTATION OF THREE-POINT PROBLEM

INSTRUCTIONS

Fill out the data, carefully construct the diagram, correct the Y's for magnification of scale, and deduce the 6 angles before computing.

Letter the known points A, B, and C from left to right with A on the observer's extreme left.

Mark angles APB as α; BPC, β; and BPD, γ.

When P is within triangle ABC, $\alpha'(ACI)=180-\alpha$ and $\beta'(CAI)=180-\beta$.

When P is outside triangle ABC, $\alpha'(ACI)=\alpha$ and $\beta'(CAI)=\beta$.

In either case, $I(AIC)=180-(\alpha'+\beta')$.

*Obtain quadrant or sign by inspection of diagram.

Apply MnS to the Y of P to obtain map grid coordinates.

SECTION XXII

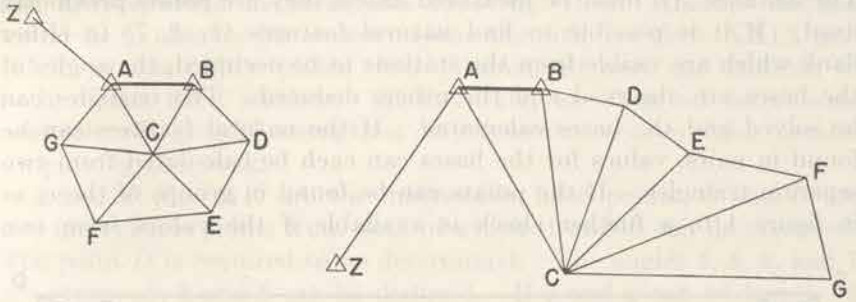
SPECIAL TRIANGULATION METHODS

	Paragraph
General	127
Single point circuit and single point net	128
Rapid triangulation	129
Inaccessible base	130

127. General.—In this section several methods adapted for obtaining quick results in grid triangulation are briefly considered. These are suggested as problems to work out in peacetime and on maneuvers, with a view of preparing to meet any emergency which might arise in war. The chief difficulties which justify such expedients are shortness of time and uncertainty as to when and in what direction the next move will be made. The field work in any grid triangulation demands the use of small and readily portable instruments, preferably of the Wild micrometer direction type since they yield excellent results so promptly. Any reconnaissance must be made in minimum time, except when the advance may be retarded. The methods of this section take further liberties in that any bases necessary would have to be measured rapidly and practically at a moment's notice; that only two stations are occupied in most of the triangles; and that natural marks are selected to serve as signals for stations owing to the frequent impossibility of making a previous reconnaissance and erecting artificial targets. On occasion some rather sharp angles will occur in the triangles, perhaps as small as 15° or even 10°. It is said that distant points can often be fixed within 100 yards by triangles of which the apexes do not exceed 4° or 5°. This weakness of figures and the failure to occupy all stations make it desirable to determine each point from more than one triangle. In most of this rapid work, observations should always be made to or from the highest point of a hill, or at least the highest ground in the neighborhood.

128. Single point circuit and single point net.—These methods may prove useful when a number of points are to be fixed in low-lying country from all of which some prominent feature is visible. The two most usual types are shown in figure 109① and ②, in both of which A and B are two known points or the distance between them is measured. Z is some point visible from A to which the azimuth is known or can be calculated. C is some prominent object visible from A and B and from points D, E, F, and G which are to be fixed. The angles at the bases of the triangles ABC, CAD, CDE, etc., are observed and the apex angles are deduced. The triangles are solved in order commencing with ABC to find the log sides BD, DE, etc. The ad-

justment and solution of the triangles of a similar figure were described in paragraph 119*b* (fig. 88). After the solution of the triangles the coordinates are computed as for a traverse, deducing the azimuths



① Circuit. ② Net.

FIGURE 109.—Single point circuit and net.

from the line *AZ*. In the single point net (fig. 109②) the base of the last triangle should close on known points or the distance should be measured as a check.

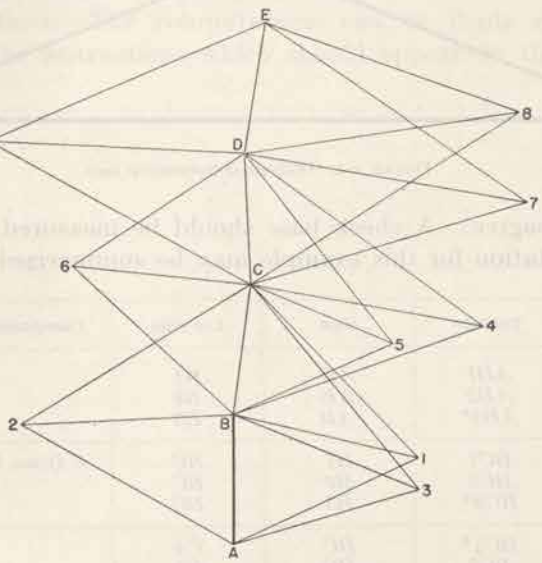


FIGURE 110.—Rapid triangulation.

129. Rapid triangulation.—Cases may arise where it is necessary to carry forward a triangulation with an advance. It may also happen that it is inconvenient to send men to the flanks, even to erect signals. In such case much greater use may be made of natural

features and checks must be introduced by seeing that bases can be calculated from two or three triangles (fig. 110). It is here assumed that observations can be made only from the points *A*, *B*, *C*, *D*, and *E*. The distance *AB* must be measured unless they are points previously fixed. If it is possible to find natural features (*1*, *4*, *7*) to either flank which are visible from the stations to be occupied, the angles at the bases are observed and the others deduced. The triangles can be solved and the bases calculated. If the natural features can be found in pairs, values for the bases can each be calculated from two separate triangles. If the points can be found in groups of three, as in figure 110, a further check is available if the values from two

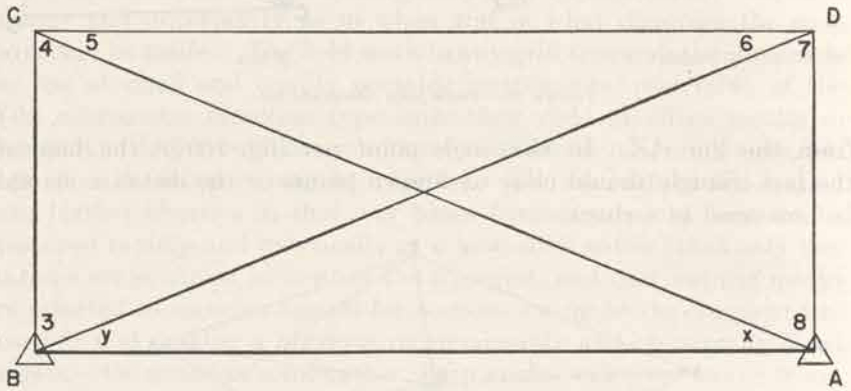


FIGURE 111.—Diagram of inaccessible base.

triangles disagree. A check base should be measured, if necessary. The computation for this example may be summarized as follows:

Triangle	Base	Log sides	Coordinates
AB1	*AB*	*B1*	
AB2	*AB*	*B2*	
*AB3**	*AB*	*B3*	
BC1	*B1*	*BC*	*C* from *B*
BC2	*B2*	*BC*	
*BC3**	*B3*	*BC*	
*BC4**	*BC*	*C4*	
BC5	*BC*	*C5*	
BC6	*BC*	*C6*	

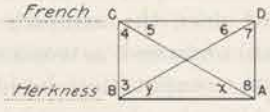
and so on.

*These triangles would be computed when two values of the same line disagree. The number of observations may be considerably reduced if the same natural features are visible to more than three of the observing stations

130. Inaccessible base.—*a. General.*—This is a method of fixing one or two points by observations taken from them when only two known stations are visible from them. Graphic solutions of this problem are explained in paragraph 124*e* (fig. 103). Another method would be to observe for an astronomic azimuth (see Secs. XXIX and XXX) at one of the points thus obtaining at once the azimuth to both of the fixed stations. At any rate, an observed astronomic azimuth will greatly strengthen any resection computation besides simplifying the work.

A and *B* (fig. 111) are the inaccessible fixed points, *C* and *D* are two intervisible points from both of which *A* and *B* can be observed. The point *D* is required to be determined. The angles 4, 5, 6, and 7 are observed; 3 and 8 can be deduced. If x and y can be found, all the angles in triangles *ABC* and *ABD* will be known, and these triangles can be solved, the azimuths of *AC*, *AD*, *BC*, and *BD* can be deduced, and the coordinates of *D*, and *C* if desired, can be computed from *A* and *B*. Either of these methods might prove invaluable if one of three signals erected for resection from the advanced positions should prove invisible. See paragraph 98*o*(7).

b. Computations.—The computations can be made on form 21 (fig. 112). The instructions which should appear on the form are given below.



Azimuth AB =	302°	08′	13″			Angle	log sines			Angle	log sines				
Log AB =	4.31742				Map Y	4	9	974	19	3	9	911	61		
AX	1383	190	4	Y	1778	099	7	—	6	9	660	27	5	9	666	83
BX	1365	603	9	Y	1789	147	6	764.2	8	9	703	57	7	9	998	50
Base Station	Proctor	MnS	15	yds. per	1000	Sum	9	338	03	Sum	9	576	94			
											-9	338	03			
	(4)	70°	26′	28″							0	238	91			
	(5)	27	40	03		tan Φ										
C=(4)+(5)		98	06	31		Φ = 60° 01′ 14″										
	(6)	27	13	06		Φ−45° = 15° 01′ 14″										
	(7)	94	45	41						log tan (Φ−45°)				5	428	68
D=(6)+(7)		121	58	47						log tan $\frac{x+y}{2}$				9	715	42
	(5)	27	40	03		(4+6+8) greater				(3+5+7) greater						
	(6)	27	13	06		log tan $\frac{x-y}{2}$ (sum)				log tan $\frac{y-x}{2}$ (sum)				9	144	10
x+y=(5)+(6)		54	53	09		$\frac{x-y}{2}$				$\frac{y-x}{2}$				7°	55′	59″
$\frac{x+y}{2}$		27	26	34		$\frac{x+y}{2}$				$\frac{x+y}{2}$				27	26	34
(8)=180−(5+D)		30	21	10		x (sum)				x (diff.)				19	30	35
(3)=180−(6+C)		54	40	23		y (diff.)				y (sum)				35	22	33

Letter	Diagram	Angles			log sines			log sides			Side
A	x	19°	30′	35″	9	523	70	3	866	93	BC
B	y+(3)	90	02	57	0	000	00	4	343	23	AC
C	(4)	70	26	28	9	974	19	4	317	42	AB
					4	343	23				
B	(3)	54	40	23	9	911	61	4	118	27	CD
C	C	98	06	31	9	995	64	4	202	30	BD
D	(6)	27	13	06	9	660	27	3	866	93	BC
					4	206	66				
C	(5)	27	40	03	9	666	83	4	081	53	AD
D	D	121	58	47	9	928	52	4	343	22	AC
A	(8)	30	21	10	9	703	57	4	118	27	CD
					4	414	70				
D	(7)	94	45	42	9	998	50	4	317	40	AB
A	x+(8)	49	51	45	9	883	38	4	202	28	BD
B	y	35	22	33	9	762	63	4	081	53	AD
					4	318	90				

Azimuth AB	302°	08′	13″	Azimuth BA	122°	08′	13″
x +	19	30	35	y −	35	22	33
Azimuth AC	321	38	48	Azimuth BD	86	45	40
(8) +	30	21	10	(3) −	54	40	23
Azimuth AD	351	59	58	Azimuth BC	32	05	17

Date May 4, 1940 Computed by: J.H.B. Checked by: J.B.H. FORM 21

FIGURE 112.—Computation of inaccessible base.

COMPUTATION OF INACCESSIBLE BASE FORMULAS

$$x+y=(5)+(6)$$

$$\frac{\sin x}{\sin y}=\frac{\sin (4)\sin (6)\sin (8)}{\sin (3)\sin (5)\sin (7)}=\tan \phi$$

If above numerator is greater than the denominator

$$\tan \frac{x-y}{2}=\tan \frac{x+y}{2}\tan (\phi-45°)$$

If the denominator is greater

$$\tan \frac{y-x}{2}=\tan \frac{x+y}{2}\tan (\phi-45°)$$

Make *MnS* correction to *Y*'s after abstracting data.

c. Checks.—The computation provides only internal checks, and the agreement of the coordinates of C and D, as computed from A and B, only proves the correctness of the computation, and is no check on the accuracy of the observed angles. Observations might also have been made at a second point C' near C, and this point included in the observations taken at D. Two computations could then be made, one with C and D observations, the other with C' and D. Agreement of the coordinates of D in both computations is an external check.

d. Exercise XXV. The inaccessible base.—The student will observe the angles from two assigned stations to an inaccessible base and perform all the computations on Form 21 and others necessary to check the coordinates of both the assigned points.

Section XXIII

MISCELLANEOUS ORIENTATION METHODS

	Paragraph
General	131
Short base	132
Subtense methods	133
Subtense variations	134
Coincidence method of obtaining distances	135
Interruption of triangulation	136
Assumption of coordinates and azimuth	137

131. General.—With the exception of the subtense bar, everything in this section may be regarded as of fairly low accuracy, until proved otherwise by trials. Most of the methods described here are something to be tested now and then and reserved for some unusual emergency. The short base methods may be better than rated and the special problem described in paragraph 132*b* is likely to occur.

132. Short base.—*a. General.*—(1) These methods assume that it is possible to extend one triangle to 5 or 6 times its base (fig. 113). Starting with a 250-foot base, the long side of the first triangle may be about 1,200 feet; of the second, about 7,000 feet. This involves apex angles around 10°, but the expansion is very rapid. (For the best methods, see (5) and *b* below.) Some armies have actually worked out detailed plans for such problems, showing just what each of the four observers does at each stage, and where the computing section is stationed.

(2) The position and elevation of the apex of first triangle are assumed.

(3) When coordinates have to be assumed by the artillery, the special short base method described below may be useful in aiding

them to utilize the temporary grid until the triangulation catches up. This method requires very close liaison. Suppose (fig. 114) that the Field Artillery have assumed an origin (X), which may be too far forward to be occupied by the topographical unit, owing to the number

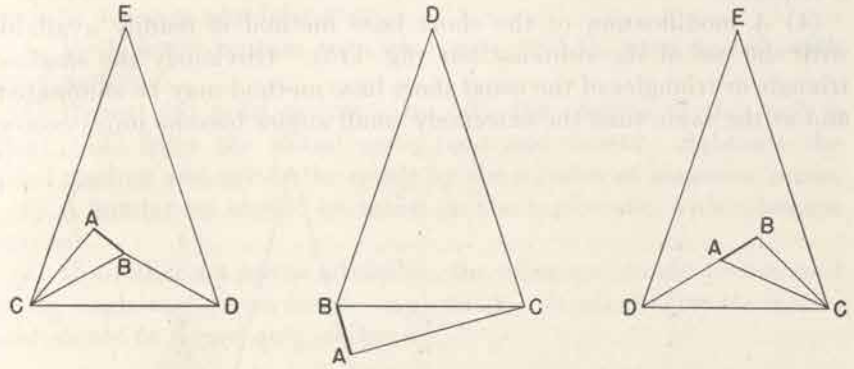


FIGURE 113.—Diagrams of short base methods.

of other men required to work there and its probable proximity to the enemy. The topographical unit's most forward station C will be selected, normally near X and visible from a short base AB, the distance CX being measured. C will often be on the reverse slope of a

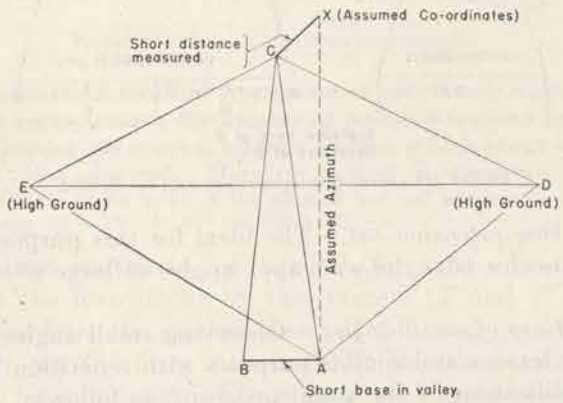


FIGURE 114.—Extension from origin or assumed coordinates.

hill to assure liaison with the artillery, with reasonable immunity from fire. AB should be close to a road, when practicable, for easy access by the computation truck. The coordinates of X and the azimuth from X to A or B having been assumed, coordinates of A and B are deduced in the same terms and the base AB is extended in the usual manner, permitting the intersection of points in the battle area in

the same terms (assumed coordinates) as the artillery, and extension forward if necessary. If the triangle ACX can be made well conditioned, it can be computed, checking by measurement of the distance CX. Other field artillery points may be included in the triangulation if required.

(4) A modification of the short base method is readily available with the use of the subtense bar (fig. 115). Obviously the smallest triangle or triangles of the usual short base method may be eliminated, and at the same time the extremely small angles become unnecessary.

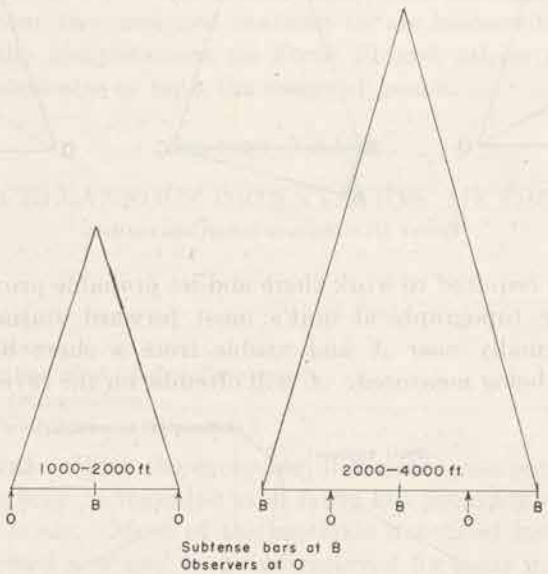


FIGURE 115.—Short base measured by subtense method.

(5) *Short base extension net.*—The ideal for this purpose is a series of similar isosceles triangles with apex angles as large as conveniently possible.

b. Observations of small angles.—Observing small angles suitable for short base extension and similar purposes with repetition instruments requires modification of the usual program, as follows:

(1) Set the circle at the required initial zero for the set, but do not record.

(2) Center on left hand target (A) with the lower motion. Unclamp the upper motion, and rotate the instrument *clockwise* several times to take up backlash, and again center on (A) with the upper motion, read, and record.

(3) With the upper motion, point at target (*B*), read, record, and secure the value for the first angle as a check against gross errors. It is not necessary to record again until the required number of repetitions have been completed, but a reading should be made after the sixth repetition and the mean compared with the first angle of a check to discover any false start.

(4) With lower motion turn clockwise to (*A*), then to (*B*) with upper motion.

(5) Repeat (4) until, pointing on (*B*), the reading of the limb is about 180° from the initial zero; read and record. Subtract the initial reading and divide the result by the number of measures made.

(6) A similar set should be taken on the explement, with telescope reversed.

(7) If another set seems advisable, the telescope should be reversed for the angle and direct for the explement. *In all of above the instrument should be turned only clockwise.*

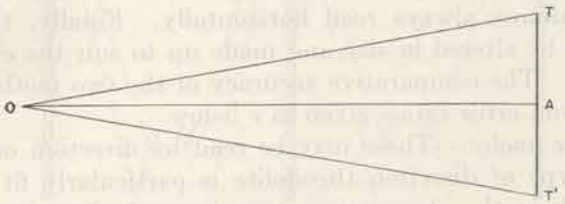


FIGURE 116.—Diagram of subtense measurement.

NOTE.—A theodolite of the Wild type cannot be used for the repetition method. For such small angles observe the number of positions required to cover about half the circle, making the interval between position initials about equal to twice the small angle being measured. Moreover, it must be remembered that all this extra trouble will be of no avail if the signals are not accurately centered over the station marks.

133. Subtense methods.—*a. General* (fig. 115).—From the angle subtended at the instrument by two targets (*T* and *T'*) at known equal distances from the station observed (*A*), and on a line perpendicular to the ray *OA* from instrument to the observed station, the distance can be calculated. The accuracy of the distance depends upon the exactness of the measurements of the subtended angle and of the distance between the targets, and partly upon the relative magnitude of the subtended angle, since the distance is calculated from the formula

$$D=\frac{d}{2}\cot\frac{a}{2}$$

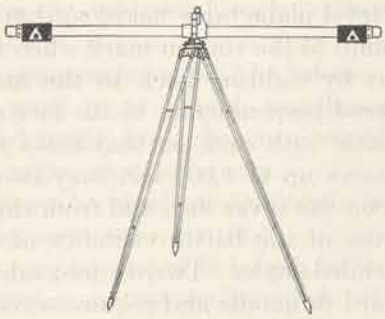
where d is the distance between the targets and a is the subtended angle. No correction for slope is necessary. Table V, TM 5–236 is convenient for the calculation though the instrument makers usually furnish tables with each bar ($d(1)$ below).

b. Differences from the stadia method.—Stadia distances are read as the intercept between two fixed stadia hairs on a specially marked stadia rod as viewed at the distant station through the transit or alidade telescope. In ordinary circumstances the length of a stadia sight is limited to 1,000 feet or less by the visibility of the divisions on the rod which are usually 0.05 or 0.10 foot each. The size and separation of the subtense targets may be varied to suit the conditions so that sights of several miles may be taken. The stadia rod is hand held in a vertical position supported on the ground with some unsteadiness and deviation from the plumb position and variable errors due to differential refraction, which may be considerable close to the ground, while the subtense targets are rigidly supported approximately at the same height above the ground as the *H. I.* Subtense angles are almost always read horizontally. Finally, the subtense targets may be altered in size and made up to suit the conditions of observation. The comparative accuracy of the two methods may be estimated from error ratios given in *e* below.

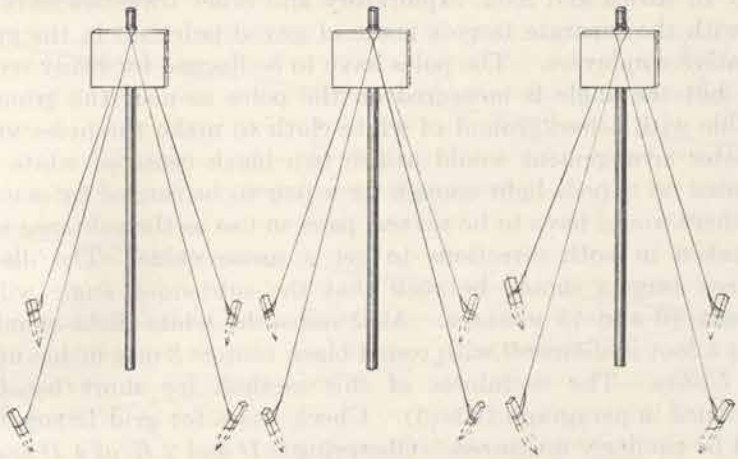
c. Subtense angles.—These may be read by direction or repetition. The Wild type of direction theodolite is particularly fitted for this class of work as the observations may be made in a fraction of the time required with other types of instrument. With a repeating theodolite at least four sets of six repetitions should be made at short distances and as many as ten such sets at the longer distances. The alternate sets should be made direct and reversed and a progressive change of zeros should be employed to overcome uneven division of the circle. If the angle is small enough to permit, say between 10 and 15 minutes, some observers prefer to leave both clamps set, turning back and forth with the upper and lower tangent screws.

d. Target types.—These include the subtense bar and two extra signals set up on either side of the station.

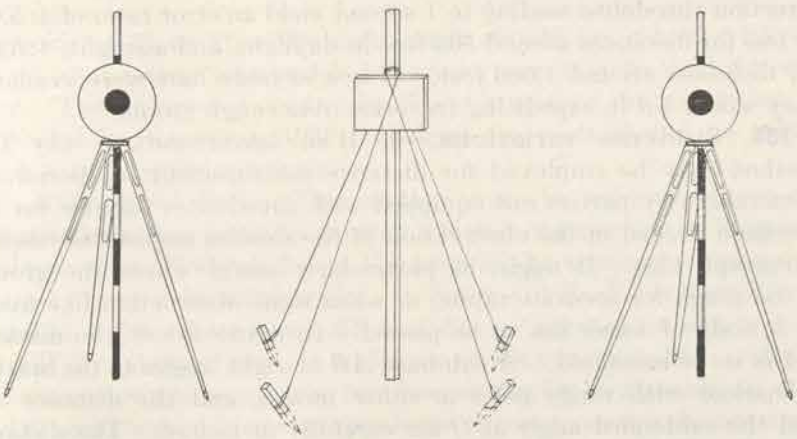
(1) As an accessory (fig. 117①), many of the European theodolites are equipped with invar steel bars about 5 feet long, with targets, universal levels, small sighting telescopes for insuring normality of the bar to the line, plumb bobs, and fittings for attachment to an extra tripod. Some of these bars have lights for night use. A subtense bar can be improvised from a level rod by putting on an extra target, securing to each target a 4-inch white disk with a solid black 1-inch disk at its center, setting the rod to exactly 10 feet, and securing the



① Wild type bar.



② African type targets.



③ Tripod targets.

FIGURE 117.—Subtense targets.

rod to the top of a level plane table board so that the line of the front of the disks will plumb to the station mark when the rod is set normal to the line of sight by sighting back to the instrument through a sighting alidade placed perpendicular to the face of the rod. If these preparations are made with care, so that disks are centered exactly 10 feet apart, distances up to 1,000 feet may be read in the daytime about as closely as on the invar bar, and from there up, they may be a little closer because of the better visibility of the targets and the increase in the subtended angle. Twenty-foot subtense bars have been used but are awkward to handle and require several supports, tripods or trestles.

(2) In Africa and Asia, exploratory and other traverses have been run with the separate targets made of guyed poles set in the ground by native employees. The poles have to be flagged for ready recognition, but the angle is measured on the poles as near the ground as possible with a background of white cloth to make the poles visible. A better arrangement would utilize two black centered white disks mounted on tripods light enough for a pair to be carried by one man, and there would have to be several pairs in use as the subtense angles are taken in both directions to get a mean value. The distance between targets should be such that the subtended angle will fall between 10 and 15 minutes. At 2 miles the white disks should be about a foot in diameter with round black centers 3 or 4 inches across.

e. Utility.—The usefulness of this method for short bases was mentioned in paragraph 132*a*(3). Check bases for grid triangulation could be similarly measured. Observing 2 D and 2 R, or 4 D and 4 R angles with a different zero for each angle, the Wild type bar and direction theodolite reading to 1 second yield an error ratio of 1:5,000 or less for distances around 500 feet in daylight, and at night, 1:5,000 for distances around 1,000 feet. If two or more bars were available, they would aid in expediting traverses over rough ground.

134. Subtense variations.—*a. With lateral station.*—(1) This method may be employed for distance measurement in special circumstances by parties not equipped with theodolites suitable for the precision needed in the observation of the smaller angles described in paragraph 133*e*. It might be particularly useful where the ground is too rough for accurate taping or when some obstruction like woods or a body of water has to be passed. In figure 118 ① the distance OA is to be measured. A sub-base AB at right angles to the line OA is marked with range poles or other means, and the distance AB and the subtended angle at O are carefully measured. The distance OA is then calculated by the formula $D = d \cot a$, where d is the

distance AB and a is the subtended angle. No correction for slope is needed.

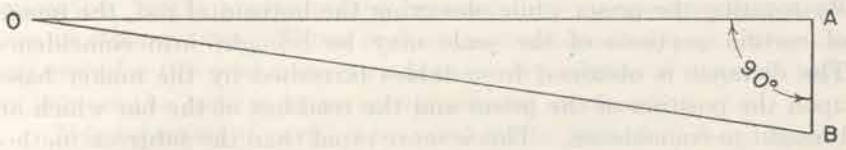
(2) The following precautions should be taken:

(*a*) The sub-base must be of suitable length and accurately taped.

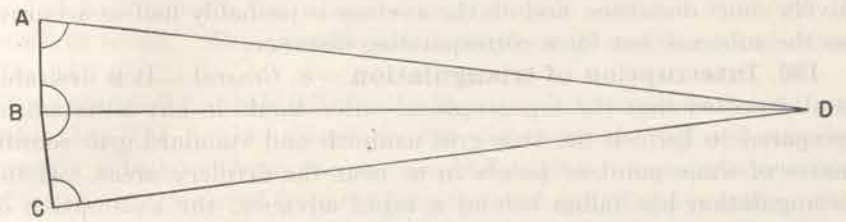
(*b*) The subtended angle must be very exactly observed, as the cotangents of small angles change rapidly.

(*c*) The distance must be checked, either by a second sub-base of a different length or by some other method.

(3) The sub-base may be varied in length to fit the ground, though excessively small angles are to be avoided. The base might be roughly $\frac{1}{20}$ or $\frac{1}{10}$ the distance OA, which would make the subtended



① Distance by lateral station method.



② Distance by lateral bases.

FIGURE 118.—Subtense variations.

angle about 3° to 5°. The right angle should be laid off with the transit, but a steel tape and 3–4–5 proportions may be used if necessary. The subtended angle should be measured by repetition as described in paragraph 107*c*. At least two sets should be taken, more with a 1-minute transit.

b. With lateral bases.—This method may be used to obtain the distance from one point to another which is undesirable or impracticable to visit. Two angles at the base of the triangle are observed instead of one subtended angle. In figure 118 ②, BD is the distance required. Two short bases AB and BC are marked on one or both sides of B, and about at right angles, or a little less, from BD. AB and BC are accurately taped; the two interior angles at B, one each at A and C are observed in enough positions or sets to secure the necessary accuracy, and the two apex angles at D are deduced. The distance is computed by the law of sines from each base separately. Observa-

tions are repeated, if necessary, until the discrepancy between the two results is so small as to indicate that the desired accuracy has been attained. A sharp distinct point should be selected for the point *D*, the station marks should be small nails set vertically in the top of the stakes, and the instrument must be exactly centered over each of the marks.

135. Coincidence method of obtaining distances.—Some of the European instruments are equipped with a special sleeve containing two prisms which can be attached at the objective end of the telescope, and a specially marked bar or rod which is fitted to be supported horizontally on an extra tripod. One of the prisms may be rotated, and a scale indicates its position relative to the other, which is fixed. By rotating the prism while observing the horizontal rod, the images of certain portions of the scale may be brought into coincidence. The distance is obtained from tables furnished by the maker based upon the position of the prism and the readings of the bar which are brought to coincidence. This is more rapid than the subtense method but requires more equipment, is less adaptable, is limited to comparatively short distances, and on the average is probably half as accurate as the subtense bar for a corresponding distance.

136. Interruption of triangulation.—*a. General.*—It is desirable and expected that the topographical units should in any situation be prepared to furnish the true grid azimuth and standard grid coordinates of some point or points in or near the artillery areas. If the triangulation has fallen behind a rapid advance, the assumption of grid coordinates (par. 137) might become necessary. If that expedient cannot be avoided, the resultant harm should be minimized, if at all possible, by approximating the true grid azimuth and coordinates so closely that later corrections are unnecessary. Methods which might be employed either individually or in combination to meet such requirements include—

(1) Approximations from the best available map.

(2) Astronomic azimuth observations, which need not take long, weather permitting.

(3) Resection on three or more points previously fixed or approximately fixed.

(4) Resection on two points previously fixed.

(5) If only one known point is visible, the occupation of two others in the advanced position at which the two angles of the triangle and an azimuth at one or both the new stations are observed, and the distance between measured, directly or indirectly.

(6) An azimuth traverse (see *b* below).

(7) A determination of position from azimuth and latitude (*c* below).

b. The *azimuth traverse* is one run primarily to carry forward the true grid azimuth in case the weather prevents observations. If suitable landmarks are available, this may be accomplished by long shots ahead on distinctly marked details of the terrain which may then be occupied, leaving cloth marks for the backsight if the object to which the foresight was made will not serve. The landmarks might be supplemented by range poles sent ahead. If the stations have to be close together, two instruments might be used, one to be set up and the angle observed while the other is being advanced the length of two sights. Extra rodmen should be provided. The distances should at least be estimated, better approximated by range pole intercepts, rope traverse, or other means. Three or four men alternating in marking the rear end of a 100-yard rope can advance very rapidly if they can keep on line, as the rope is being moved almost constantly.

c. Determination of position by azimuth and latitude is feasible only when astronomic observations can be made for azimuth and latitude, and if a previously fixed landscape feature is visible within 45° of north or south. The computation is simple on geographic coordinates (fig. 119 ①).

The distance OA equals OB divided by cos α, OB being calculated from table XI, TM 5-236. Similarly the difference in longitude may be roughly calculated from the relation $AB=OB$ tan α and the seconds of longitude calculated from the longitude column of above table. On the grid (fig. 119 ②) the coordinates of C could be converted to geographic (par. 119*g*), the distance OC calculated as above for OA, and the coordinates of O computed from the azimuth and distance OC on form 14. Ultra refinement in these calculations is not warranted, as the error of the observed latitude may be ±50 yards.

137. Assumption of coordinates and azimuth.—This step is always initiated by the Field Artillery as a means of coordinating the fire of various units, whenever standard grid azimuth and coordinates are not available in their areas of operations. The establishment of such temporary data is tactically undesirable since it diminishes the fire effect of any batteries operating with the data, delays units coming into action, and interferes with continuous operation during the conversion from temporary to standard data. The topographical units are interested in all of this as some of their forward elements in cooperation with the artillery may have started grid triangulation on the temporary grid for the purpose of aiding them in keeping their several units on the same relative control datum, temporary though it may be.

As the topographical units will have to make further use of these points when the grid triangulation comes forward, the conversion from temporary to standard grid data should be accomplished on Form 17 (par. 119*h*), as soon as a common point and the azimuth difference have been established. This not only causes extra work and duplication of effort, but tends to confuse the grid triangulation and the control data. Some of the methods of avoiding these conversions have been outlined

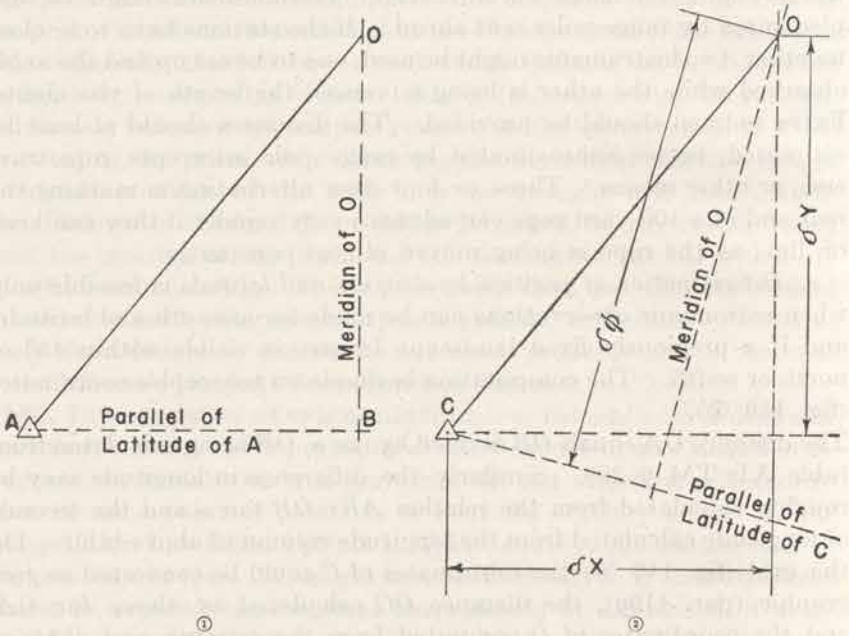


FIGURE 119.—Determination of position from latitude and azimuth.

in paragraph 136. At all times such close liaison should be maintained with the artillery units concerned that their intentions may be known in advance and provision made for their needs, thus avoiding the necessity for assuming temporary data. Before assuming temporary data the artillery are expected to consult with the topographical units, with a view of securing the best possible assumption so as to minimize the later conversion of temporary data wherever practicable.

SECTION XXIV

PLANE TABLE AND TELESCOPIC ALIDADE—ADJUSTMENTS

Paragraph
General description 138
Adjustment of plane table 139
Use of plane table 140

138. General description.—*a. Board.*—The plane table now used in the Army consists of a board 24 by 31 inches, built up of ¾-inch white pine strips glued and cleated at the ends with 2-inch maple strips. At the center of the under side of the board is attached a circular brass plate, which has at its center a circular hole, threaded to take the upper end of the clamping bolt of the tripod movement. Eight clamp screws and sockets are attached to the board to hold the paper in place on the upper face of the board. The board is provided with a flexible case of waterproof canvas.

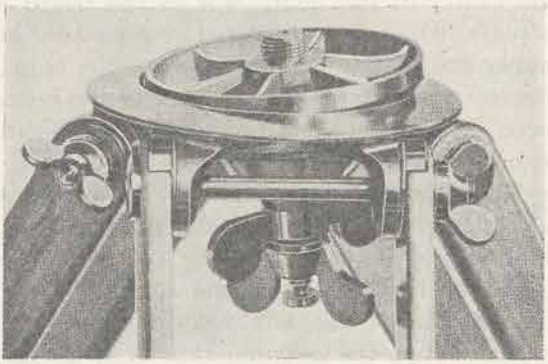
b. Tripod.—The tripod consists of full-length split legs of hardwood, or collapsible legs, with pointed steel shoes, hinged to the Johnson movement ball-and-socket head. (See fig. 120.) This head provides for the motions of leveling and horizontal turning. The upper clamp (*A*) controls the leveling of the table while the lower clamp (*B*) controls the azimuth. When the instrument is to be set up both clamps are loosened. The table is leveled and clamp *A* tightened, which forces the level cup *c* against the tripod head *d*, friction holding it in this position. In practice, the upper clamp is seldom loosened, as the table is more quickly leveled by moving the feet of the tripod. The table is then turned to the required azimuth and the azimuth clamp *B* tightened, which forces the azimuth cup *e* against the tripod head *d*, thus preventing the table from turning in azimuth. The nut *B* acts as a check or lock nut on *A*. The tripod with head weighs 9 pounds complete.

c. Plumbing arm.—The plumbing arm is a device for plumbing a point on the plane table over the station point. The arm is made of oak, shaped so that while the upper member rests on top of the board, the lower member extends at an acute angle below the board. The lower end of the lower member has a brass hook for the suspension of a plumb bob directly under the brass index on the free end of the upper arm.

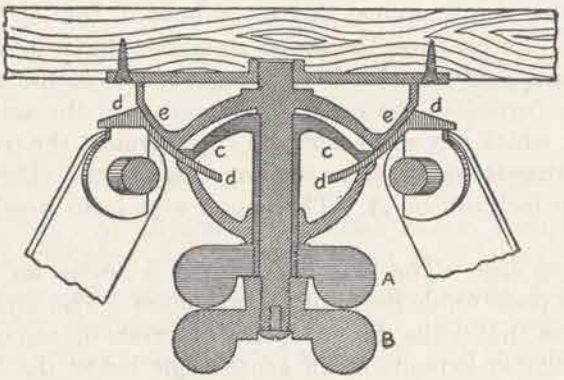
d. Alidade.—The telescopic alidade (fig. 121) with all accessories should be neatly fitted and secured to fixtures on the inside of a hardwood case which has a hinged cover, two brass hooks, and a heavy leather strap with roller buckle. The plumbing arm and plumb bob

are packed in the case or in the plane table case, depending upon the manufacturer. The accessories usually furnished are—

1 extra tangent screw for the stadia arc.
1 extra tangent screw complete with socket for the telescopic axis.
1 extra striding level vial.
1 pocket magnifying glass.
1 suitable screw driver.
1 sunshade.
2 steel adjusting pins.



① View.



② Section.

FIGURE 120.—Johnson plane table head.

e. Use.—The same care and precautions should be taken in the use of the plane table alidade as for the transit and levels. Protection should be provided for the alidade and map in case of sudden showers or windstorms. All parts should be well cleaned at the end of each day's work by being wiped with a soft, dry cloth.

f. Beaman stadia arc, (fig. 122① and ②).—(1) This provides a rapid and exact solution of the stadia problem and facilitates use of the plane table. With it, differences in elevation and reduced horizontal distances can be determined with great rapidity and without intricate calculation. The arc is attached to the vertical limb of the alidade and carries two scales having zero points marked 0 and 50, respectively, either scale being read by an index. The scale graduations of the Beaman arc are so spaced and numbered as to give simple multiples of the rod interval.

(2) To obtain differences in elevation between instrument and rod, the scale marked *V* is used. The index point of this scale is marked 50, so a scale reading of less than 50 indicates that the telescope is depressed, while a reading greater than 50 shows that the telescope is

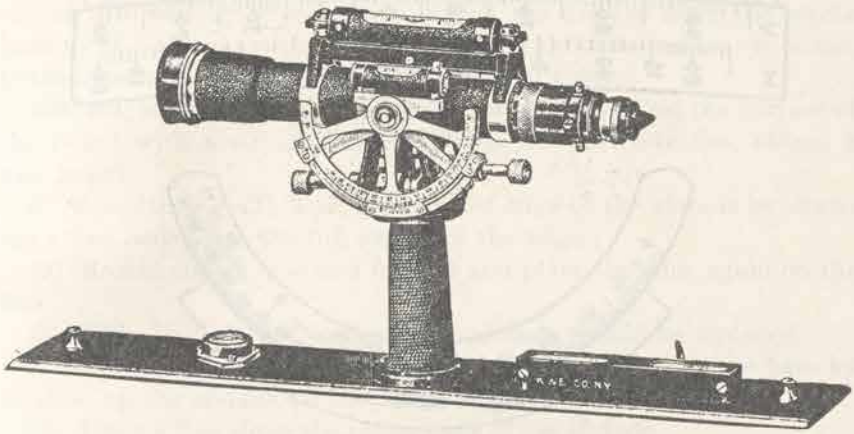


FIGURE 121.—Plane table alidade.

elevated. Only such inclinations of the telescope need be used as will give a whole number reading on the *V* scale, the fractional part of the elevation being shown by the rod reading.

(3) To obtain the desired multiple, sight anywhere on the rod (it does not matter where) so that a whole number reading is obtained on the *V* scale.

(4) For example, suppose the observed subtended stadia reading on the rod to be 6.40 (640 feet), to obtain a whole number for the *V* scale reading, the telescope is inclined so that the *V* scale reads 33, at which setting the middle wire reads 7.30 on the rod. Then the desired multiple equals

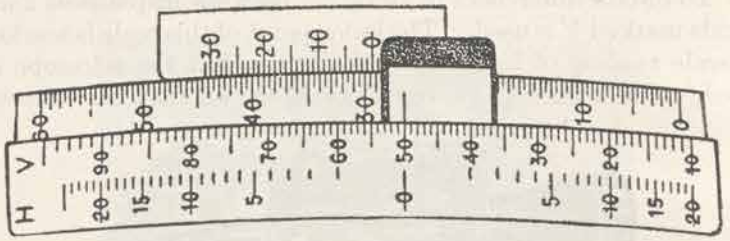
$$33-50=-17 \text{ and}$$
$$-17\times 6.40=-108.8$$

Difference in elevation between instrument and base of rod is then

$$-7.30-108.8=-116.1 \text{ feet}$$

This subtracted from the *H. I.* of the instrument will be the elevation of the new point. For accurate work it is necessary to correct the distance for K, and $(c+f)$ if any, before calculating the difference of elevation.

(5) The horizontal distance is found by means of the scale marked H, which gives at the same pointing which sets the V scale a direct



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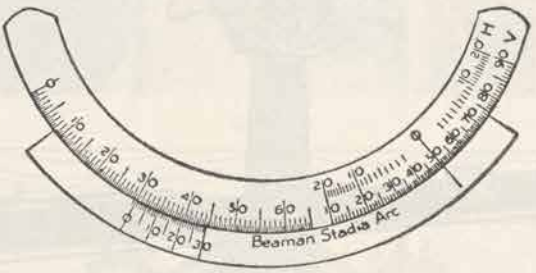

② New style.

FIGURE 122.—Beaman arcs.

reading of the percentage of correction (always negative) necessary to reduce the observed stadia reading to the true horizontal distance.

(6) For example, at the setting for the above V scale reading the scale (H) would read 3, or 3 percent.

3 percent of 640 feet=19.2 feet
640−19.2=621 feet=horizontal distance

When the instrument has a stadia constant affecting the accuracy of the work the rod interval must be corrected for same before recording distance.

(7) The angular values of the divisions of the Beaman arc are as follows:

Division:	Angular value °	′	Division—Continued.	Angular value °	′
1	0	34. 4	15	8	43. 7
2	1	8. 8	20	11	47. 3
5	2	52. 2	25	15	00. 0
10	5	46. 1	30	18	26. 1

(8) Test the accuracy of the graduations of the Beaman arc by comparison with the regular degree arc as follows:

(*a*) Set the degree arc at 30° with main clamp and tangent screw.

(*b*) Set the Beaman arc index at 50 by means of the attached index spring screw.

(*c*) Set the Beaman arc on any graduation it is desired to test, using the main clamp and tangent screw only.

(*d*) Take the reading on the main arc, using 30° as the reference point. The angle, if for an even five division, should correspond with one of the readings given above, which are derived from the stadia formula $H=D$ (constant) $\frac{1}{2}\sin 2A$, H being the difference in elevation, D the observed distance, and A the vertical angle.

139. Adjustment of plane table.—*a. Board.*—Test the surface of the board with a straightedge. If it is not perfectly flat, obtain a new board.

b. Straightedge.—(1) Test the beveled edge of the alidade by drawing a fine pencil line the full length of the edge.

(2) Rotate the alidade end for end and place the edge again on the line.

(3) If the lines do not coincide, the alidade should be replaced.

c. Blade level.—(1) Test the spherical level attached to the base by leveling up the alidade on the plane table.

(2) Draw a line along the ruling edge of the alidade.

(3) Rotate the alidade by turning it end for end and placing it on the same line.

(4) If the level is out of adjustment, correct by bringing the bubble halfway back to its center position by means of the screws at the base of the housing.

d. Striding level.—(1) Place the striding level on the rings of the telescope and bring the bubble to the middle of the tube by means of the vertical tangent screw.

(2) Reverse the level end for end and place it again on the rings.

(3) If the bubble moves from the center position, bring it halfway back by means of the adjusting screw under one end.

(4) Repeat until the bubble remains in the center of the tube. (Check striding level adjustment before start of each day's work.)

e. Arc index level.—(1) Place alidade on a horizontal surface (the plane table board may be used) and level alidade by the striding level.

(2) Set Beaman arc index at 50.

(3) If the bubble of the arc index level is not in the center, move it there by means of the adjusting screw at one end.

(4) Reverse the instrument (180°) and retest the adjustment, repeating same until the bubble remains in center.

f. Vertical wire.—To make the vertical wire vertical when the base is leveled—

(1) Level the plane table carefully by means of the striding level.

(2) Sight on a plumb line or rock the telescope on its horizontal axis while sighting on a well defined point.

(3) Loosen the reticle screws and rotate the reticle to correct error.

(4) Tighten the reticle screws and retest.

g. Line of sight.—To make the line of sight parallel to the axis of the rings—

(1) Loosen the knurled collar in front of the longitudinal telescope bearing.

(2) Focus the telescope on a distant point and revolve the telescope 180° in its collar.

(3) If the cross wires do not remain on the point, correct by moving the reticle one-half of the distance back to the point.

(4) Retest the vertical wire.

h. Beaman arc.—Proceed as described in paragraph 138*f* (8).

i. Determination of stadia constant (K) *and* ($c+f$).—The stadia constant and ($c+f$), if any, for the telescopic alidade of a plane table is determined as described for the transit in paragraph 70.

j. Exercise XXVI.—Make the adjustments to a plane table as described in this paragraph.

140. Use of plane table.—The plane table is used mainly in triangulation reconnaissance and certain mapping operations. The map or drawing paper is fastened to the top of the table, which is set horizontally. The telescopic alidade, resting on the table can be moved thereon from point to point. When the telescope is pointed at an object, a pencil line along the ruling edge of the alidade gives the direction to the object; the distance is then measured by tape or stadia. Since the plotting and the field work are carried on simultaneously, the instrument man can sketch in the ground forms as he goes. Rarely, if ever, will a topographer or surveyor be required to make a *complete* topographic survey with the plane table to a scale smaller than 1:10,000. Only when aerial photographs are unavailable

or when parts of photographs are so indistinct as to be useless for interpretation will the topographic detail to a smaller scale be plotted in the field. Plane table surveys are normally limited to large scale maps, or plats required for construction work.

SECTION XXV

GENERAL INSTRUCTIONS—PLANE TABLE METHODS

	Paragraph
General	141
Equipment for field work	142
Field sheets	143
Setting up and orientation	144
Beaman (stadia) arc record	145
Typical procedure of locating points	146
Application of plane table	147
Supplementary (plane table) control	148
Radiation	149
Traversing	150
Intersection and resection	151
Magnetic orientation	152

141. General.—Plane table surveys provide the advantages of—

a. Dispensing with the usual field notes and hence avoiding mistakes in recording measurements.

b. Visualizing the progress of the map in the field, thus easily preventing the omission of necessary data.

c. Easy location of inaccessible points without trigonometric calculation.

d. Quick determination of the position of any occupied point with respect to three known visible points.

e. Fewer men required.

142. Equipment for field work (fig. 123).—Besides the equipment described in section XXIV, the plane table man needs dividers, scale, No. 10 needles, pencils, eraser, Beaman arc record book, and waterproof cover for board and instrument. The equipment for each rodman should include stadia rod; hatchet, machete, or brush hook, if necessary; colored crayon; and strips of red and white signal cloth.

143. Field sheets.—Field sheets (or plane table sheets) which indicate the data required for control are usually prepared in the office and given to the topographer. A plane table sheet to be used in the field must show at least two stations, intervisible from each other, which have been accurately plotted with at least one elevation. The ideal situation would be to have sufficient control shown on the plane table sheet to enable the topographer merely to occupy one station after another and sketch in the surrounding detail. This, however,

cannot be expected under normal conditions. A cover sheet with an opening large enough to expose the area being worked should be fastened over the field sheet to protect it from hands and alidade.

144. Setting up and orientation.—*a. Setting up plane table.*— Set up the tripod with the board approximately level, two legs being placed downhill if the set-up is made on a slope. Some plane tables are equipped with three leveling screws. This is an advantage in leveling the board but involves additional weight. If the table is so equipped, place the alidade in the center of the table and by means

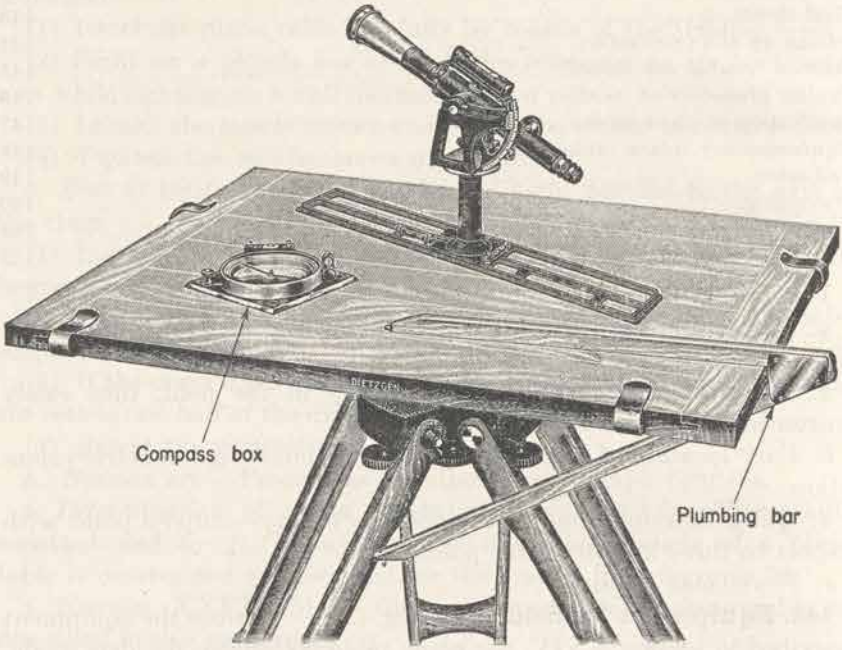

FIGURE 123.—Plane table accessories.

of two of the leveling screws, bring the bubble of the circular level to a line through the center of the level and perpendicular to the line of the two screws. Then, with the third screw, bring the bubble to the center of the level. When the table is equipped with a ball-and-socket joint or Johnson head, the board is moved by hand until the bubble is approximately in the center and the final adjustment made by tapping with the finger; the wing nut is then tightened. Experienced plane-table men usually level the board by adjusting the tripod legs, as the board seldom needs to be precisely level.

b. Orienting plane table.—To orient the table, place the edge of the alidade along the magnetic arrow or meridian, and then turn the table until the north end of the compass needle is at *N*, checking that the magnetism in the needle has not been reversed. If the station occupied and some distant and conspicuous point have been plotted on the map, a more accurate orientation is obtained by placing the alidade edge on these two plotted points, turning the table until the vertical wire of the telescope falls on the distant point, and clamping. This may be called the "backsight" method. Other plotted stations which are in view from the station occupied afford a check upon the position of the instrument. When working on a field scale of 1:20,000 or smaller, the point on the table need not be set exactly over the corresponding point on the ground, as under these circumstances the area of the whole table may be considered as a point without introducing serious error. When working on scales 1:10,000 and larger, the point on the table should be brought closely over its corresponding ground point by dropping small stones from the under side of the board or by use of the plumbing arm and bob; the accuracy of centering required is directly proportional to the scale.

145. Beaman (stadia) arc record.—To minimize any extensive repetition of field work in case of an occasional error in plotting distances or calculating elevations, the topographer should keep a record of such observations that may serve as a check and assist in a ready discovery of accidental errors. The Beaman (stadia arc) record in figure 124 shows notes on a standard page of Form 9–913A, United States Geological Survey. The headings of the various columns make the notes self-explanatory. Paragraph 146*b* explains the method of recording.

146. Typical procedure of locating points.—*a. General.*—The procedure outlined in this paragraph is typical of observations relating in general to radiation (par. 149) and traversing (par. 150), assuming that the plane table is to be oriented by the "backsight" method, the plotted station visible from the location of the plane table being utilized. In regions where local magnetic attraction is not too prevalent, magnetic orientation is employed in all plane table traversing at scales of 1:10,000 and smaller, thereby eliminating half the set-ups and giving the topographer the choice in locating the board on the spot best fitted for sketching the terrain. (See par. 152.)

b. Method.—To obtain the location of new points and their elevations from a known point—

Date: 2/10 1940 Traverse from ⊡A2 to ⊙D7

	Distance	Beaman Arc of		Product	Rod Correction	Diff. Elev.	H.I.	Elevation
1						(3.8)	137.4	133.6
F 2	640	33	(3)	− 108.8	7.3	− 116.1		21.3
3	210	54		+ 8.4	6.3	+ 2.1		139.5
4	190	58		+ 15.2	4.8	+ 10.4		147.8
5	230	40	(1)	− 23.0	5.6	− 28.6		108.8
6	270					− 8.2		129.2
7	185					− 0.9		136.5
8	205	35	(2)	− 30.8	7.1	− 37.9		99.5
9						(3.6)	24.9	21.3
C 10	640	68	(3)	+ 115.2	6.4	+ 108.8		133.7
F 11	490	60	(1)	+ 49.0	4.3	+ 44.7		69.6
12	160	60	(1)	+ 16.0	3.2	+ 12.8		37.7
13	80					− 3.2		21.7
14	110					− 6.3		18.6
15	220	57		+ 15.4	6.2	+ 9.2		34.1
16						(3.2)	72.8	69.6
C 17	490	40	(1)	− 49.0	2.4	− 51.4		21.4
F 18	520	53		+ 15.6	3.2	+ 12.4		85.2
19	130	46		− 5.2	7.1	− 13.3		60.5
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FIGURE 124.—Beaman (stadia arc) record.

(1) Set up on station, level board so that spherical level bubble is *approximately* in center, get *HI*, and compute elevation of instrument. Send rodman to rear station.

(2) Reverse alidade on orientation ticks for line "new station to previous station." Signal rodman.

(3) Sight on rod and clamp table. Send rodman to new station and signal.

(4) Pivot alidade and sight on rod. Signal rodman. Read distance, correct for stadia constant (*K*), if any, and record as "Distance." (See sample notes, fig. 124.) Draw line to new station and a short "tick" at each end of alidade.

(5) Level telescope.

(6) *If middle wire cuts rod*, use telescope as a level. (Record rod reading as "Diff. elev." and subtract from *HI* to obtain elevation of new point.)

(7) *If middle wire does not cut rod*, depress or elevate telescope until middle wire cuts rod, and clamp.

(8) Center arc index level, look at *V* scale, and with vertical slow motion further depress or elevate telescope until it reads exactly one of the full divisions.

(9) Read *V* scale and record as "arc." If *V* scale reads 40 or less, or 60 or more, read *H* scale and record. This *H* scale reading is the *percentage* to be subtracted from the stadia distance (4) to obtain horizontal distance.

(10) Read rod at center hair middle wire and record as "rod correction." Signal rodman "down."

(11) The difference between 50 and the *V* scale reading (9), multiplied by 0.01 of the stadia distance (4) gives the difference of elevation between the instrument and the point on the rod cut by the center hair (10). This difference is recorded (with proper sign) as "product."

(12) If the *V* scale reading (9) is less than 50, the sign of the product is minus, if greater than 50, it is plus. Add, algebraically, product and rod correction and record as "diff. elev." Apply the latter to "*HI*" to obtain elevation of new point, recording each step in proper column.

(13) Plot shot and enter elevation beside it to read from bottom of sheet.

NOTE.—On stations which are part of a plane table traverse read distance and *V* scale both fore and back and take mean of results, as at *C* in figure 124.

147. Application of plane table.—The plane table lends itself readily to the determination of the location of points by the following methods:

a. Radiation (par. 149).
b. Traversing (par. 150).
c. Traversing and radiation.
d. Intersection (par. 151).
e. Resection (par. 151).

Similarly, several methods of plane table leveling may be used for determination of elevations. The choice of methods depends upon the particular problem in hand.

148. Supplementary (plane table) control.—Frequently it is necessary for the topographer to supplement the initial control, plotted on the plane table sheet, with additional points. The amount of such control will depend on the required density of control station, the character of the area to be mapped, and the scale of the plot. All such supplementary control is plotted graphically and may be established by traversing (par. 150) or by graphic triangulation (par. 151). Elevations for supplementary control stations are obtained from stadia distances and Beaman arc readings (sometimes vertical angles) in the case of traversing, and scaled distances plus Beaman arc readings in the case of graphic triangulation. Supplementary control points for filling in topography are occasionally marked by plain stakes without witness or reference marks.

149. Radiation.—To plot points by radiation is identical with what is known as plotting "side shots." This method (fig. 125) is as follows: Set up at *O*, orient, and clamp the board. *O* is usually the adjusted position of some control traverse station or similar control point. Measure the height of the telescope above the ground for the *H.I.* Pivot the alidade with its edge at the point occupied and sight on a rod held at some salient point, as *A*. Read the Beaman arc; then read the distance by the stadia method and lay off to proper scale the distance *Oa*, marking the elevation near the point. Continue this method until all desired points have been located on the sheet. Ground features can be sketched in as the work progresses.

150. Traversing.—It is seldom possible to plot all the desired points within an area from one point; hence the plane table must be moved from one station to another. This method is called traversing. Assuming that it is desired to plot the tract of land *ABCD* (fig. 126) with a ridge along the line *AC*, making *D* invisible from *B*, it is first necessary to set up over the point *A*, which has been assumed on the drawing paper. Draw radial lines from A toward *B* and *D* and make

"ticks" at the ends of the alidade for better reorientation. Then measure the distances to these points. Lay off these distances from A, to scale, on their respective lines, thus locating points b and d. Next, move the table and set it over B. Place the alidade along the plotted line (ticks) bA and sight on A by rotating the table. When the rod, held on A, is bisected, clamp the board. The plane table is now oriented. Draw a radial line from B toward C, measure the distance from B to C, and lay off this distance to scale from b. This gives the position of C. Now move to C and orient the table by placing the alidade along the line (ticks) cB. sighting on B. Draw

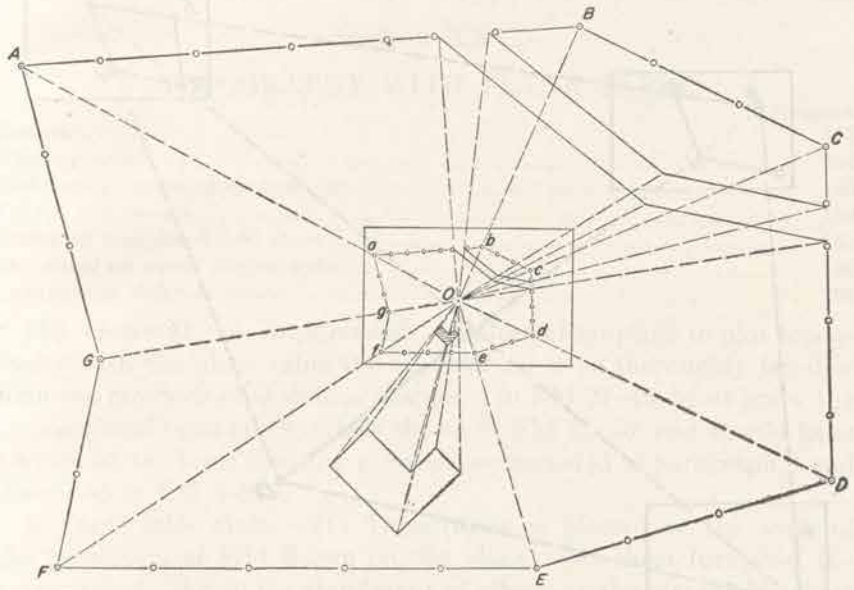

FIGURE 125.—Radiation.

a radial line from C toward D and measure the distance. When this distance has been plotted to scale on the line cd, the point d thus located should fall on the point d previously located from station A. If it does not check on returning to the original point, the traverse must be adjusted as explained in paragraph 62.

151. Intersection and resection.—*a. Intersection.*—The process of locating a point by radial lines from two or more known stations at which the table has been set up is called intersection. After the point has been located on paper to the satisfaction of the topographer, distances from it to the original points are scaled. These distances, together with their corresponding vertical angles or Beaman arc

readings, are used for determining the elevation of the new point. (See also par. 121.)

b. Resection.—When a point is located by setting up on it and sighting at visible stations which have been plotted on the field sheet, the process is known as resection. Usually not less than three radial lines from well distributed stations around the occupied point to be located are used. (See also par. 124.)

152. Magnetic orientation.—While this method is sufficiently accurate for traversing, it cannot be employed with the longer sights

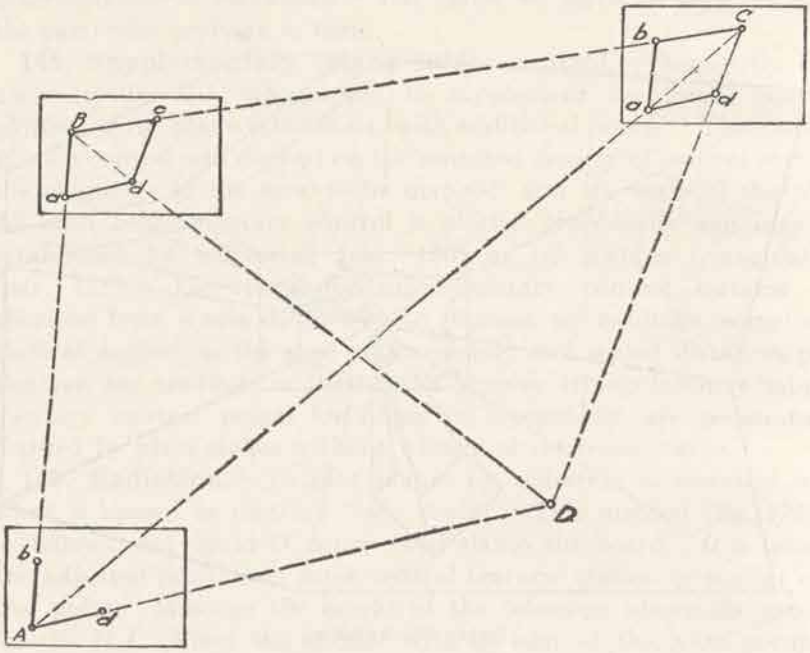

FIGURE 126.—Traversing.

required for plane table triangulation, except as a means of securing preliminary orientation in resection. In traversing, the *H. I.* is obtained by backsight, which requires the signs of the product and rod correction to be reversed from those used for foresights and side shots as described in paragraph 146. To avoid confusion the line for each backsight should be marked *B* in the Beaman arc record. The signs to be affixed to "product" and "rod correction" are determined according to whether the observation is a *B. S.* or *F. S.*, by following the rule of universal application, namely—

Observation.	Product	Rod correction
B. S.	Opposite sign to that indicated by arc reading	+
F. S.	Same sign as that indicated by arc reading	−

For example, arc reading 54 indicates +; therefore the sign of product is − for *B. S.* and + for *F. S.* Note that the sign of "rod correction" is the same as in leveling. When line of sight is level, arc reading is 50, and therefore only entry is rod reading, entered as "rod correction," whose sign follows above rule.

SECTION XXVI

TOPOGRAPHY WITH PLANE TABLE

	Paragraph
General	153
Plotting detail	154
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Inking field sheets	159
Checking completed field sheet	157
Sketching on aerial photographs	158
Completing skeleton sheets	156

153. General.—*a. Requirements.*—Before attempting to plot topography with the plane table the operator must be thoroughly familiar with the methods of sketching discussed in FM 21–35, must know the conventional signs and symbols shown in FM 21–30, and should have performed the basic drafting exercises enumerated in paragraph 2 and described in TM 5–230.

b. Plane table sheets.—(1) Topography is plotted to the scale of the projection or grid shown on the plane table sheet furnished the topographer. From the standpoint of efficiency the plane table sheet is the least satisfactory portion of the plane table equipment. Owing to its hygrometric nature it is very susceptible to atmospheric changes, expanding and contracting unceasingly. This would be but an insignificant source of error or annoyance if it were equal in all directions. The map or plan would then simply change its scale, for which an allowance could readily be made. But the objectionable feature is the unequal expansion and contraction which changes the relative distance and directions of the points. It has been determined by experiment that strips cut longitudinally from drawing paper vary from 10 to 25 percent more than strips cut transversely from the same paper.

(2) Various other materials are used for plane table sheets, such as paper mounted on aluminum and "Pyralin" (white opaque). They

are quite satisfactory and the latter can be used in rainy weather if necessary.

(3) Most plane table sheets, however, are made of two sheets of drawing paper, mounted with the grain at right angles, and with cloth between them. To reduce the distortion to a minimum, a sheet of this type should be thoroughly seasoned before it is taken to the field or a projection laid down on it. This is effected by exposing it alternately to a very damp and a very dry atmosphere. On testing a sheet after a week of such exposure it will be found to have much less tendency to expand or contract unequally. Paper stored away, piled in stacks, does not properly season.

c. Organization of party.—In organizing a party for field work it is well to have the recorder carry the table. A part of his duty is to remain constantly with the instrument and never to leave it unguarded. He should be taught at the beginning of the work the correct method of setting the table over a point and taking it up. In the first instance he grasps firmly two legs of the tripod and "flips" the third one until it reaches the ground at the proper distance from the point; he then places the other two in position. The distances from the point will vary according to the slope of the ground. In taking up the table, two legs should be grasped firmly and raised, pivoting upon the other leg; the two held legs are closed and the table is raised in place upon the shoulder. At least one rodman is needed and if the topographer is experienced he should be able to keep two rodmen busy. Rapidity of execution is largely dependent upon their efficiency. When well trained they should be able to recognize the salient points of the features to be mapped, so that the topographer can draw in details correctly from the least number of readings. The amount of assistance an aide can give to his chief is limited only by his skill and experience, the logical inference being that he is in training to become a topographer himself, and can take charge of an increasing share of the work as he becomes more and more familiar with the methods employed.

154. Plotting detail.—*a. Method.*—Detail is located by the method described in paragraph 146*b*. At the larger scales, several side shots may be needed at each set-up for plotting the works of man and terrain features. As each point and its elevation are marked on the sheet, an effort should be made to add part of the contour above and below the point, as the topographer can estimate their positions best while the point is still marked by the rod. On succeeding shots the contours should be completed as the rod is moved, if practicable. In this manner the contours are finished as the salient features are located and additional shots are seldom necessary for contouring. At 1:48,000

and smaller scales, side shots, except a few local intersections, are scarcely ever required.

b. Use of conventional signs.—All topographic and military features will be shown by conventional signs and symbols conforming to those in FM 21–30. Features for which no sign or symbol is prescribed may be shown by special symbols provided they are explained in a legend on the sheet.

c. Contouring.—(1) No rule can be laid down as to the number of elevations that should be determined from each plane table station or for a given area. It will depend on the skill of the topographer and the configuration on the ground. It would indicate careless and slovenly work if contours were found to deviate frequently from their true position on the sheet by more than half an interval for a slope of less than 5° in open country. When the slope is steeper, and in wooded regions, a greater latitude is permissible, but even here, in representing the crests of ridges, prominent hilltops, and valley floors, this limit of half an interval should not be departed from.

(2) The topographer will be assisted in sketching contours, where the modeling is intricate, by lightly drawing a skeleton composed of ridge and valley lines in their proper positions around the station. On the ridge lines will be found the extreme outward or convex bends of the contours and on the stream lines the extreme inward or concave bends.

(3) *Contours* (fig. 128).—(*a*) A contour never splits (5), nor do two contours run into one (6), nor will they cross each other except in the rare instance of an overhanging cliff (8). When an auxiliary contour is introduced, no more of it is drawn than is sufficient to delineate the special feature which makes it necessary. A principal contour, on the other hand, should not have an end within the map; if it commences at one edge it must terminate at another.

(*b*) A closed contour encircled by one or more closed contours is either a hill, as shown in (1), or a depression, as shown in (2), the arrows showing the direction in which water would run. The summits of all the hills of importance should have their elevations determined and marked on the map.

(*c*) A series of contours, as shown in (3), is either a croupe (the end of a ridge or promontory) or a valley. If a croupe, the contours will have their concave sides toward the higher ground; if a valley, the contours will have their concave sides toward the lower ground.

(*d*) A combination of four sets, as in (4) with convex sides turned toward each other, represents a dip in a ridge or the junction of two ridges, and is called a saddle.

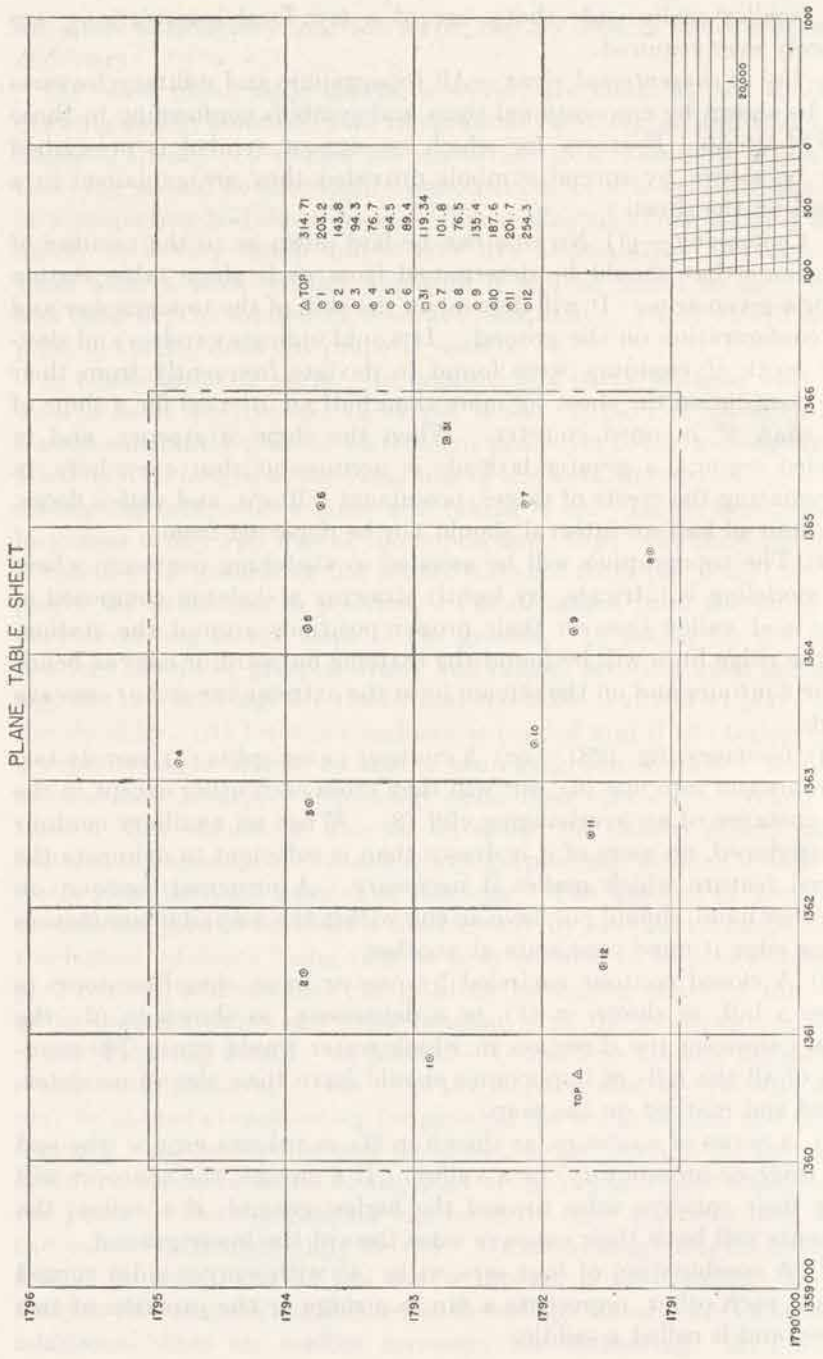

FIGURE 127.—Plane table sheet.

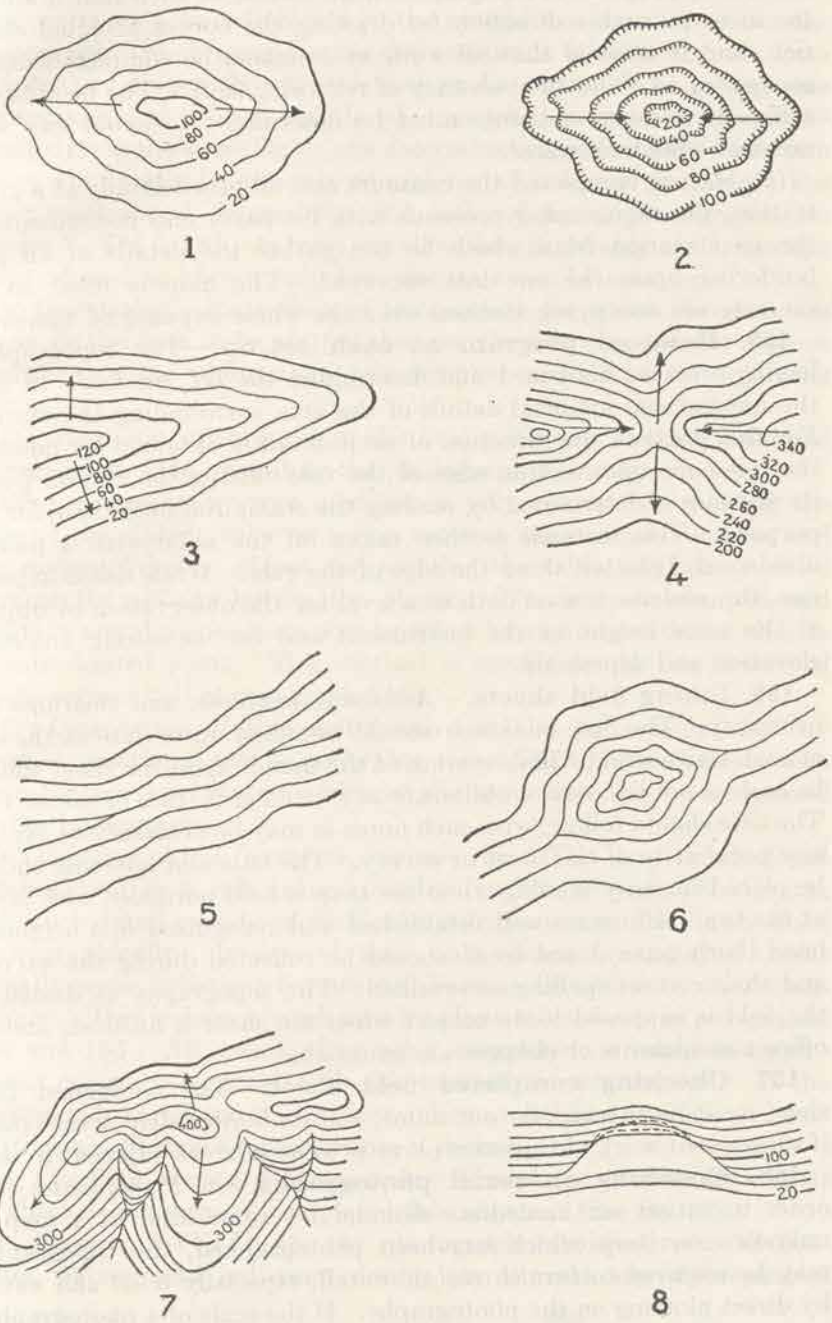

FIGURE 128.—Correct and incorrect contour groups.

(4) The progress of topographic work should be such that it affords the most favorable direction for drawing the curves of equal elevation, and it is usual that all work at a station be completed during occupancy to avoid the necessity of returning to it. The heights of a sufficient number of points must be determined to avoid any wide range of visual estimates.

(5) Having completed the contours and all other details at a given station, the topographer proceeds with his party and instruments to the next station from which he can gather the details of an area bordering upon the one last surveyed. The map is filled in by successively occupying stations over the whole expanse of the sheet.

155. Habitual program at each set-up.—The topographer, having oriented his board and determined the *HI*, proceeds to map the natural and artificial details of the area surrounding the station. For this purpose the direction of each detail is obtained by pointing the telescope upon it, the edge of the rule cutting the station point; its distance is determined by reading the stadia rod held there for the purpose. This distance is then taken off the scale with a pair of dividers and plotted along the edge of the rule. While this is in progress, the alidade is used both as a level for the observation of objects of the same height as the instrument and for measuring angles of elevation and depression.

156. Inking field sheets.—Accuracy, neatness, and clearness are necessary. The map or sketch should be inked up to date at the end of each day's work. The location of the names upon the sheet should be such as not to cover or obliterate any detail or feature of the survey. The title should follow, with such notes as may be necessary to explain any peculiarity of the sheet or survey. The title and lettering should be placed for easy reading when the map is held normally with north at the top. All names well established and recognized in a neighborhood, both general and local, should be collected during the survey, and their correct spelling ascertained. The topography as drawn in the field is supposed to be correct when the sheet is finished, and no office amendments or changes are admissible.

157. Checking completed field sheet.—The completed field sheet must be checked for omissions, and its border strip areas traced if subsequent work of the same party is to be in areas adjacent to it.

158. Sketching on aerial photographs.—*a. Prerequisites.*—In order to satisfy an immediate demand for some kind of a map in unknown territory which has been photographed, the topographer may be required to furnish certain detail, especially relief and names by direct plotting on the photographs. If the scale of a photograph is

unknown, the topographer may have to determine it by comparative measurements between objects on the ground and their images on the photograph. Such comparative determination should be made from at least two diagonally opposed lines or distances. If more than one photograph is to be used and the photographs have been made successively during the same flight, one determination of the scale (from two lines) will usually be accurate enough for all practical purposes. For other methods of determining the scale of aerial photographs, see section V, TM 5–230. At least one elevation, preferably of the starting point, should be known unless an elevation can be assumed.

b. Orientation.—Regardless of the purpose for which the sketch is made, or the data to be plotted thereon, the board is set over a point identified in the photograph and oriented by one of the following methods:

(1) Magnetic orientation is identical with the method described in paragraph 144*b*, regarding the photograph as a map.

(2) Orientation by true meridian is accomplished by placing the edge of the alidade ruler on a line between the occupied station and the distant (visible) object and rotating the table until the vertical wire of the alidade bisects the object. The table is clamped and sights in any direction may now be taken by sighting with the alidade at any desired point. This method is similar to the one described in the second half of paragraph 144*b*.

(3) Orientation of the photograph can also be accomplished by graphic resection if two or preferably three distinct objects can be seen from the occupied station which at the same time can be recognized on the photographs.

c. Plotting, detail, contours, names, etc.—The horizontal detail (planimetry) as a rule being complete on the average photograph, only such detail need be plotted as is indistinct or cannot be obtained photographically. In general, then, only contours, names, and occasionally some indistinct feature will have to be added by the topographer. This is done as explained for plane table sheets in paragraphs 154 and 155. All detail, after being completed, is inked in appropriate colors to contrast with the tone of the photograph. If the photograph is not too dark, black ink may be used exclusively, permitting quick, direct reproduction, if desired.

d. Checking completed photographs.—Completed photographs must be checked the same as completed field sheets. (See fig. 129.)

e. Exercise XXVII.—On the photograph furnished, plot all contours, names, and indistinct detail; also correct any discrepancies

FIGURE 129.—Completed (inked) photograph.

found in the horizontal detail that may be due to construction or developments subsequent to the date the photograph was taken. Ink and check the completed photograph.

159. Completing skeleton sheets.—*a*. Skeleton plane table sheets are prepared from aerial photographs and control data. They show the military grid, triangulation and traverse stations, bench marks, elevations of control points, and all planimetric detail obtainable by

tracing or transferring from photographs. Orientation of a skeleton sheet over the occupied station is described in paragraph 144*b*. The plotting of detail, including contours, is identical with the methods described in paragraph 146*b*. The topographer, after completing the sheet, will check every detail as explained in paragraphs 156 and 157 before turning them over to the office for compilation.

b. Exercise XXVIII.—On the skeleton sheet furnished, plot all details, contours, and names as required and complete and check the sheet as explained in this paragraph. Figure 130 shows a part of a completed skeleton sheet.

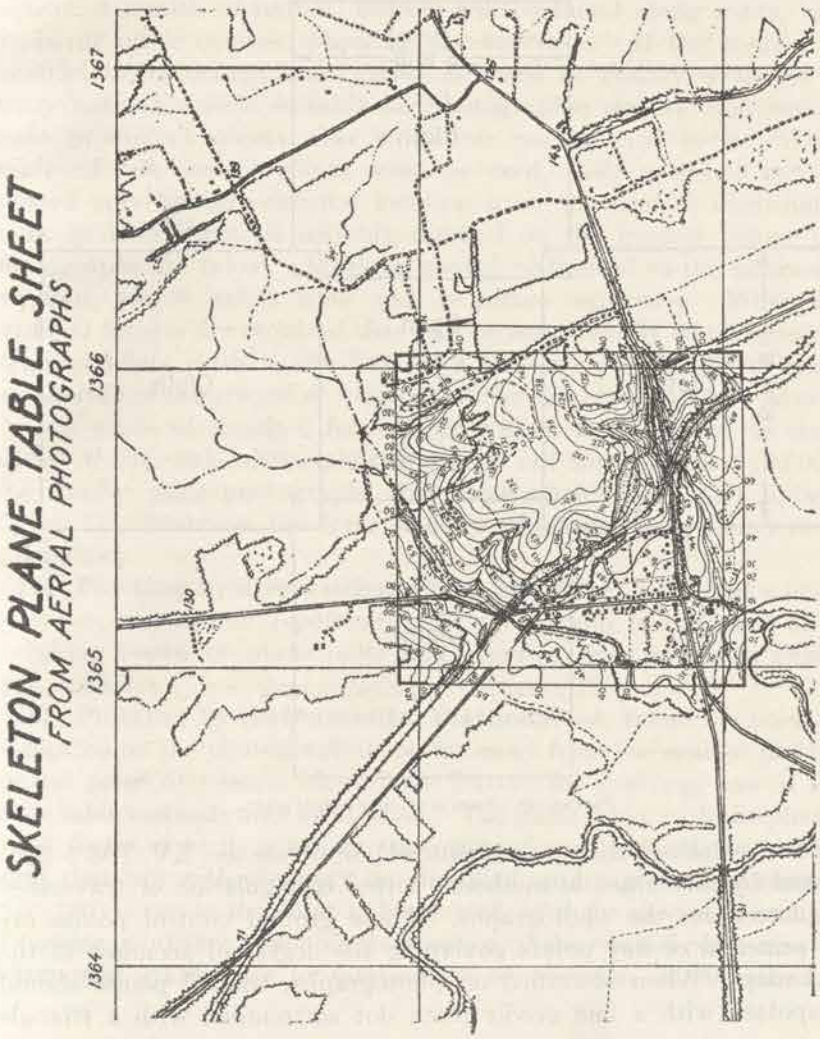

FIGURE 130.—Part of completed skeleton sheet.

SECTION XXVII

PLOTTING CONTROL POINTS ON PHOTOGRAPHS IN FIELD

Paragraph
Importance and necessity 160
Plotting by direct orientation (spotting) 161
Plotting by instrumental methods 162

160. Importance and necessity.—Methods used in the construction of maps from aerial photographs make it mandatory, if precise

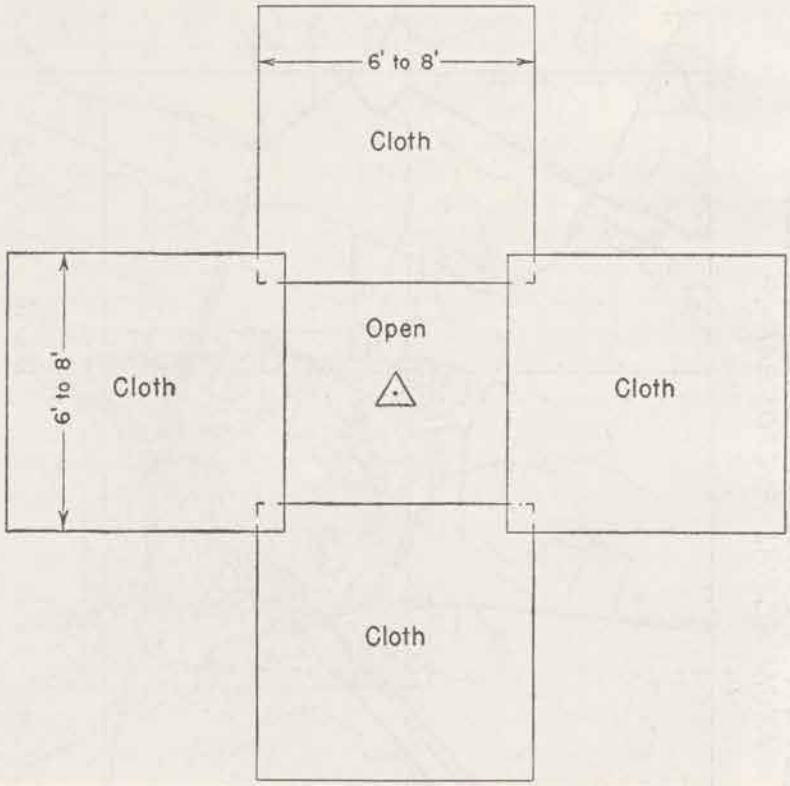

FIGURE 131.—Form of paneling control point.

results are desired, that ground control points (see sec. XV, TM 5–230) located by instrumental methods—either triangulation or traverse—be plotted on the photographs. These ground control points are the principal or key points governing the degree of accuracy of the final map. When identified on photographs, control points should be spotted with a fine needle prick dot surrounded with a triangle

and labeled. The triangles should be comparatively large (sides about 0.3 inch long) in order not to obscure the detail immediately around the point. If circles are used instead of triangles, they, too, should be large (diameter about 0.2 inch). When the ground control is established, special effort should be made to ascertain that all points selected can be readily identified on the photographs. In this case, the plotting of all ground control points is done in the drafting room; additional field work is eliminated. In selecting natural objects for points of control, preference should be given to those of fixed and permanent nature. If traverses are run to provide control, a wealth of natural objects will be found along roads, railroads, or other courses taken for the traverses. If the control be established by triangulation, there will not be readily available so many natural objects suitable for photographic targets, but usually some prominent object near a station can be tied to it. When unidentifiable control points must be used, their positions can be plotted only by instrumental location from the nearest identifiable point or they must be suitably marked on the ground before the photographs are taken. Marking should be limited to the minimum required, for it takes time and is rather expensive. Whenever artificial targets are required they can be most readily constructed of strips of white cloth in the form of a hollow cross, tacked to short pegs driven in the ground or weighted at the corners with heavy stones. Crosses made of panels 6 feet square will be large enough to show plainly if the scale of the photographs is not smaller than 1:20,000. For smaller scale photographs the panels should be 8 feet square. Figure 131 illustrates the form of cross which has been found most satisfactory.

161. Plotting by direct orientation (spotting).—To plot a point by direct orientation (spotting), the photograph is mounted on a sketching board or plane table and oriented upon a known object (par. 158*b*).

162. Plotting by instrumental methods.—*a.* When the point to be plotted on the photograph is too far away from the nearest plotted control point or nearest identifiable feature for spotting, one of the plane table methods may be utilized. The plane table, with the photograph fastened to it, is set up and oriented over the nearest picture point that can with certainty be identified, and a *plane table traverse* (par. 150) is run to the point to be plotted; or if the necessary number of picture points are available the location of the control point may be determined graphically by intersection or resection, as described in

paragraph 151. Another method of marking a control point on a photograph is by determining its position from a stadia traverse (par. 80) and plotting the point by polar coordinates, or computing its coordinates and plotting the point as described in section XIII, TM 5–230. The plotting, in either case, will have to be done to the scale of the photograph.

b. Exercise XXIX.—The description and elevations of certain ground control points being given, locate and plot them on the photographs furnished, using either plane table methods or the stadia-traverse method as outlined above.

Section XXVIII

ELEMENTS OF FIELD ASTRONOMY

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Determination of hour angle and declination	166
Astronomic triangle	167
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163. General.—*a. Scope.*—This section is intended to provide certain elementary instruction in field astronomy and should be read carefully as a guide to the methods best adapted to various circumstances. The determination of time and longitude, and the more precise latitudes and azimuths are treated in Special Publications of the United States Coast and Geodetic Survey (No. 14, Determination of Time, Longitude, Latitude, and Azimuth; and No. 109, Wireless Longitude).

b. Ephemeris.—The *American Ephemeris and Nautical Almanac*, issued by the United States Naval Observatory, will be needed for some of the methods of section XXX. The *American Nautical Almanac*, of the same issue and of one-third the bulk and one-fifth the cost, could be adapted for most purposes by changing the forms; but its ephemeris of the stars omits certain data, and the special tables on Polaris are missing. "The Ephemeris of the Sun and Polaris and Tables of Azimuths and Altitudes of Polaris," published by the General Land Office, gives condensed tables satisfactory for the sun and Polaris, but contains nothing of the other stars. The small pocket books issued by some instrument makers have practically the same contents as the Land Office Ephemeris.

c. Meridians.—(1) The azimuth of a line is its angle, measured clockwise, from a base direction line, which may be the true meridian, the magnetic meridian, or a north and south grid line. In third-order triangulation, azimuths are measured from true south; in grid triangulations, from grid north. Astronomic azimuths will be considered as clockwise from the true north. Due to inequalities in the density of the earth's crust, the plumb line does not point to the center of the earth at all points on its surface. Geodetic azimuth is azimuth corrected for error due to local deflection of the plumb line from the true vertical. It varies slightly from the uncorrected value known as astronomic azimuth. The azimuth of a celestial body is the spherical angle at the zenith between the meridian and the vertical circle of the celestial body, or the angle in the plane of the horizon between the plane of the meridian and the plane of the vertical circle through the celestial body. The azimuth of the celestial body *S* (fig. 132) is the spherical angle *PZS*, which is the same as the plane angle *NOJ*.

(2) The basis of azimuth determination is the location of the true meridian of the observer. True north for any observer is represented on the earth by the arc of a great circle between the observer and the North Pole, created by the cutting of the surface of the earth by a plane through the observer and the North and South Poles. This great circle is known as the observer's true meridian. The meridians defined in this paragraph are the meridians of the earth's surface, that is, terrestrial longitude lines.

(3) Azimuth determination by observation of a celestial body may be divided into two main operations:

(*a*) Measurement of the angle between the position of the sun or star and a datum point, and recording that angle together with the exact time of day.

(*b*) By referring to tables for the observed body in the ephemeris, its actual azimuth can be computed for the exact time and date of the observations. Having found this true azimuth of the sun or star and the measured angle between it and the datum point, the azimuth of the datum point is established.

d. Terrestrial coordinates.—(1) The *longitude* of a place is the arc of the Equator intercepted between the meridian of that place and the meridian of Greenwich, England, adopted as the zero meridian, from which all longitudes are measured east or west up to 180° of arc. The difference in longitude between two places may also be defined as the angle between the planes of their respective meridians. Since the earth rotates through 360° in 24 hours, this difference can also be

expressed in time instead of arc, 1 hour being equal to 15° of arc. As 1 second of time is therefore equivalent to 15 seconds of arc (over 1,100 feet at 40° of latitude), the astronomic determination of longitude with fair accuracy requires knowledge of the exact time and means of timing observations at fractions of seconds.

(2) The *latitude* of a place is described in several ways. The *geodetic latitude* of a place is the inclination of the line, normal to the spheroid, to the plane of the Equator, measured in the plane of the meridian. Since the earth is a spheroid, the normal line, except at the poles and on the Equator, does not pass through its center. The *astronomical* latitude is the inclination of the plumb line to the plane of the Equator. Because of the deflection of the plumb line, geodetic and astronomical latitudes are seldom in agreement, and may differ by several seconds. This is not enough to cause any material error through taking the latitude of a place from a good map for use in azimuth determination. On the other hand, the simple methods of latitude determination mentioned in paragraph 168c may be expected to average several seconds in error from the geodetic latitude, and so do not afford a good check on position.

e. Motions of earth and solar system.—(1) The earth has two motions, both clockwise when looking toward the north; it rotates about its own axis, and it revolves around the sun. The earth moves around the sun once a year in an orbit which lies practically in one plane, and whose form is that of an ellipse, the sun being at one of the foci. Since the earth is held in its position by the force of the sun's gravity, its speed along its orbit depends on its distance from the sun. The orbit being an ellipse, this distance is constantly changing. It can therefore be said that the motion of the earth in its orbit around the sun is not uniform. This fact is of importance in its effect on solar time. The axis of rotation of the earth is inclined to the plane of the orbit at an angle of about 66½°; that is, the plane of the earth's Equator is inclined at about 23½° to the plane of the orbit. This latter angle is known as the *obliquity of the ecliptic*. The direction of the earth's axis is practically constant and it therefore points at nearly the same place in the sky throughout the year and from year to year.

(2) The sun appears to be farthest north about June 22, at which time the days are longest and the sun's rays most direct in the Northern Hemisphere. In winter, due to the inclination of the earth's axis, the sun is actually below the plane of the earth's Equator; in summer it is above this plane. It must therefore cross the plane of the Equator in going from below to above (winter to summer), and again

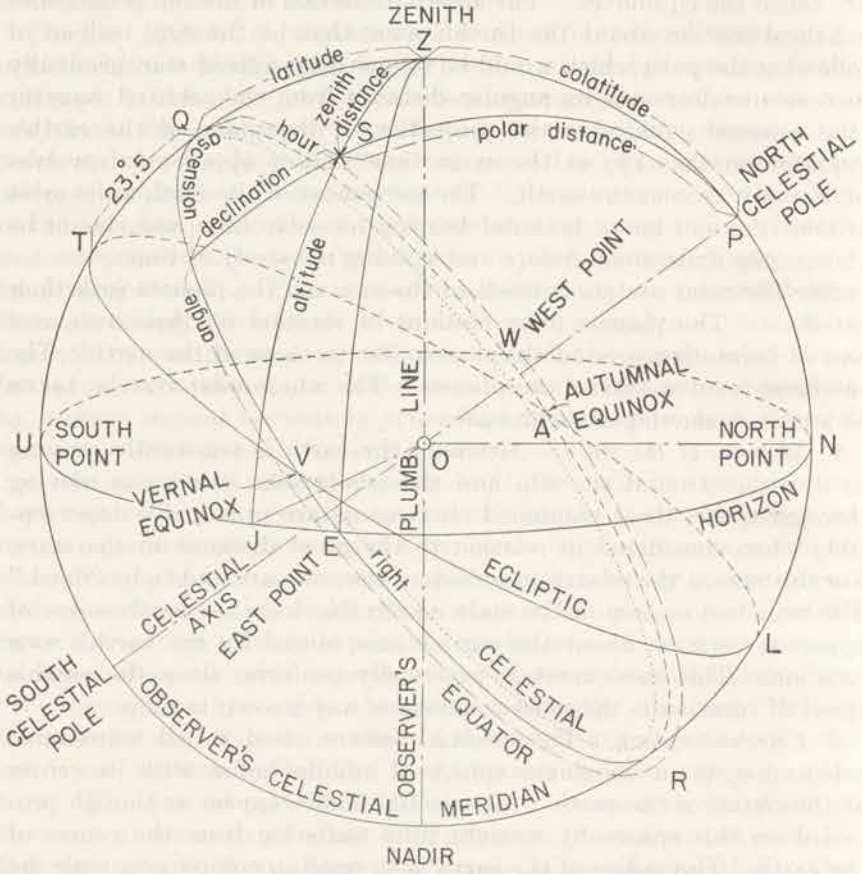
going from above to below (summer to winter). These two crossings occur about March 21 and September 22. On these dates the sun is in both the plane of the earth's Equator and the plane of the earth's orbit, and the days and nights are of equal length. These two points are called the equinoxes. The apparent motion of the sun is therefore a helical motion about the earth's axis; that is, the sun, instead of following the path which would be followed by a fixed star, gradually increases or decreases its angular distance from the celestial equator (the celestial equator is the projection of the plane of the earth's Equator on the sky) at the same time that it apparently revolves once a day around the earth. The movement of the earth in its orbit around the sun has a material bearing on solar time and should be thoroughly understood before undertaking the study of time.

(3) The solar system consists of the sun and the planets with their satellites. The planets have motions of rotation on their own axes and of revolution around the sun similar to those of the earth. The satellites revolve around the planets. The whole solar system, taken as a unit, is moving through space.

f. Motions of the stars.—Although the earth is constantly moving in its orbit around the sun and the whole solar system is moving through space, these combined movements are practically imperceptible when considered in relation to the great distance to the stars. For this reason the relative positions of the stars are said to be "fixed." The principal motion of the stars discernible from the earth is one of apparent rotation about the earth's axis caused by the earth's own rotation. This movement is practically uniform, since the earth's speed of rotation is the most constant of any known motion.

g. Celestial sphere.—The celestial sphere, used in all astronomic calculation, is an imaginary sphere of infinite radius with its center at the center of the earth. All celestial bodies appear as though projected on this sphere by straight lines radiating from the center of the earth. The radius of the earth is so small in comparison with the distances to the fixed stars that the position of an observer on the surface of the earth does not affect the projection of a star on the celestial sphere. However, the accuracy of the projection of the sun, as well as that of the planets, is affected by the fact that the observer is not at the center of the earth but on its surface. Therefore, the position of the observer is considered the center of the celestial sphere and the apparent positions of the sun, planets, and satellites are corrected for parallax due to the distance from the surface to the

center of the earth. Figure 132 represents the celestial sphere. The radius of the earth is so small compared with the infinite radius of the celestial sphere that *O* may be regarded as any point on the surface of the earth.



Astronomic lines and points are named in capitals, angles in lower case letters, except for the following observer, *O;* a star, *S;* portions of vertical circles, *NZU, ZE, ZSJ;* azimuth of star, spherical angle *PZS,* or plane angle *NOJ.*

FIGURE 132.—The celestial sphere.

h. Apparent motions of all celestial bodies.—All celestial bodies appear to revolve about the earth, but their motion (except a part of the motion of the sun, moon, and planets) is only apparent and is the direct result of the rotation of the earth itself. However, for convenience the earth is considered to remain stationary at the center of the celestial sphere and the latter is assumed to rotate about an axis which is the infinite prolongation of the earth's polar axis. The stars

are fixed on this celestial sphere except for motions not capable of measurement during a short period. The sun and planets are displaced on this sphere according to their motions as viewed from the earth. Each fixed star will describe an apparent daily path from east to west around the earth, its apparent orbit being a plane perpendicular to the axis of the earth. The sun, in addition to its apparent motion due to the rotation of the earth on its axis, moves slowly each day along the ecliptic. This motion is the cause for the continual change in declination of the sun. The apparent daily path of the sun varies slightly, therefore, from a circle in a plane perpendicular to the earth's axis.

164. Definitions.—A general comprehension of the fundamentals of practical astronomy is necessary before the solution of azimuth problems can be thoroughly understood. Nearly all of the astronomic points, lines, and angles defined below are named in figure 132, which should be studied with this paragraph. The common abbreviations for some of these terms are shown in parentheses after the names in the definitions. Other abbreviations are given in paragraph 167*d*.

a. A *great circle* is one whose plane passes through the center of a sphere. Other circles are *small circles*. In figure 132, all circles shown are great circles.

b. The *north and south celestial poles* (*P* and *SP*) are the points where the prolonged polar axis of the earth intersects the celestial sphere.

c. The *celestial equator* is the great circle on the celestial sphere cut by a plane through the center of the earth perpendicular to the earth's axis.

d. The *observer's vertical axis* is the direction of the plumb line at rest, or the prolongation of that line toward the celestial sphere from the observer's point.

e. The *zenith*, (*Z*) and *nadir* for any place on the earth's surface are the two points where the prolonged plumb line intersects the celestial sphere. The zenith is the point directly overhead and the nadir is the point directly underneath.

f. The *horizon* for any place on the earth's surface is the great circle cut on the celestial sphere by a plane perpendicular to the plumb line at that place.

g. A *vertical circle* is any great circle on the celestial sphere, which passes through the zenith.

h. The *meridian* of any observer is the great circle on the celestial sphere which passes through the celestial poles and the observer's zenith.

i. The *prime vertical* for any place on the earth's surface is the vertical circle perpendicular to the meridian. It intersects the horizon at the points directly east and west.

j. The *ecliptic* is the great circle cut on the celestial sphere by the plane of the earth's orbit. Its plane is the plane of the earth's path in traveling around the sun. If we could look past the sun and see the stars we would see the sun and stars moving slowly across the sky. The sun would gain slightly on the stars each day. In a year, the sun would have traced a line through the heavens crossing certain constellations. This line is the ecliptic, and the constellations in which the sun's projection is at any time is one of the constellations in the zodiac.

k. The *equinoxes* are the two points where the ecliptic intersects the celestial equator. The point where the sun crosses the Equator in passing from south to north is called the *vernal equinox* (*V*) The other is called the autumnal equinox and is diametrically opposite the *vernal equinox.* It should be remembered that each equinox is a definite point in the heavens. The equinoctial points are moving slowly westward along the ecliptic at a rate of about 50 seconds a year. The phenomenon is known as the *precession of the equinoxes,* and slight periodic changes in the precession are called *nutation.* As a result, all the fixed stars gradually change their positions with reference to the Equator and the vernal equinox.

l. Celestial coordinates.—The location of a celestial body may be defined by one of several systems of coordinates used in connection with the celestial sphere. There are but three commonly used systems which need consideration in connection with azimuth problems.

(1) The *horizontal system of coordinates* is that in which the horizon, meridian, and zenith are used as references. The two coordinates in this system are *altitude* and *azimuth.* These two coordinates being known at any instant, the celestial body is definitely located for that instant.

(2) The *equatorial system of coordinates* is that in which the celestial equator, vernal equinox and celestial poles are used as references. The two coordinates in this system are *right ascension* and *declination.* Knowledge of the value of these two coordinates definitely locates the body.

(3) The *meridional equatorial system of coordinates* is a variation of the equatorial system, the only difference being that the meridian, instead of the vernal equinox, is one of the references. The two coordinates in this system are the *hour angle* and the *declination.* The body is also definitely located if these are known.

m. The *latitude* of any place on the earth's surface is the angular distance of that place measured from 0° to 90°, north or south of the Equator. It may be defined astronomically as the angle between the plane of the Equator and the line observer–zenith, which is the same as the angle between the plane of the horizon and the line observer–celestial-pole. The complement of the latitude is called the *colatitude.*

n. An *hour circle* is any great circle through the celestial poles. The meridian is the hour circle through the zenith and is known as the zero hour circle of the place.

o. The *hour angle* (*HA*) of a celestial body is the angle between the plane of the observer's meridian (zero hour circle) and the plane of the hour circle through the body. This angle is similar to differences in longitude on the earth's surface. It is measured westward from the meridian (counterclockwise looking from the earth toward the north celestial pole). It is generally considered as an arc measured along the celestial equator toward the west and is expressed in time from 0 to 24 hours. It may also be expressed in arc of the celestial equator, 0° to 360°.

p. The *right ascension* (*RA* or α) of a celestial body is the arc of the celestial equator measured from the vernal equinox eastward (clockwise when looking from the earth toward the north) to the hour circle through the body. It is measured in units of time from 0 to 24 hours.

q. The *declination* (δ) of a celestial body is the angular distance from the celestial equator to the body, measured along the hour circle of the body. Declination is given the positive sign when the body is north of the celestial equator and negative when south.

r. The *polar distance* (p) of a celestial body is the arc of its hour circle measured, in the Northern Hemisphere, from the north celestial pole to the body. It is the algebraic complement of the declination, that is, 90° minus the declination if the declination is positive, and 90° plus the declination if the declination is negative. It is measured from 0° to 180°.

s. The *altitude* (h) of a celestial body is the arc of its vertical circle measured from the horizon to the body, or it is the vertical angle at the observer's station between the horizon and body. The *observed* altitude of any celestial body is affected by *refraction*, and the observed altitude of the sun or planets is affected by the parallax of the body, owing to the fact that the observer is on the surface of the earth and not at its center. (See x and y below.)

t. The *zenith distance* ($90° - h$) of a celestial body is the complement of the altitude of the body.

u. The *azimuth* (A) of a celestial body is the angle at the zenith between the meridian and the vertical circle of the body. It is actually measured as an arc in the plane of the horizon and may be east or west of north. In figure 132, the azimuth of the star is NJ (measured east of north).

v. The *culmination* or *transit* of a celestial body is the passage of that body across the meridian of the observer. As the meridian is a great circle, any celestial body has two transits or culminations; passage across the upper arc of the meridian ($PZQU$), or above the pole, is called *upper culmination;* passage across the lower arc of the meridian (PNR), or below the pole, is called *lower culmination.*

w. The *elongations* of a celestial body are the two points in its apparent orbit at which the bearing from the meridian is the greatest. A star is said to be at *eastern elongation* when its bearing to the east of the meridian is the maximum and at *western elongation* when its bearing to the west of the meridian is greatest.

x. The *refraction* of a celestial body is the apparent displacement of the body due to the bending of the rays of light passing through layers of air of varying density. The effect of refraction is noted directly in measuring altitudes, the celestial body always appearing higher than it really is. As the observed altitude is thus too great, the correction is always subtracted from observed altitude. Refraction has no apparent effect on the azimuth of the body. (See par. 169*e*.)

y. The *parallax* of a celestial body is the difference in direction of the body as seen from the center of the earth and from a station on the surface of the earth. There is no apparent parallax of the fixed stars, but the parallax of the sun and planets is measurable. Parallax affects only the altitude of the body and makes it appear lower than it really is. Consequently, the effect of parallax is opposite in sign to that of refraction and the correction is always added to an observed altitude. (See par. 169*f*.)

165. Time.—*a. Kinds of time.*—(1) In dealing with the location of celestial bodies by a measured position referred to the time of observation, we must deal with two general classes of time—*sun or solar time,* and *star or sidereal time.* Solar time is further subdivided into *apparent time, mean time,* and *standard time.* The reason for two general classifications of time is shown in figure 133.

(2) The basis of each kind of time is the day. A *solar day* is the time between successive transits of the meridian by the sun; the

sidereal day is the time between successive transits of the meridian by a fixed star. In figure 133, C and C_1 represent the center of the earth in its orbit around the sun on two successive days. If O_1 represents the position of the observer on the first day, when a certain star (S') on a line with the sun crosses his meridian, the same star (S') will cross his meridian on the following day when the earth has made exactly one revolution about its axis, and his position will be represented by O_2. This period of time is called a sidereal day and is subdivided into 24 sidereal hours. If O_1 represents the position of the observer on the first day when the sun crosses his meridian, it is obvious from figure 133 that the earth must make more than one complete

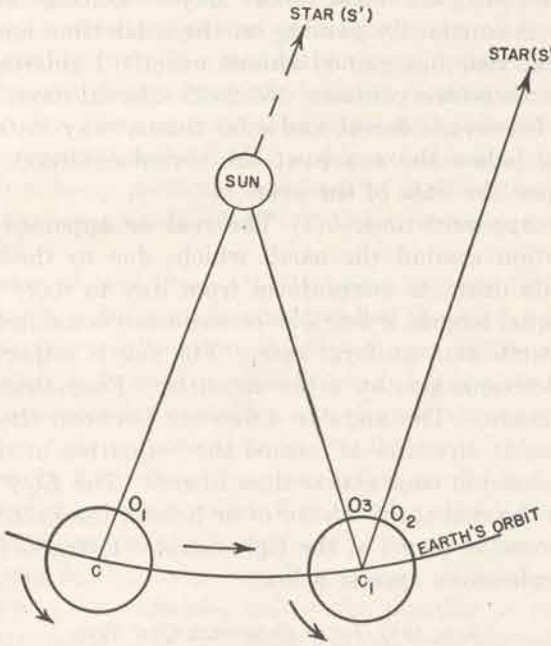


FIGURE 133.—Effect of orbital motion of earth on time.

revolution before the sun crosses his meridian on the second day. The position of the observer when this occurs is represented by O_3, and the period of time is known as a solar day consisting of 24 solar hours.

(3) It thus becomes apparent that a solar day is longer than a sidereal day. This difference in time averages about 3 minutes and 56 seconds per day. If two clocks, one regulated to mean solar time and the other to sidereal time, were started at the same instant, both reading the same time, the sidereal clock would gain about 10 seconds per hour on the solar clock. A solar day is the interval of time between

two successive lower transits of the sun's center across the same meridian. When the center of the sun is on the upper side of the meridian (upper transit), it is noon. When on the lower side, it is midnight. The instant of midnight is taken as 0 hour or the beginning of the solar day. Since the earth revolves around the sun in an elliptical orbit in accordance with the laws of gravitation, the apparent angular motion of the sun is not uniform, and the days, as determined by successive transits across the lower meridian, are therefore of different lengths at different seasons.

(4) The period of time that it takes the earth to make one complete revolution around the sun is known as a *tropical year*. The tropical year contains 365.2422 mean solar days. During this year the sidereal time is constantly gaining on the solar time and at the end of one tropical year has gained almost exactly 1 sidereal day. The tropical year therefore contains 366.2422 sidereal days. The difference in time between sidereal and solar time at any instant therefore varies from 0 (when the sun is at the vernal equinox) to 24 hours, depending upon the date of the year.

b. True or apparent time.—(1) The real or apparent sun has an apparent motion around the earth which, due to the travel of the earth along its orbit, is nonuniform from day to day. In order to get days of equal length, a *fictitious* or *mean sun* is assumed to revolve around the earth at a uniform rate. The sun is either ahead of or behind the fictitious sun by a few minutes. Four times a year the two suns coincide. The angular difference between the positions of these two suns at any time is termed the "equation of time" (*EQT*), and it is expressed in time rather than in arc. The *EQT* is a measure of how much the real sun is ahead of or behind the fictitious or mean sun, and is found in part I of the Ephemeris. Extracts from page 16 of the 1941 ephemeris appear below.

Sun, 1941, for 0h Greenwich Civil Time

Date	Day of week	Apparent right ascension			Apparent declination			Equation of time, apparent—mean			Sidereal time (right ascension of mean sun+12h)		
		h	m	s	°	′	″	m	s		h	m	s
Dec. 11	Th	17	10	14.31	−22	57	5.1	+7	1.13		5	17	15.4
				s 264.18			″ −299.8			s −27.62			
12	Fr	17	14	38.49	23	2	4.9	6	33.51		5	21	12.0

The plus sign in the equation of time for December 11 and 12 indicates that the real (or apparent) sun was ahead of the fictitious or mean sun

on these days. The equation of time (*EQT*) varies between about $+16m$ and $-14m$. The daily change in the equation of time may be as much as 30 seconds; consequently, the exact equation of time for any place and instant must be interpolated between tabular values. Some ephemeris tables express equation of time differently and with the opposite sign. In entering an unfamiliar table, first look at the heading of the column to make sure how the signs should be applied.

(2) The earth's rotation on its axis is a uniform motion, but because of the variation in velocity of the earth's progress in its orbit, and because of the obliquity of the ecliptic, apparent solar days vary in length. Therefore, clocks cannot be regulated to apparent solar time. However, observations on the sun will give its hour angle; add 12 hours to obtain apparent solar time. In observing the sun for azimuth and latitude, it is necessary to use apparent time because it is the real sun which appears in the heavens, and on which the instrument is pointed. The apparent solar time of the observer is referred to as "local apparent time" (*LAT*); apparent solar time at some other point is referred to by prefixing the name of the locality, for example, "Greenwich apparent time" (*GAT*). (Other abbreviations are given in par. 167*d*.)

c. Mean solar or civil time.—(1) In order to establish a solar day of uniform length, a fictitious luminary called the *mean sun* is assumed to move uniformly in the celestial ecliptic, completing the circuit in one tropical year. As used in the *American Ephemeris and Nautical Almanac* a mean solar day coincides with the civil day of the same date, and mean solar time is referred to as "civil time." Mean solar time cannot be directly determined by observation on the sun, but may be derived by applying certain corrections to solar or stellar observations.

(2) Civil or mean time is a measure of the position of the mean sun with respect to the lower meridian of any particular spot on the earth's surface. For example, unless the position is exactly on an even 15° longitude line, the civil time will not coincide with the standard time for that geographic position. If the locality is east of the central time meridian by, say, 5°, then the sun will have passed the local meridian $\frac{5}{15}$ hour, or 20 minutes, before it reaches the meridian of the central time meridian. If the geographic position is west of the standard time meridian by 3°, the sun will not reach the local meridian until 12 minutes after it has passed the meridian of the center of the standard time zone. It would therefore actually be earlier in civil time at the latter place than the standard clock indicates. The mean solar (civil) time of the observer is referred to as "local mean time" (*LMT*); of another point, by prefixing the name

of the place, for example, "Greenwich mean time" (*GMT*), or "Greenwich civil time" (*GCT*), which is the time for the zero meridian of longitude.

d. Standard time (*ST*).—(1) Mean solar time may be either *local time* or *standard time*. Local mean solar time varies with the longitude of the place. At a given instant it is later at a place to the east of the observer and earlier at a place to the west, the variation being 4 minutes for each degree of difference in longitude. Tables for the conversion of time into arc and of arc into time are given in TM 5–236. It would be impracticable for each particular locality to have its own local mean time and for this reason a series of *standard time belts*, each covering 15° of longitude, has been adopted. The standard time for each belt is the same as the local mean solar time for the middle meridian in the belt, which has a longitude from Greenwich of some multiple of 15°. Standard times in two adjacent belts differ by 1 hour. The following table gives standard time data:

	Longitude	Standard time (comparative)
Greenwich (*GST*)	0°	Noon.
Eastern (*EST*)	75° west	7:00 a. m.
Central (*CST*)	90° west	6:00 a. m.
Mountain (*MST*)	105° west	5:00 a. m.
Pacific (*PST*)	120° west	4:00 a. m.
Hawaiian (*HST*)	157°30′ west	1:30 a. m.
Philippine (*PhST*)	120° east	8:00 p. m.

(2) The difference between local mean time at any place and standard time is simply the difference in longitude expressed in time between the place and the standard time meridian of that particular time belt. A place at 78°39′ *W* longitude is $5^h\ 14^m\ 36^s$ from Greenwich or $14^m\ 36^s$ from the *EST* meridian. In the United States, standard time is in universal use and is counted from 0 to 12 hours before noon (a. m.) and from 0 to 12 hours after noon (p. m.), comprising 24 hours from midnight to midnight, which is called a civil day. Standard time is now in use in all the principal countries of the world; in most cases the systems of standard time are based on the meridian of Greenwich.

e. Sidereal time (*Sid. T.*).—(1) Apparent time is sun time, and is used when dealing with the sun. Sidereal time is used when dealing with the stars and could be termed "star time." There are on the celestial equator two points where the ecliptic (or sun's apparent yearly path through the heavens) intersects the celestial equator.

These intersections are called the vernal (or spring) and autumnal (or fall) equinoxes. They may be considered for present purposes as being approximately fixed points in the heavens. The vernal equinox (*VE*) has the same apparent motion as the stars. The simplest conception of time is that of sidereal time. The rotation of the earth on its own axis is practically a uniform motion. The interval of time between two successive upper transits of the vernal equinox over the same meridian is called a "sidereal day" and is equal to $23^h\ 56^m\ 04.091^s$ of mean solar time. The sidereal day at any point begins at the passage of the vernal equinox over the meridian of the point and ends at the next passage of the vernal equinox over the same meridian. A sidereal day is divided into 24 sidereal hours, each 9.83^s of mean solar time shorter than a mean solar hour. Astronomic clocks are usually regulated to sidereal time. The hour angle of the *VE*, or the counterclockwise angle between the upper meridian and the *VE*, as viewed from the south celestial pole, is called sidereal time. Sidereal time then may be considered to be a measure of the amount by which the *VE* has traveled since it last passed the upper meridian. This is the same as the right ascension of the meridian at that instant; the hour angle of the vernal equinox is measured from the meridian west to the vernal equinox and the right ascension of the meridian is measured from the vernal equinox east to the meridian. It is therefore evident that the right ascension of any celestial body when at upper culmination is the sidereal time at that instant and that the right ascension of the mean sun at mean noon is the sidereal time at mean noon.

NOTE.—A sidereal day begins when the *VE* crosses the *upper* meridian, while a solar day begins when the sun crosses the *lower* meridian.

(2) It is impossible to convert directly some particular instant of solar time into some particular instant of sidereal time, but a period of mean solar time can be converted into a period of sidereal time by applying a correction for the difference expressed in sidereal time between a mean time and a sidereal time period. An hour of mean solar time is 9.856^s of sidereal time longer than a sidereal hour, consequently the correction is always additive and 9.856^s times the time interval expressed in mean solar hours. The ephemeris gives this correction in convenient form (table III). The ephemeris gives the sidereal time at 0^h Greenwich civil time, which is the right ascension of the mean sun $+12^h$. Sidereal time at 0^h civil time at any point is found by correcting the sidereal time at 0^h Greenwich civil time by

the variation for the difference in longitude from Greenwich, the correction being the product of the hourly change in the right ascension of the mean sun (9.856^s) and the longitude of the place expressed in hours and fractions. The correction is plus for longitudes west and minus for longitudes east of Greenwich. The sidereal time at any local mean time is found by correcting the sidereal time at local mean midnight by the sidereal time interval before or after local mean midnight.

(3) Sidereal time at some particular instant may be converted to mean solar and apparent solar time by a similar procedure. The ephemeris (table II) gives data for conversion of sidereal time into mean solar time.

f. Conversion of time.—(1) *Standard time to mean solar (civil) time.*—Reduce standard time to mean solar time by adding or subtracting the time equivalent of the difference in longitude from the standard meridian, 4 minutes of time for each degree, 4 seconds of time for each minute of longitude, and 1 second for each 15 seconds of longitude, accordingly as the place is east or west of the standard time meridian.

(2) *Mean solar (civil) time to apparent solar time* and apparent solar time to mean solar (civil) time conversions will not be explained here in detail, as the azimuth methods of section XXIX are based on the meridian of Greenwich. Methods of determining Greenwich civil, apparent, and sidereal time are described in *h*, *i*, and *j* below.

g. Determination of standard time.—(1) In the United States all watches or clocks are set to standard time (sometimes daylight saving time). Standard time is used in recording the time of observation on any celestial body regardless of the method to be used in determining azimuth. In work of this type it is essential that the time as shown by the watch be absolutely correct. In the sun-hour-angle method of determining azimuths an error of 1 second in time may introduce an error of as much as 25 seconds in azimuth. In determining azimuth by Polaris, accuracy of time is not so important due to the slower azimuthal motion of the star, that is, an error of 1 minute in time may introduce an error of 30 seconds in azimuth. The correct standard time is determined at the Naval Observatory, Washington, D. C., and is broadcast by naval radio stations in the United States and its foreign possessions. The more distant stations are controlled through automatic reradiation with an error of less than one tenth of a second. See table XX, TM 5–236.

(2) *United States Navy time signals.*—The schedule for the East Coast is given below. Schedules for other areas and future years' information should be obtained from the Navy Department.

All signals sent from NAA (Arlington, Va.)

Hour	Frequency (kilocycles)	Hour	Frequency (kilocycles)
0 midnight	113 and 9,425.	12 Noon	113 and 9,425.
1 a. m	113 and 9,425.	1 p. m	113.
2 a. m	113 and 9,425.	2 p. m	113 and 9,425.
3 a. m	113 and 9,425.	3 p. m	113.
4 a. m	113 and 9,425.	4 p. m	113 and 9,425.
5 a. m	113.	5 p. m	113.
6 a. m	113.	6 p. m	113 and 9,425.
7 a. m	113.	7 p. m	113 and 9,425.
8 a. m	113 and 9,425.	8 p. m	113.
9 a. m	113 and 9,425.*	9 p. m	9,425.*
10 a. m	113 and 9,250.	10 p. m	113 and 9,250.
11 a. m	9,425.*	11 p. m	9,425.*

*Signals so marked may be sent via NSS (Annapolis, Md.).

(3) All U. S. Naval time signals are broadcast in the following standard manner:

The signals begin 5 minutes before the hour and consist of a dash on each second, except that there is no signal on the 29th second of any minute nor on certain seconds at the ends of the minutes, as shown in the following table:

Minute	Seconds										
	50	51	52	53	54	55	56	57	58	59	60
55	–		–	–	–	–					–
56	–	–		–	–	–					–
57	–	–	–		–	–					–
58	–	–	–	–		–					–
59	–										

The dashes in the above table indicate seconds on which signals are transmitted. The dash on the beginning of the hour (shown on 59 minutes 60 seconds above) is much longer than the others. In all cases the exact second is denoted by the beginning of the dash, the end being without significance. It will be noted that the number of dashes sounded immediately following the single second omission and preceding the long omission at the end of each minute indicates the number of minutes of the signal yet to be sent. For instance, the signal for 56 minutes and 52 seconds is omitted and then 3 seconds are sounded, indicating that 3 minutes of the signal remain to be transmitted. These time signals, if received directly and automatically, are seldom in error by as much as 0.20 second. The average error is less than 0.030 second. This is well within the most rigid requirement of accuracy for orientation.

(4) Time signals cannot be received directly on commercial radios except within a few hundred miles of station NAA, Washington, D. C. However, any radio sergeant can build a small long-wave regenerative radio set for their reception. Some commercial radio broadcasting stations broadcast time signals, but the accuracy of these signals is not sufficiently dependable for our use.

(5) The Western Union Telegraph Co. furnishes time signals once a day to all its offices in the United States. These signals are transmitted on a direct wire from the Naval Observatory to a master clock in the Western Union Telegraph Office at Washington, D. C. The signals are then transmitted from this clock to all parts of the country by telegraph at noon, E. S. T. Reliable standard time can therefore be obtained from any Western Union Telegraph Office within 2 or 3 hours after the signals have been received.

(6) Accurate time also can be determined by using a chronometer if one is available. In order to avoid confusion in reading the correct time, a watch should be set so that the minute hand reads the even minute when the second had is at 60, regardless of the amount the watch may be fast or slow within that minute. The correction in seconds should then be noted and applied to the "watch time of observation." A good 17-jewel watch can be adjusted so that its errors will not exceed 5 seconds in 24 hours. A 21-jewel railroad watch can be adjusted so that its error will not exceed 3 seconds in 24 hours.

h. Determination of Greenwich civil or Greenwich mean time (GCT or GMT).—Used in all azimuth determinations, *GCT* is the reading of the standard clock at Greenwich at the moment the observation is taken at any place. To obtain the *GCT* of the observation, it is necessary only to correct the standard time of observation by the time difference between the standard time zone in which the observation was made, and Greenwich. If the observation was after noon, add 12 hours to convert to the 24-hour clock. Also add (or substract) the number of hours the *ST* is earlier (later) than *GCT*. If the result is over 24 hours, drop that amount, and add 1 day to the date.

For example, suppose that an astronomical observation had been taken at 6:20:00 p. m. at 78°39′00″ *W* longitude on December 11, 1941. Determine the *GCT* of the observation.

		m	*s*
EST of observation p. m. Dec. 11, 1941	6	20	00
Correction to 24-hour clock	+12	00	00
Correction for 75° *W* longitude time zone	5	00	00
GCT of observation Dec. 11, 1941	23	20	00

i. Determination of Greenwich apparent time (GAT).—This consists of converting the *ST* of observation to *GCT* of observation as in *h*

above and correcting the *GCT* by the equation of time to obtain *GAT*. Find the time difference, *EQT* with sign, between the real and the fictitious suns at the beginning of day of *GCT* of the observation in the ephemeris, or in the extract of same given in (1) above. The tabular *EQT* has to be corrected for the change during the time between 0^h and *GCT* by the calculation:

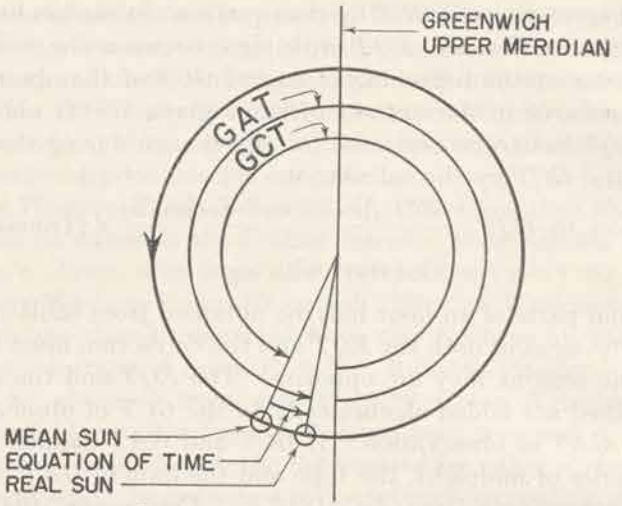
$$\pm\text{correction to } EQT = \frac{GCT \text{ (hours and decimals)}}{24} \times (\text{Tabular change for day, with sign})$$

The decimal parts of an hour may be obtained from table XLV, TM 5–236. The signs of both the *EQT* and the correction must be shown, for at some seasons they are opposite. The *EQT* and the correction thus obtained are added algebraically to the *GCT* of observation, resulting in *GAT* of observation. If *GCT* and *GAT* happen to fall on opposite sides of midnight, the time and the date of *GAT* have to be changed accordingly (see fig. 134①). Continuing the example from *h* above:

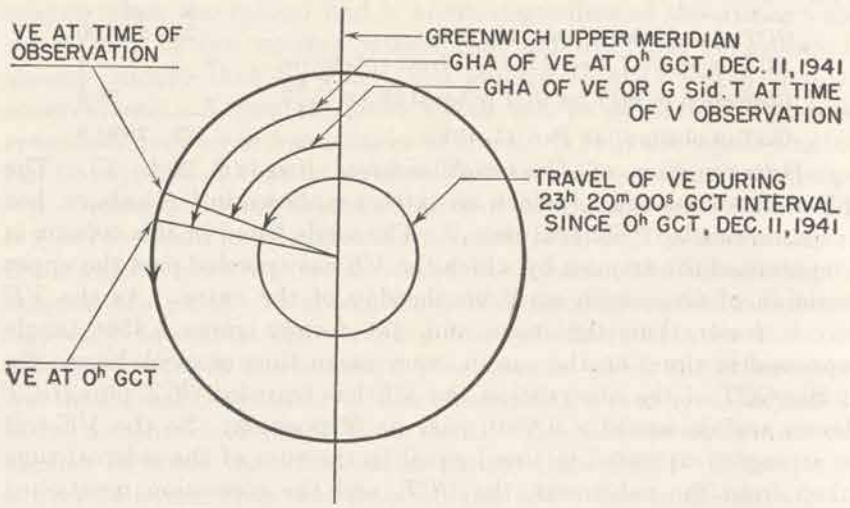
		h	m	s
GCT of observation Dec. 11, 1941		23	20	00
EQT for 0^h *GCT* Dec. 11, 1941, from ephemeris	+		7	1.1
Correction to *EQT* for *GCT* interval $23.3 \times \frac{27.62}{24}$	−			26.8
GAT of observation Dec. 11, 1941		23	26	34.3

j. Determination of Greenwich sidereal time (*G Sid. T*).—The ephemeris of the sun, of which an extract is shown in *b* (1) above, has a column headed "Sidereal time." The angle listed in this column is a measure of the amount by which the *VE* has traveled past the upper meridian of Greenwich at 0^h on the day of the entry. As the *VE* travels faster than the mean sun, the former gains 9.856^s (angle expressed in time) on the sun in every mean time or clock hour. So at the *GCT* of the observation the *VE* has traveled *GCT* plus [*GCT* (hours and decimals) $\times 9.856^s$] past its 0^h position. So the *VE* will be at angle (expressed in time) equal to the sum of the sidereal time taken from the ephemeris, the *GCT*, and the correction mentioned above. The result represents the hour angle of the *VE*, with respect to the upper meridian of Greenwich at the time of the observation, and is the *G Sid. T* of the observation (see fig. 134②). Again continuing the example of *h* above:

	h	m	s
GCT of observation Dec. 11, 1941	23	20	00
Sidereal time at 0^h *GCT*, from ephemeris	5	17	15.4
Gain of *VE* during 23^h20^m, from table III, ephemeris		3	50.0
G Sid. T. of observation	28	41	05.4
Subtract 24 hours	24	00	00.0
G Sid. T of observation Dec. 11, 1941	4	41	05.4

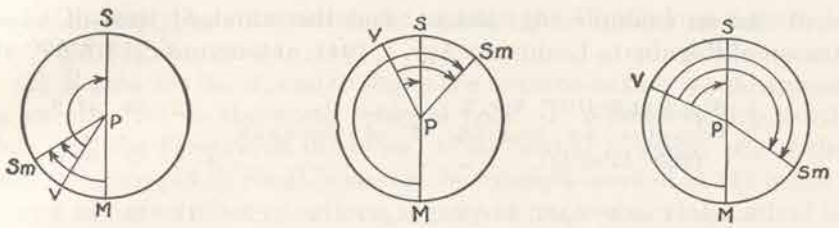


① *GAT* from *GCT*, showing *EQT*.



③ *G Sid. T* at time of observation.

FIGURE 134.—Determination of time.



③ Sidereal interval of star at upper transit after mean 0^h.
Angle $VPS=RA$ of star (and of meridian).
Angle $VPSm=RA$ of mean sun.
Angle $MPSm$=sidereal interval.

④ Sidereal interval of star at upper transit before mean 0^h, first case.

⑤ Sidereal interval of star at upper transit before mean 0^h, second case.

FIGURE 134.—Determination of time—Continued.

k. Time of upper transit of star.—(1) The right ascension of a star at upper culmination is the sidereal time at its upper culmination. The difference between the sidereal time at upper culmination of the star and the sidereal time at 0^h local mean time is the sidereal interval between the upper culmination and 0^h local mean time. It is always necessary to consider the relative positions of the vernal equinox, the star, and the mean sun in order to know whether the interval is before or after mean midnight. Figure 134 ③, ④, and ⑤ represents different combinations with respect to a star at upper culmination. Looking toward the North Pole, P represents the pole, PM the meridian, V the vernal equinox, and S the star.

(2) The sidereal interval before or after 0^h local mean time may be converted into a mean time interval which will give the local mean time of upper culmination. This in turn may be converted into standard time. The mean time interval between upper and lower culmination is one-half the mean time interval between two successive upper culminations. This varies slightly, but is approximately $11^h58^m02^s$. The times of culmination are independent of the latitude of the place.

(3) As an example (fig. 134③), find the standard time of upper transit of Regulus (α Leonis) on Apr. 1, 1941, at longitude 91°15′00″ W.

		h	m	s
1. *Sid. T* at 0^h *GCT*, Apr. 2		12	39	47.2
2. Correction for longitude *W* of Greenwich (6^{h}5^m) table III	+		1	00.0
3. *Sid. T* at 0^h *LMT* Apr. 2		12	40	47.2
4. *RA* of Regulus, Apr. 1	−	10	5	15.5
5. Sidereal interval at culmination before 0^h *LMT* (3−4)		2	35	31.7
6. Correction *Sid. T* to mean time interval, table II	−		0	25.5
7. Mean time interval before 0^h *LCT*, Apr. 2 (5−6)		2	35	06.2
8. *LCT* of culmination, Apr. 1 (p. m.) (12 hours−7)		9	24	53.8
9. Correction for longitude from 90th meridian	+		5	00.0
10. *CST* of upper culmination (p. m.) (8+9)		9	29	53.8

166. Determination of hour angle and declination.—*a. Coordinates* are provided for stars somewhat similar to the geographic coordinates for points on the surface of the earth. The declination of a star (or the sun) is measured in degrees, minutes, and seconds of arc north (plus) or south (minus) of the celestial equator, as the terrestrial latitude of a point is measured from the earth's Equator. The terrestrial longitude of a point as measured from the meridian of Greenwich is similar to the right ascension of a star, except that the latter is measured always eastward, and in hours, minutes, and seconds of time.

b. (1) The hour angle of a star depends on the sidereal time and the right ascension of the star. When a star is on the meridian of any point its hour angle is zero and its right ascension is the same as the right ascension of the meridian. When a star is not on the meridian its hour angle is measured from the meridian westward to the star and its right ascension is measured from the vernal equinox eastward to the star. The right ascension of the meridian (or the sidereal time) is measured from the vernal equinox eastward to the meridian. It is therefore evident that at any place and instant *the hour angle of a star is always equal to the local sidereal time at that instant minus the right ascension of the star.* Working from Greenwich meridian, *G Sid. T* is most easily determined, and *GHA* of the star is found by deducting the *RA* of the star from *G Sid. T.* The *GHA* is converted

to *LHA* simply by subtracting (or adding) the *W* (*E*) longitude from the *GHA*.

(2) Figure 135 ①, ②, and ③ shows the heavens as they would appear to an observer at the south celestial pole. *P* represents the North Pole, *PG* the Greenwich meridian, *V* the vernal equinox, and *S* the star. Figure 135 ① roughly shows the example worked in (4) below; ② and ③ depict other situations, merely to prove that this method is applicable to all stars and times.

(3) *To determine the LHA of a star*, calculate the *G Sid. T* of the observation (par. 165*j*); subtract from that the *RA* of the star obtained from the ephemeris, thus obtaining the *GHA* of the star at time of observation, which is corrected to the *LHA* by subtracting west or adding east longitude. This procedure applies to all stars but not to the sun, which is discussed in *d* below.

(4) Suppose, for example, that an observation had been taken on Altair at 6:20 p. m. at 78°39′ W. longitude on December 11, 1941. The first step is to find *G Sid. T* of the observation, which was determined in paragraph 165*j* to be $4^h41^m05.4^s$.

Extract from ephemeris (p. 316), apparent places of stars

α Aquilae (*Altair*)								
Greenwich upper transit	Right ascension				Declination			
	h	m	s		°	′	″	
Dec. 6. 6	19	47	55. 73	4	+8	42	61. 0	14
16. 6	19	47	55. 69		8	42	59. 6	

Interpolating in the 1941 ephemeris, or in the above extract, the *RA* of Altair for December 11, 1941, is $19^h47^m55.7^s$; the tabular difference is so small that interpolation only to the nearest date is necessary. The *GHA* is obtained, and reduced from time to arc by table XIX, TM 5–236. Subtracting the west longitude of the observer, the *LHA* is found as shown:

	h	m	s
G Sid. T at time of observation, Dec. 11, 1941	4	41	05. 4
RA of Altair for Dec. 11, 1941	−19	47	55. 7
GHA of Altair at time of observation	8	53	09. 7
	°	′	″
GHA of Altair in arc (table XIX, TM 5–236)	133	17	26
	h	m	s
Longitude of observer (*W*)	−78	39	00
LHA of Altair at time of observation, Dec. 11, 1941	54	38	26

c. The declination of Altair for the moment of observation is interpolated from the ephemeris for the nearest date, as the tabular difference is only 1.4 seconds for 10 days. For the evening of December 11, 1941, the declination of Altair is +8°43′00″.

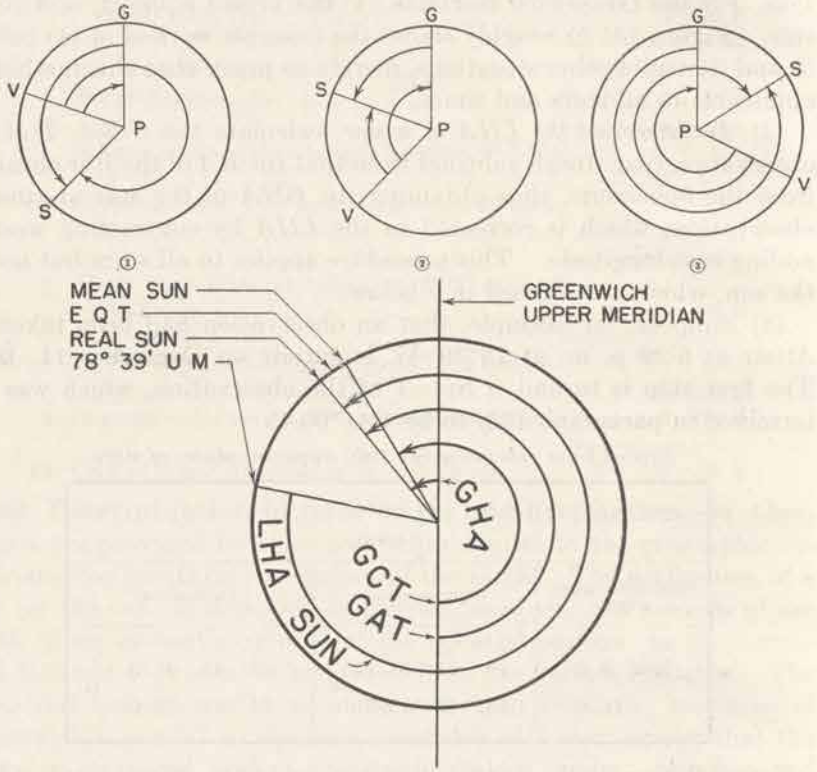


FIGURE 135.—①, ②, and ③, right ascension and hour angle as determined by reference to vernal equinox and meridian.
Angle $VPG = RA$ of Greenwich meridian.
Angle $VPS = RA$ of star.
Angle $GPS = VPG - VPS$ = hour angle of star.
④, *GHA* and *LHA* of the real sun.

d. Determination of LHA of the sun.—(1) The first step is to find *GAT* of the observation (par. 165*i*). Figure 135④ shows *GCT*, *GAT*, the positions of the real and mean suns, and the meridians of Greenwich and 79°39′ *W* longitude for the example in (2) below. *GAT*—12 hours is the *GHA*, and subtracting the *W* longitude from the *GHA*, expressed in arc, gives the *LHA* of the sun at time of observation.

(2) For example, suppose that an observation is taken on the sun from 78°39′00″ *W* longitude at 9: 00: 00 a. m. *EST* on December 11, 1941. (See extract of ephemeris, par. 165*b*(1).)

	h	m	s
EST of observation AM, Dec. 11, 1941	9	00	00
Correction for 75° *W* longitude time zone	5	00	00
GCT of observation	14	00	00
EQT for 0^h *GCT*, Dec. 11, 1941, from ephemeris	+	7	1.1
Correction to *EQT*, $14\times\frac{27.62}{24}$	−		16.1
GAT of observation	14	6	45.0
Subtract 12 hours	12	00	00.0
GHA of sun at time of observation	2	6	45.0
	°	′	″
GHA of sun (arc) by table XIX, TM 5–236	31	41	15.0
	h	m	s
W longitude	−78	39	00.0
LHA of sun at time of observation, Dec. 11, 1941	313	02	15.0

e. Declination of sun.—From the *Ephemeris of the Sun*, the declination is −22°57′5.1″ at 0^h on December 11, 1941, and is increasing negatively at the rate of 299.8^s per day. The declination at the time of observation is calculated:

	°	′	″
Declination at 0^h *GCT*, Dec. 11, 1941	−22	57	5.1
Correction for 14 hours *GCT* $14\times\left(\frac{-299.8}{24}\right)$	−	2	54.9
δ at time of observation, Dec. 11, 1941	−23	00	00.0

167. Astronomic triangle.—*a. Definition.*—Most astronomic computations are based on the solution of the *astronomic triangle*. For any celestial body the astronomic triangle is a spherical triangle the vertices of which are the *celestial pole*, the *zenith*, and the *celestial body*. In the Northern Hemisphere the north celestial pole is always used even if the celestial body is south of the Equator. The sides of the astronomic triangle are arcs of great circles passing through each pair of vertices mentioned above. In figure 136, *P* represents the north celestial pole, *Z* the zenith, *S* the celestial body, and *O* the observer. The astronomic triangle is the spherical triangle *PZS*. It is considered to be the triangle with the elevated pole.

b. The *elements* of the astronomic triangle are the sides and angles of the triangle. Since the arcs *PQ*, *JZ*, and *MP* each equals 90°, and the arcs *QZ*, *JS*, and *SM*, respectively, represent the latitude,

altitude, and declination, the elements of the triangle have the following significance:

Side $\begin{cases} PZ = 90° - \phi, \text{ or the colatitude} \\ SZ = 90° - h, \text{ or the zenith distance } (z) \\ PS = 90° - \delta, \text{ or the polar distance } (p) \end{cases}$

Spherical angle $\begin{cases} A = \text{the azimuth of the celestial body} \\ t = \text{the hour angle of the celestial body} \\ q = \text{the parallactic angle} \end{cases}$

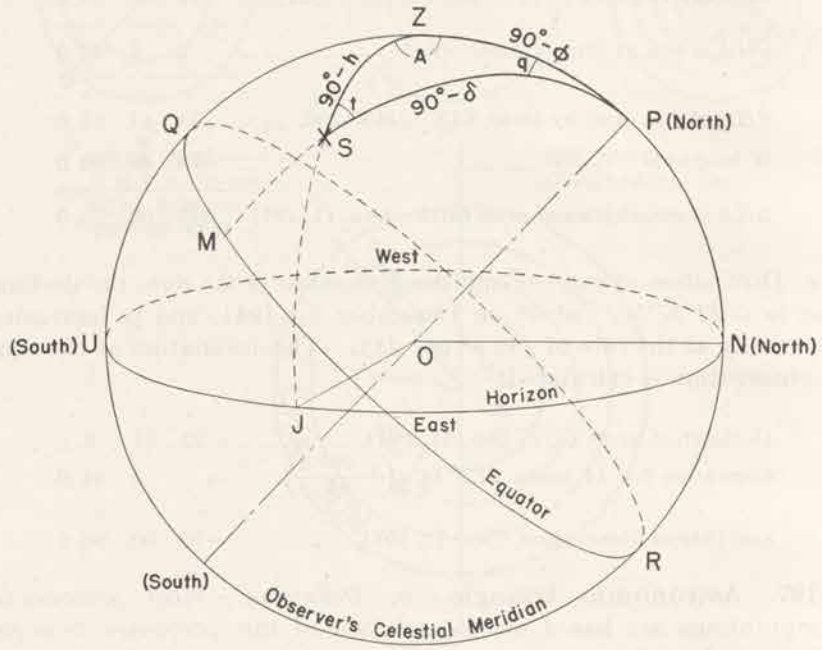


FIGURE 136.—Astronomic triangle.

The sign of the declination must be taken into account, as the polar distance is the arithmetical sum, $90° + \delta$, in case of a negative declination. Azimuth (A) is here used in the special sense, meaning the bearing east or west of the pole from 0° to 180°. The spherical angle at P is the hour angle (t) of the body if the body is west of the meridian, or is $360° - t$, if the body is east of the meridian.

c. Solution of triangle.—According to the principles of spherical trigonometry, if any three elements of a spherical triangle are known the other three can be found. Therefore, the azimuth of a celestial body may be computed if the *latitude* of the place, the *declination* of the body, and any one of the other three elements are known. For the ordinary azimuth determination, the latitude of the place may

be taken with sufficient precision from any good map. The declinations of all of the principal celestial bodies are given in the ephemeris. Consequently, two of the required elements are always known. The parallactic angle (q) cannot be measured from the earth; consequently, the *azimuth problem resolves itself into measuring either the hour angle or the zenith distance.* These two methods of azimuth determination are called the hour-angle method and the altitude method.

d. Astronomic notation.—With few exceptions the notation given below is that generally employed in astronomic work and is used in this text and in the ephemeris:

GHA=Greenwich hour angle.
LHA=Local hour angle.
EST=Eastern standard time.
GCT=Greenwich civil (or mean) time.
GAT=Greenwich apparent time.
LCT=Local civil (or mean) time.
LAT=Local apparent time.
EQT=Equation of time.
$Sid.\ T$=Sidereal time.
$G\,Sid.\ T$=Greenwich sidereal time.
VE=Vernal equinox.
RA=Right ascension.
0^h-=Midnight beginning date.
ϕ=Latitude of place of observation.
λ=Longitude of place, measured from meridian of Greenwich.
δ=Declination of celestial body.
h=Altitude of celestial body.
t=Hour angle of celestial body.
A=Azimuth of celestial body (east or west of north).
z=Zenith distance of celestial body (or $90° - h$).
p=Polar distance of celestial body (or $90° - \delta$).
q=Parallactic angle.
$s=\frac{1}{2}(h+\phi+p)$.
°=Degrees.
′=Minutes of arc.
″=Seconds of arc.
^{h}=Hours.
^{m}=Minutes of time.
^{s}=Seconds of time.

e. Astronomic formulas.—(1) The following formulas are used in connection with azimuth determinations. The choice of a formula should depend on the precision with which the angle is defined by the trigonometric function. If the angle is very small, it is more accurately found through its sine than through its cosine. If the angle is near 90°, the reverse is true. On account of the rapid variation of the tangent, an angle is always more accurately determined by use of this function than by use of either the sine or cosine. The formulas employed in the forms of sections XXIX and XXX have been chosen as those best fitted to the general case.

(*a*) *To find azimuth, given latitude, declination, and local hour angle.*

$$\tan\frac{1}{2}(A-q)=\frac{\sin\frac{1}{2}(\phi-\delta)}{\cos\frac{1}{2}(\phi+\delta)}\cot\frac{1}{2}t$$

$$\tan\frac{1}{2}(A+q)=\frac{\cos\frac{1}{2}(\phi-\delta)}{\sin\frac{1}{2}(\phi+\delta)}\cot\frac{1}{2}t \qquad (1)$$

The algebraic sum of the values $\frac{1}{2}(A-q)$ and $\frac{1}{2}(A+q)$ equals azimuth from the north.

$$\tan A=-\frac{a\sin t}{1-b\cos t} \qquad (2)$$

where $a=\sec\phi\cot\delta$, $b=\tan\phi\cot\delta$, and A=azimuth east or west of north.

(*b*) *To find azimuth, given latitude, declination, and altitude.*

$$\cos\frac{1}{2}A=\sqrt{\frac{\cos s\cos(s-p)}{\cos\phi\cos h}} \qquad (3)$$

$$\tan\frac{1}{2}A=\sqrt{\frac{\sin(s-h)\sin(s-\phi)}{\cos s\cos(s-p)}} \qquad (4)$$

where $s=\frac{1}{2}(h+\phi+p)$ and A=azimuth east or west of north.

(*c*) *To find azimuth, given local hour angle, altitude, and declination.*

$$\sin A=\sin t\sec h\cos\delta \qquad (5)$$

where A=azimuth east or west of north.

(*d*) *To find azimuth of a star at elongation.*

$$\sin A = \cos \delta \sec \phi \tag{6}$$

where A=azimuth east or west of north.

(*e*) *To find altitude, given latitude, declination and local hour angle.*

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t \tag{7}$$

(*f*) *To find altitude of a star at elongation.*

$$\sin h = \sin \phi \csc \delta \tag{8}$$

(*g*) *To find local hour angle, given latitude, declination and altitude.*

$$\cos t = \frac{\sin h - \sin \phi \sin \delta}{\cos \phi \cos \delta} \tag{9}$$

(*h*) *To find local hour angle of a star at elongation.*

$$\cos t = \tan \phi \cos \delta \tag{10}$$

(*i*) *To find local hour angle and azimuth of a star on the horizon.*

$$\cos t = -\tan \phi \tan \delta \tag{11}$$

$$\cos A = -\sec \phi \sin \delta \tag{12}$$

where A=azimuth east or west of north.

(*j*) *To find local hour angle and altitude of a star at transit over prime vertical.*

$$\cos t = \tan \delta \cot \phi \tag{13}$$

$$\sin h = \sin \delta \csc \phi \tag{14}$$

(2) Formula (1) is used in the hour angle method for sun and stars; (2) is used frequently for hour angle observations on circumpolar stars; (3) is preferred for the altitude method, as results are identical with those from (4) and the computation is somewhat simpler; (6) and (8) may be useful in preparing to observe circumpolar stars. The others may be of use in special cases occurring too infrequently to warrant detailed discussion. Some formulas, better adapted to machine computation, have been omitted. In all of the forms, five-place logarithms are used, as complete interpolation will give results correct to the second, except when all the data and computations are in tenths of minutes.

168. Comparison and choice of methods.—*a. General information on astronomic observations.*—(1) Before undertaking any survey operations, the observer and his recorder must have enough practice to gain some skill in their work, to form certain habits of procedure which will insure accuracy in the results, and to learn to choose the method most suitable to the work at hand. Signals, targets, and rods are immovable while being observed, but celestial bodies appear to move, often quite rapidly. This makes the pointing more difficult, and in some programs the exact time of centering has to be read and recorded. The use of the prismatic eyepiece and working at night seem strange at first. The computations are perhaps nore varied than in triangulation, and often seem rather abstract to the beginner. However, all of these difficulties are completely overcome through knowledge and experience.

(2) Before taking the field, the following have to be prepared or arranged in advance:

Adjusted instrument.
Known error of watch.
Flashlights, if at night.
Ephemeris and TM 5–236 (tables).
Notebook and blank forms.
Descriptions of stations to be occupied and observed.
Signals or signal lamps to be posted.

(3) The instrument should be in good adjustment. The plate bubbles should have special attention as the only error not overcome by the combination of direct and reversed readings is the deviation of the vertical axis from the plumb line. Some observers prefer to rely upon the striding level or telescope level for the final leveling of the instrument. In most of the programs the instrument cannot be releveled during a set of observations (3 D/R). To allow it to become acclimatized, the instrument should be set up, centered, and leveled at least 20 minutes before observations are to begin. The tripod feet should be located so as to be least in the way during the observations. For best results, a surveyor's umbrella should protect the instrument from the hot sun. As most of the instruments not of the Wild type have no interior illumination for the cross hairs at night, this is accomplished by having an assistant direct a flashlight, held at such distance as to secure the desired intensity, at a piece of white paper inside of and extending beyond the sunshade. This flashlight must be shielded from the observer's eyes.

(4) The watch should be preferably a large-sized, 21-jewel railroad watch, with a large second hand. The error to the nearest second is

easily obtained from radio time signals. The minute hand should be set to the nearest minute, and in exact correspondence with the second hand, reading the even minute when the second hand is at 60. No attempt should be made to set the second hand. The recorder does not attempt any computations during observation. He is responsible that the observer follows the prescribed program, for immediately requesting a check on any readings which seem discordant, and for reading the second of each observation where a space is provided. When the observer calls, "Ready," the recorder starts counting the seconds on the watch to himself. When the observer calls, "Tip," at the instant of centering, the recorder immediately records the time and is then ready to repeat and record the circle readings reported by the observer.

(5) A notebook should be carried at all times, in case the desired blank forms are not at hand. The forms, of which filled out samples are shown in the next two sections, have spaces for recording the observations. It will be noticed that most of the azimuth observations are to be made in three separately computed sets, each of 3 D/R. If these do not provide at least two sets giving a close check, the observations should be repeated. With the Wild type of theodolite, each of the three sets may be reduced to only 1 D/R, as this will provide more accurate readings than 3 D/R on most of the instruments. In either case, the sets should be begun on different zeros about 60° apart. The recorder is responsible for advising the observer when to reverse, etc. It will be noticed that space is left for recording only one vernier reading. This is sufficient for the intended purpose, and time is saved.

(6) In all astronomic observations, the telescope must be focused for infinity so as to give the sharpest possible image of the celestial body. As the focus should not be changed during the observations, the signal or mark light should not be less than 2 miles distant from the observer, but may be brought to a half-mile distance if more is impracticable, as accurate centering is possible, even on a slightly blurred image. For accuracy in pointing, the terrestrial object should subtend between 2 and 4 seconds at the instrument. A signal box containing a flashlight or bulb and dry cells should be prepared, having an adjustable vertical slit or series of slits of varying width, one of which can be accurately plumbed over the mark while the light is directed exactly toward the instrument. If such a box is not at hand, a bare flashlight bulb may be made to serve. If the light seems too bright, the intensity may be reduced at the instrument by placing a piece of thin transparent paper in front of the objective.

b. Choice of method.—(1) While the sun is up, it is the only practicable celestial body available. For several reasons pointed out in paragraph 169, the sun is more difficult to observe than are the stars. However, if all of the precautions given in that paragraph can be taken, results from the sun can be surprisingly good, and well within military requirements. With the sun below the horizon, Polaris can be observed. The infinity focus must be exact, and a black cylinder of paper half a foot long secured around the sunshade with a rubber band helps in picking up the star. The approximate azimuth and altitude must be known and computed and set on the circles. Once the star is in the field, the direction may be marked by noting a landscape feature on line, and only the altitude need be set for further pointings.

(2) *Polaris* is the most useful celestial body for many purposes. Its position and appearance are known to nearly every one. Through the telescope, no other star of comparable magnitude is in the field.

(3) Other stars can be used for obtaining astronomic data when Polaris is obscured. In fact, by means of some programs or combinations, more accurate data may be obtained from them than from Polaris. A careful study of the heavens should be made in conjunction with the star charts (figs. 138, 139, and 140), so that at least a few of the more useful stars may be indentified on sight, no matter how small the area of the sky actually visible may be.

(4) The purpose of military astronomic observations must have full consideration in the choice of method. They are as follows:

(*a*) Latitude and longitude to establish an initial point for starting a survey in unmapped territory.

(*b*) Determination of time.

(*c*) Azimuths for checking computed azimuths in extensive systems of triangulation and adjusting traverse of third-order or greater accuracy.

(*d*) Azimuths for checking grid triangulation.

(*e*) Azimuths as a basis for assuming grid coordinates and azimuth.

(*f*) Azimuths for subsidiary purposes, such as checking traverses and aiding resection.

(*g*) Latitude for use in astronomic computations.

(*h*) Latitude to aid assumption of coordinates.

(*i*) Latitude for subsidiary purposes, such as the determination of position from latitude and azimuth and the interpolation of grid azimuth corrections.

(*j*) Longitude for (*g*), (*h*), and (*i*).

Method (*a*) requires special equipment and training not likely to be found in a topographic unit. Method (*b*) is no longer needed, since radio time signals are so accurate and frequent. While azimuths of (*c*) order are easily obtained with the 1-second theodolites of the Wild type and the method of sections XXIX and XXX, ordinary instruments require additional refinements in both observation and computation to yield such results. Methods (*d*), (*e*), and (*f*) should all be satisfied by azimuths within one-half minute of the correct one. All of the methods described and computed on the forms will give closer results if skillfully executed. At the same time, the forms have been made as simple as possible in order to promote rapid completion of the work. However, the value of close results for (*d*), (*e*), and (*f*) will be easily appreciated, and opportunity should be made for improving the skill and technique required for these procedures. Methods (*g*), (*h*), and (*i*) can be fairly well fulfilled by the methods mentioned in *c* below. Method (*j*) is hard to accomplish by simple means. None will be described here. However, longitude is of less importance than latitude, for neither longitude nor time is needed for one of the most accurate methods of azimuth determination, altitude on east and west stars (par. 178*d*(2)). One point in regard to latitude and longitude cannot be over-emphasized. In determining the "Corrections for the reduction of geographic azimuths to grid azimuths" from the table in United States Coast and Geodetic Survey Special Publication No. 59, the interpolation must be as close as possible, which involves an accurate knowledge of geographic coordinates (par. 117*f*(5)).

(5) *Comparison of azimuth methods* may be tabulated as follows:

Method	Observation	Computation	Remarks
Sun altitude method.	Requires practice.	Very simple.	Reliable and accurate for skilled observer. Time within few minutes. Best on or near prime vertical altitude over 15°. At least 2 hours from apparent noon. No check except more sets or wait until afternoon.
Sun hour angle method.	Simpler than above.	Longer than above.	Best on or near prime vertical, but may be used when sun is visible and over 1 hour from apparent noon. Time within 1 second. No check except more sets or wait until afternoon.

Method	Observation	Computation	Remarks
Polaris at elongation.	Very easy	Almost none	Best method for beginner. Partially self-checking. Exact time not required. Elongation may be at inconvenient time, or only during daylight.
Polaris at any hour.	do	Easy, with tables.	Reliable when Polaris is visible. Time within 10 seconds. No check except more observations.
Other stars altitude.	Quite easy	Very simple	Need only the date of observation. Star should be on or near prime vertical. Requires some practice, as star moves rapidly in altitude. Check by taking two sets on E (or W) star and two on W (or E) star. Altitude over 15°.
Other stars hour angle method.	Simpler than next above	Longer than altitude method.	Time within 1 or 2 seconds. Star should be on or near prime vertical. Same check as for altitude method.
Circumpolar stars (except Polaris) altitude or hour angle.	Simple	Same forms (no special tables.)	Should be near elongation. No check but more observations. Computation better with special formulas but no forms are provided.

c. Observations for latitude.—(1) The latitude is to be calculated from the declination (δ) of a celestial body on the meridian obtained from the ephemeris for the moment of observation and the zenith distance (z), which equals 90° minus the observed altitude (h), corrected for refraction. The general equation is $\phi = z + \delta$, but the signs depend upon the position of the observed body.

The four cases in which the *arithmetical equations* change are shown below:

(a) For upper transit of body below Equator

$$\phi = z - \delta$$

(b) For upper transit of body between Equator and zenith

$$\phi = z + \delta$$

(c) For upper transit of body between zenith and pole

$$\phi = -z + \delta$$

(d) For lower transit of body

$$\phi = 180° - z - \delta$$

These relations are apparent in figure 132, and may be recalled when needed by making a hasty diagram.

(2) The altitudes may be observed on the following:

(*a*) Limb of sun at transit.

(*b*) Polaris at upper or lower culmination.

(*c*) Polaris at any time.

(*d*) Other stars on observer's meridian.

(*e*) Circummeridian altitudes.

The refraction should be corrected for pressure and temperature before being applied to the observed altitudes. The sun's apparent semidiameter is found in the ephemeris, and the transit is at apparent noon. Shortly before that time, get the limb of the sun on the center horizontal hair and keep it there until the image begins to move in the opposite direction; then read the vertical angle, which must be corrected for index error. The times for upper and lower culmination of Polaris are given in the ephemeris, table VII. The method of observing the altitude is similar to that described for the limb of the sun. It is evident that methods (*a*) and (*b*) are only approximations, as the single reading possible precludes even a D/R observation, except that the telescope should be quickly plunged and a reversed reading made as a rough check on the direct one.

(3) Method (*c*) is fairly accurate and fully explained with an example in the heading of table I, *American Ephemeris and Nautical Almanac*. With a vertical circle reading to 30 seconds or more, greater accuracy in the latitude can doubtless be obtained by rapidly making a set of 6 D/R, checking the centering of the collimation or vertical vernier bubble, and recording the time for each reading, finally meaning all of the vertical angles and the times in calculating the true altitude and the hour angle. No horizontal angles are necessary.

(4) Method (*d*), while consisting of single observations of stars on the observer's meridian, has one advantage over (*c*), in that an equal number of north and south stars can be observed, thereby balancing out any error of the refraction correction. The north and south stars should be in pairs at about the same altitude. The N observations should be alternately D and R, and the same for the S. Any stars which can be identified from the ephemeris can be observed, but the altitudes should be between 25° and 60°. The latitudes from each star are calculated separately and the mean latitude adopted. The time of transit of the first star to be observed can be calculated by the method explained in paragraph 165*k*. The approximate time for the others can be roughly calculated by comparing the right ascensions. Except in the Tropics, the limits of permissible declination will be materially different for the north and south stars.

(5) Method (*e*) is most accurate of all as it permits of observing 6 D/R on each star, but the computation is rather long and will not be explained here.

169. Observations on sun.—*a. General.*—Some general remarks on astronomic observations are contained in paragraph 168*a*. As most surveying is done by daylight, when observations on stars are impracticable, the surveyor should seize every chance to use observations on the sun until he is confident of obtaining a reliable azimuth at nearly any time the sun is visible. By following correct principles, good azimuths may be secured the first day, and the knack of making exact tangencies on the limbs of the sun will soon make possible results as close and reliable as those expected from the stars.

b. The most favorable position of the sun for either method of observation for azimuth is on or near the prime vertical, due east or west of the observer. The sun is then moving most slowly in azimuth, and the horizontal angles are certain to be closer to the true angles. This holds good for both the azimuth and hour angle methods as in the latter the instant of tangency can be more easily perceived and the effect of any error in time is minimized by the slowness of the movement.

c. The *prismatic eyepiece* with dark sunglass yields more accurate tangencies than can be obtained from the image intercepted on a card. Exact focus on a hand-held card is difficult to maintain. At the moment of tangency the image of the cross hair disappears from the field. Finally, in order to focus the image sharply on the card, the eyepiece has to be thrown out of focus with the attendant danger of residual parallax when sighting at the mark through the telescope. Every instrument utilized for solar observations should be equipped with a prismatic eyepiece. Before sighting at the sun, the dark glass must always be turned into place over the aperture, as even one sight at the sun through the clear telescope can be very harmful to the eye.

d. Pointing at the sun can best be accomplished by letting the sun make its own tangency. For the altitude method both the vertical and the *center* horizontal hairs have to be tangent to the sun's limbs simultaneously. Near the prime vertical the sun changes azimuth so slowly that it is comparatively easy to keep the vertical hair tangent to the limb of the sun while awaiting the sun, which is set in such relation to the horizontal hair that the limb will soon move into position tangent with the center hair, as shown in figure 137①. This same operation is repeated for all three of the direct pointings. For the three reversed pointings the sun must be placed in the *diagonally opposite* quadrant as shown in figure 137②. In this case the center

hair is placed so as to partially cover the sun's image, and the vertical hair is kept tangent to the other limb while the sun appears to move up and off the center hair. At another time of day the sun may appear to be moving in the opposite direction when it might be more convenient to make the three direct pointings as at (*c*) and the three reversed pointings as at (*d*). Which quadrant is used for the first three really does not matter, just so the quadrant diagonally opposite is used for the second three. This is one place where the usual precaution "to turn against the spring" is not observed. This is justifiable if it improves the tangencies, for they are not repetitious, but individual readings. Closer to the meridian, where the sun is moving faster in azimuth than in altitude, some observers prefer to keep the limb of the sun tangent to the center hair with the elevation tangent

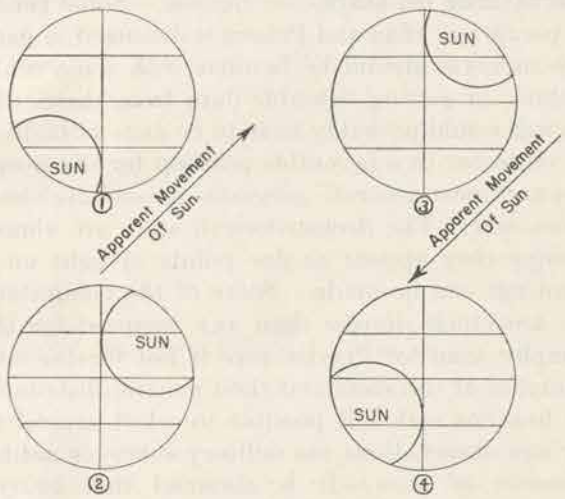


FIGURE 137.—Images of sun before tangency, altitude method.

screw, while the sun makes its tangency with the vertical hair. There is no objection to such procedure, though most observers can probably operate the upper plate tangent screw more smoothly, and with less tendency to pull the instrument out of position.

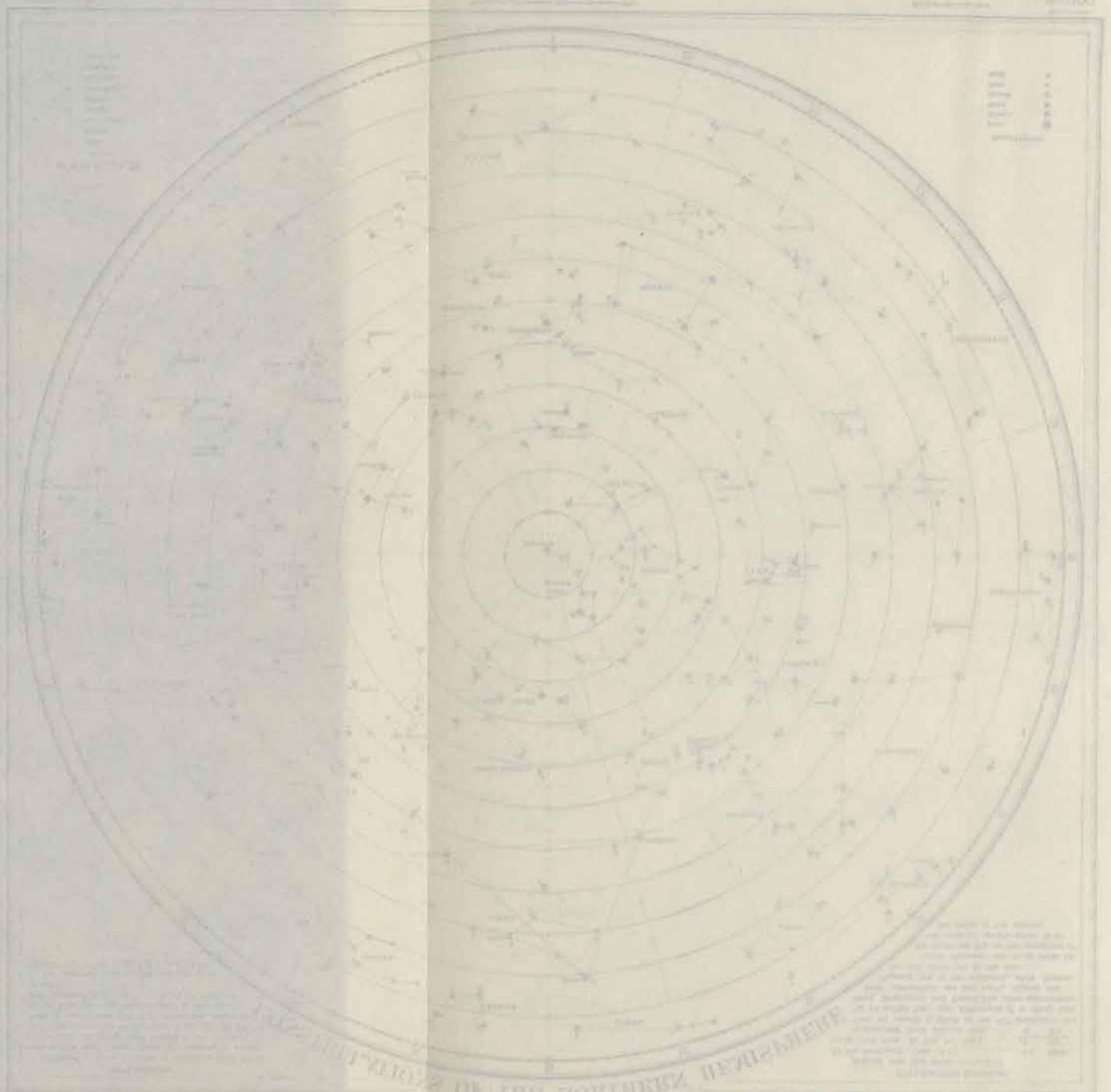
e. Refraction.—Any observed altitude, whether of the sun or a star, must be corrected for refraction. Table XXI, TM 5–236, computed for mean temperature and mean barometric pressure, gives mean values of this correction. In observations of a precise nature, all latitude and close azimuth observations, this mean value needs two small corrections, one for temperature and one for barometric pressure. These corrections are included in table XXI, TM 5–236. (Note that these are corrections to mean refraction and not to the altitude.)

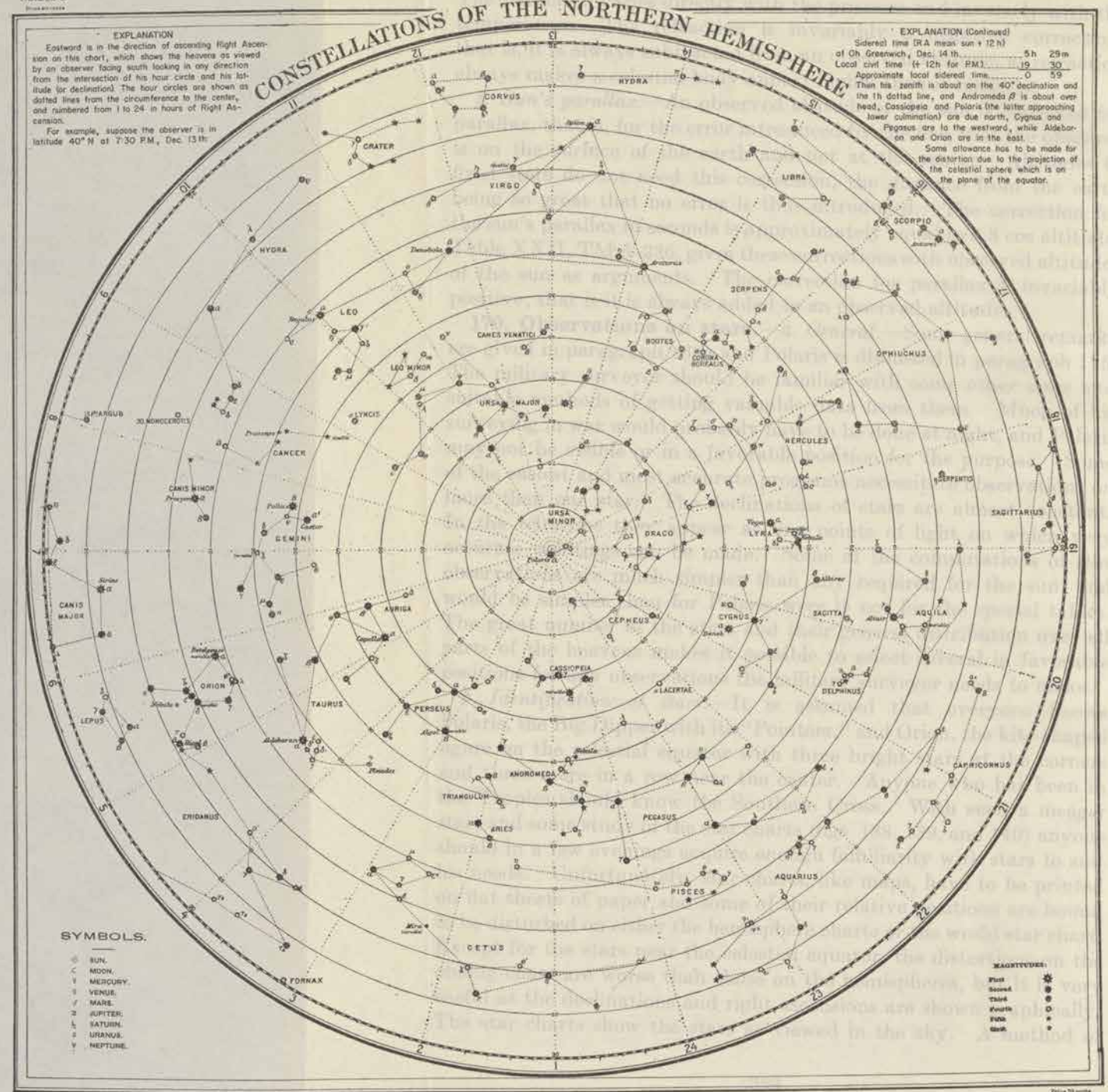
The refraction varies directly with the pressure and inversely with the temperature. The refraction is invariably a negative correction; that is, it is always subtracted from an observed altitude, as refraction always makes a celestial body appear higher than it really is.

f. Sun's parallax.—An observed altitude of the sun is corrected for parallax, that is, for the error introduced by the fact that the observer is on the surface of the earth and not at the center. Altitudes of fixed stars do not need this correction, the distance from the earth being so great that no error is thus introduced. The correction for the sun's parallax in seconds is approximately equal to 8.8 cos altitude. Table XXII, TM 5–236, gives these corrections with observed altitudes of the sun as arguments. The correction for parallax is invariably positive; that is it is always added to an observed altitude.

170. Observations on stars.—*a. General.*—Some general remarks are given in paragraph 168*a* and Polaris is discussed in paragraph 175. The military surveyor should be familiar with some other stars and suitable methods of getting valuable data from them. Much of his surveying in war would probably have to be done at night, and Polaris may not be visible or in a favorable position for the purpose. Some of the easiest and most accurate programs necessitate observations on more than one star. The declinations of stars are almost constant. In the telescope they appear as fine points of light on which very accurate pointings can be made. Some of the computations of star observations are much simpler than any required for the sun, and would be simpler than for Polaris were it not for the special tables. The great number of the stars and their general distribution over all parts of the heavens makes it possible to select several in favorable positions for any observations the military surveyor needs to make.

b. Identification of stars.—It is assumed that everyone knows Polaris, the Big Dipper with its "Pointers," and Orion, the kite-shaped figure on the celestial equator with three bright stars at the corners and three more in a row near the center. Anyone who has been in the Tropics should know the Southern Cross. With such a meager start and some study of the star charts (figs. 138, 139, and 140) anyone should in a few evenings acquire enough familiarity with stars to suit his needs. Unfortunately, star charts, like maps, have to be printed on flat sheets of paper and some of their relative positions are bound to be disturbed on either the hemisphere charts or the world star chart. Except for the stars near the celestial equator, the distortions on the oblong chart are worse than those on the hemispheres, but it is very useful as the declinations and right ascensions are shown graphically. The star charts show the stars as viewed in the sky. A method of





Washington, D.C., published Dec. 1936 by the Hydrographic Office, under the authority of the SECRETARY OF THE NAVY.
EDITION: 9th, Aug. 1938
Price 20 cents
No.2100

FIGURE 138.—Star chart, Northern Hemisphere.

CONSTELLATIONS OF THE SOUTHERN HEMISPHERE

No. 2101

Price 10 cents

CONSTELLATIONS OF THE SOUTHERN HEMISPHERE

EXPLANATION FOR USE OF CHART.

Mount the chart on a light drawing board. Pivot a string in the center and fit a loop in the end of the string outside of the circumference. This string represents the Meridian of the observer. Desiring to know where the different Heavenly bodies should be looked for at any time, plot the position of the Sun, Moon, and Planets by means of the Right Ascension and Declination taken from the Nautical Almanac. The position of the Sun will indicate at once what Stars and Planets will be above the Horizon and those below it at sunset, with certain modifications, due to a further consideration of latitude and declination.

Wishing to know what bodies are at or near the Meridian at a certain hour, say 7 p. m., get the Right Ascension of the Mean Sun for that time, add 7 hours, and we have the Local Sidereal Time or Right Ascension of the Meridian.

EXPLANATION FOR USE OF CHART (Continued).

Place the string in this position; it will then be seen at once what bodies are on or near the Meridian, as well as those lying to the eastward and westward of it. Plot the position of the Zenith by its Declination which is equal to the Latitude of the observer. Since the Horizon is 90° from the Zenith, the approximate altitude of any body on the Meridian may readily be seen. The position of the well-known bodies, the Sun and Moon will assist greatly in identifying unfamiliar Stars and the Planets, not easily followed in their ever-changing positions, can readily be recognized by their location in regard to well-known constellations, the Sun and Moon, or with reference to the Meridian at any time. The chart, being a projection of the fixed stars on the plane of the Equator, is eccentric to the Zenith of the observer in every case when not located at the Pole. Remember that eastward is always in the direction of the increasing Right Ascension.

SYMBOLS.

- ☉ SUN.
- ☾ MOON.
- ☿ MERCURY.
- ♀ VENUS.
- ♂ MARS.
- ♃ JUPITER.
- ♄ SATURN.
- ♅ URANUS.
- ♆ NEPTUNE.

MAGNITUDES:

- First
- Second
- Third
- Fourth
- Fifth
- Sixth

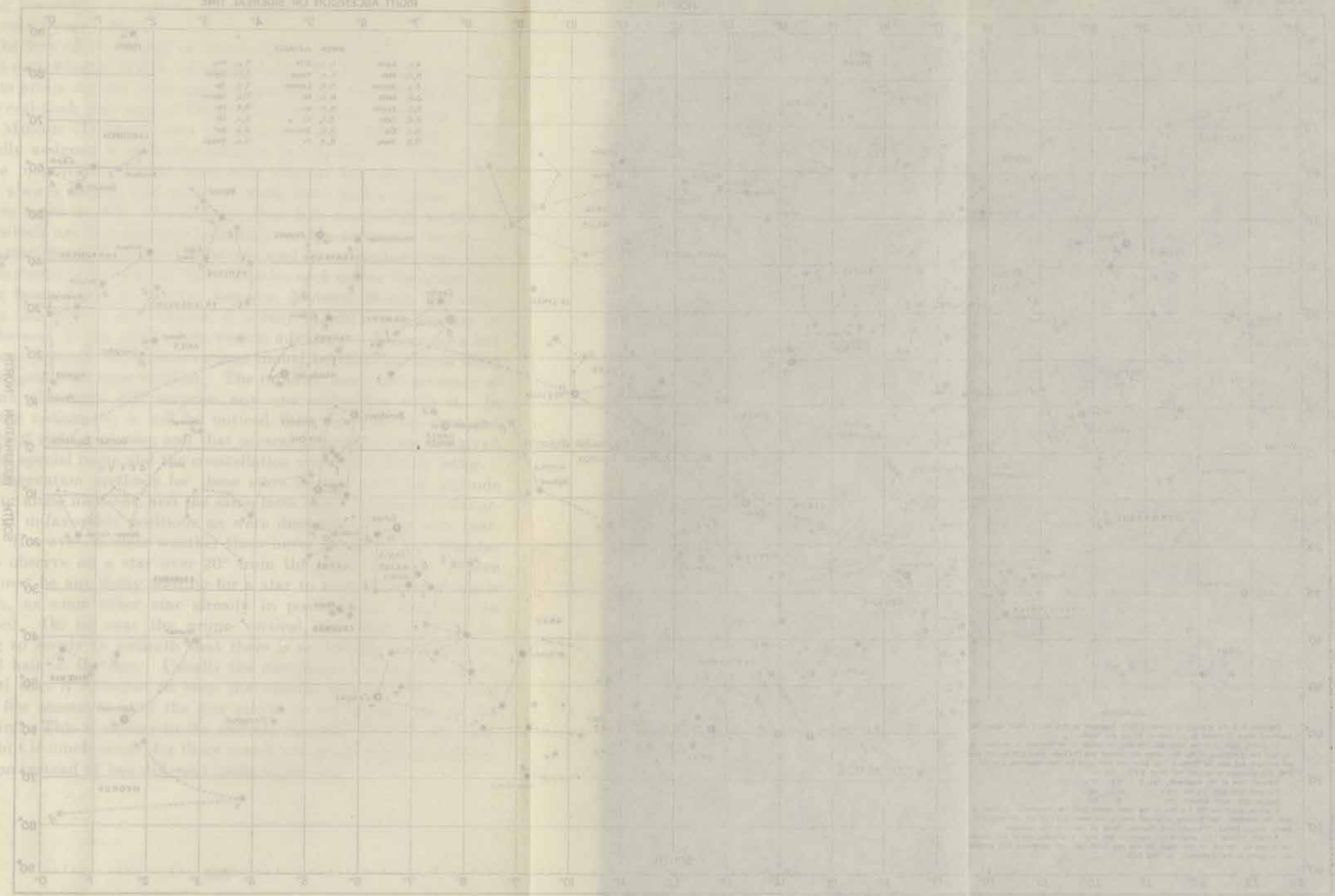
Washington, D.C., published Sept. 1900 at the Hydrographic Office, under the authority of the SECRETARY OF THE NAVY.

EDITION: 3d. Feb. 1918

Price 10 cents

No. 2101

FIGURE 139.—Star chart, Southern Hemisphere.



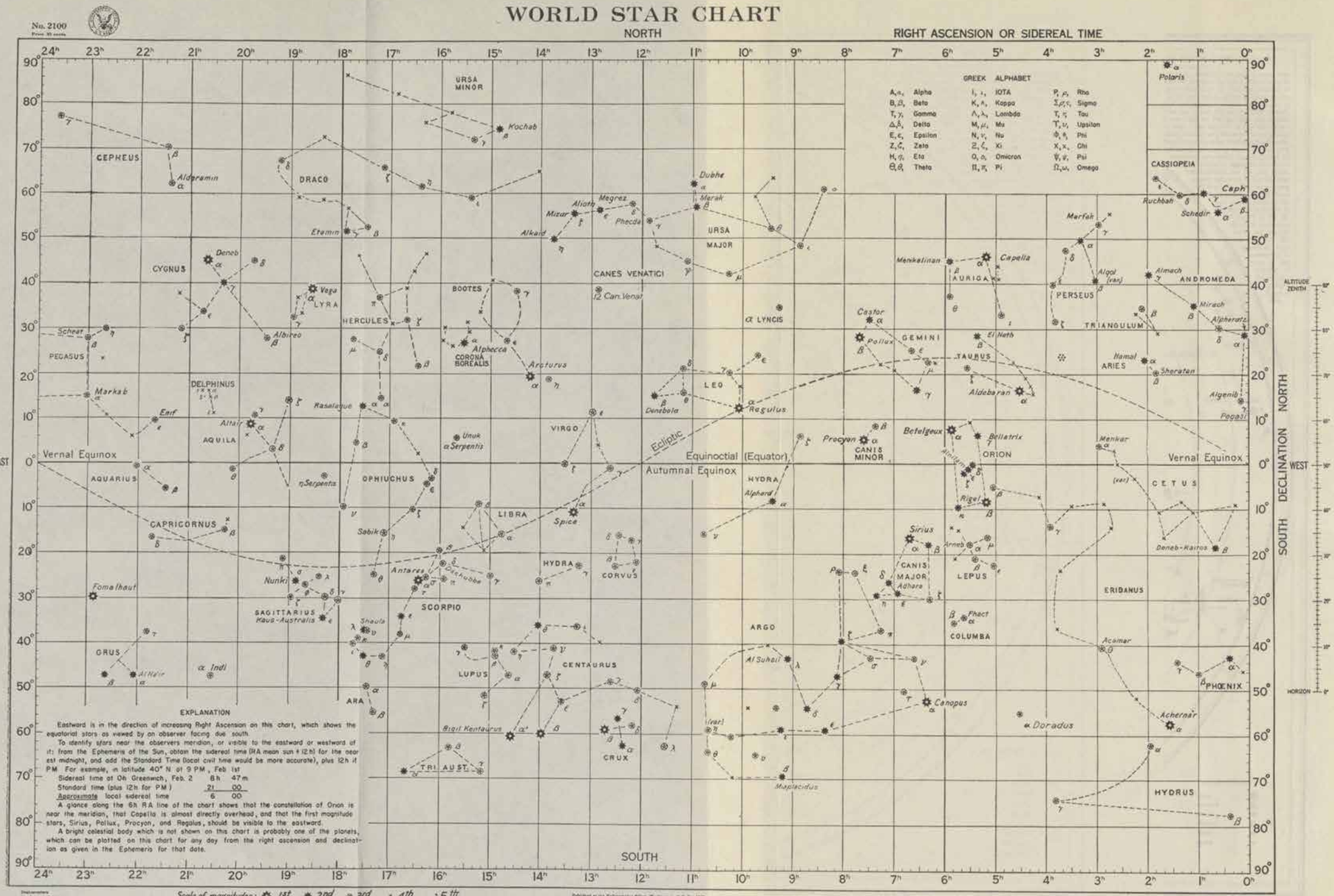


FIGURE 140.—World star chart.

locating the observer's meridian and zenith on the charts is explained in figures 138 and 140.

c. The first efforts should be concentrated on learning a half dozen stars in each 6 hours of *RA*, which would be useful for observing on or near the prime vertical or as east and west stars. For instance, there are several such just west of Orion forming a large pentagon with α Canis Minoris (Procyon) near the center. The Greek letters were originally assigned in each constellation to the stars in order of the relative magnitude. These were only relative but the α stars are nearly always bright, and many of them have special names. The stars are rated in order of brightness from first magnitude to fifth or sixth, which are the dimmest ordinarily visible without a telescope. On the star charts, the magnitude is indicated by conventional signs, and it also is shown at the head of the column for each star in the ephemeris. Having learned Castor, Pollux, Regulus, Alphard, Sirius, and other stars in this part of the heavens, it is easy to add others coming up from the east, as the night or the season advances. As soon as a few have been learned, some observations should be taken on them when they are near the prime vertical. The rapidity, ease, and accuracy of work on the stars will surprise any one unfamiliar with it. In using the ephemeris, it will be noticed that the stars are arranged in order of right ascension and that several important ones are given both the special name and the constellation name with Greek letter.

d. Observation methods for these stars include both the altitude and hour angle methods, and the same facts apply in regard to favorable and unfavorable positions as were described for the sun (par. 169*b*). However, in clear weather there never should be any temptation to observe on a star over 20° from the prime vertical. There never need be any delay waiting for a star to move into a favorable position, as some other star already in position can doubtless be identified. On or near the prime vertical, the stars seem to be moving so slowly in azimuth that there is no trouble in setting the vertical hair on the star. Usually the movement in azimuth will be so rapid that it is better to keep the vertical hair on the star and wait a few moments until the star moves to the intersection of the two hairs. This is similar to the method described for the sun (par. 169*d*), but is much easier, for there is only one point of light requiring attention instead of two different limbs of the sun.

SECTION XXIX

AZIMUTHS FROM OBSERVATIONS ON SUN

	Paragraph
Altitude method	171
Ageton method as applied to azimuth problem	172
Hour angle method	173
Equal altitude method	174

171. Altitude method.—*a. General.*—(1) In the altitude method of determining azimuths, the observation consists of measuring the horizontal angle between the mark whose azimuth is desired and the celestial body whose azimuth can be computed. The known elements of the astronomic triangle are—

(*a*) The side from the zenith to the pole, which is the complement of the latitude of the place of observation.

(*b*) The side from the pole to the celestial body (polar distance), which is the complement of the declination of the body.

The element obtained from observation is the side from the zenith to the celestial body (zenith distance), which is the complement of the observed altitude. As in the hour angle method, the desired element in the triangle is the angle at the zenith which is the azimuth of the celestial body.

(2) Following are two formulas suitable for the logarithmic computation of the angle at the zenith, either one of which may be used, although the first is simpler:

$$\cos \frac{1}{2}A=\sqrt{\frac{\cos s \cos (s-p)}{\cos \phi \cos h}} \qquad (1)$$

$$\tan \frac{1}{2}A=\sqrt{\frac{\sin (s-h) \sin (s-\phi)}{\cos s \cos (s-p)}} \qquad (2)$$

where

h=observed altitude, after all corrections have been applied.
ϕ=latitude of the station.
p=polar distance of the celestial body (90°−declination).
$s=\frac{1}{2}(h+\phi+p)$.
A=azimuth of the celestial body (less than 180°) from the north.

(3) Particular attention should be paid to securing the best available data. Roughly, an error of 1 minute in either latitude, declination, or altitude may cause a corresponding error of over half a minute in the azimuth. Time is less important here, because a difference of 5 minutes cannot change the azimuth over ⅒ minute. A set of $3D/R$

can be made in less than 10 minutes without hurrying, and the elapsed time should not be much more. All field notes and computations may be completed on form 22 (fig. 141).

b. Procedure in observing.—(1) Set up the transit over the station and level carefully. It is common practice to use the striding or the telescope level for greater accuracy.

(2) Attach the prismatic eyepiece and check the parallax adjustment. Lightly clamp the horizontal axis and leave it so until set is completed. Slip the dark glass over the prismatic eyepiece, and point the telescope at the sun. This can be done quickly by holding the hand behind the eyepiece and moving the telescope horizontally and vertically while watching the shadow on the hand. Focus until the sun's image is sharply defined. Watch the sun for a moment and decide in which quadrants, *as seen by the observer through the eyepiece,* and which tangent screws should be used for the direct and reversed pointings (par. 169*d*). Have the recorder mark the quadrants as they are to be observed.

(3) Set the *A* vernier at 00, turn the dark glass away from aperture, and sight on the mark indicating the far end of the direction line with the lower motion. The lower motion is not touched again and the instrument cannot be releveled until the set is completed.

(4) Slide the colored glass over the prismatic eyepiece, unclamp the upper motion, and point the telescope (direct) at the sun.

(5) With the sun in the field of the telescope, clamp the upper motion. With both hands operating upper horizontal and vertical tangent screws simultaneously, set image of the sun into the selected quadrant and tangent to the vertical cross hair, and in position to make contact with the center hair by its own movement. Call, "Ready." Using only the horizontal motion, hold the vertical hair tangent to the sun while it moves toward the horizontal hair. (See fig. 137.) Just as the edge arrives at tangency the observer calls, "Tip," at which the recorder takes and records the time. The observer then reads to the recorder the *A* vernier and the vertical vernier, which are recorded.

(6) The operations of (5) are now repeated twice, except that the time is not taken on these readings.

(7) The telescope is reversed and a set of three readings on the *B* vernier taken as in (6) except that the sun's image is placed in the quadrant of the cross hairs diagonally opposite the quadrant used in (5) and (6). On the third reversed pointing at the sun the time is taken and recorded as in (5).

(8) The telescope is now plunged and sighted back to the mark direct. The vernier should again read 00.

(9) Read and record magnetic bearing to the mark. This completes one set of observations, which should be followed by two more sets immediately.

(10) The results may be roughly checked by testing the proportions of changes in horizontal and vertical angles between the first and second, second and third, fourth and fifth, and fifth and sixth pointings at the sun. If the observations are made rapidly and evenly, there will not be much variation in this relationship. Perhaps the simplest way is to record the four differences of horizontal and vertical angles in columns on scratch paper, ascertain the fractional change to bring each of the horizontal differences to 20, and apply these fractional changes to the respective vertical angles. This can be done mentally, and quickly gives a rough indication of any gross error which would require a repetition of the observations.

c. Computation.—The three sets are computed separately in order to secure an independent check on the work.

(1) Compute the mean values of the time of the first and last solar pointings, of the horizontal angles recorded, and of the vertical angles recorded.

(2) The mean of the vertical angles is corrected for refraction of light due to the earth's atmosphere and for parallax by values obtained from tables XXI and XXII, TM 5–236.

(3) The mean ST of the set is converted to GCT by adding 12 hours to p. m. observations, and the difference in time between Greenwich and the ST used.

(4) Obtain the sun's declination from the ephemeris for 0^h GCT. Correct the declination for the GCT of the observation.

(5) If δ is (+), the polar distance (P) equals $90°-\delta$; if δ is (−), $P=90°+\delta$.

(6) ϕ is taken from the best available map, if not otherwise known.

(7) $s-p$ is the arithmetical difference between s and p, and is always positive.

(8) The azimuth of the sun from the north is then derived by substituting in the formula

$$\cos \tfrac{1}{2} A=\sqrt{\frac{\cos s \cos (s-p)}{\cos \phi \cos h}}$$

The true azimuth of the sun is deduced from A.

(9) The mean horizontal angle between the mark and the sun must be subtracted from the azimuth of the sun to give the true azimuth of the line to the mark.

(10) The azimuth of the mark is then converted to true grid azimuth by applying the "Correction for the reduction of geographic azimuth to grid azimuth" interpolated in the table in United States Coast and Geodetic Survey Special Publication No. 59. The magnetic declination from grid north may be obtained by deducting the grid azimuth from the magnetic azimuth.

(11) The instructions which should appear on the form are given below.

AZIMUTH DETERMINATION—ALTITUDE METHOD

INSTRUCTIONS

Observation.—Before set, adjust parallax, focus sun sharply, choose quadrants for D and R. Do not touch lower motion, leveling screws, or horizontal axis clamp during set.

Computation.—Three sets are computed separately for check. Refraction and parallax from tables XXI and XXII, TM 5–236.

p. m.=add 12 hours for p. m. observations.
TZC=time zone correction to Greenwich time.

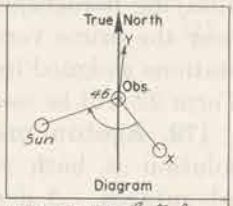
$$\delta \text{ correction } GCT \times \frac{\text{(variation per day)}}{24}$$

If δ is (+), $p=90°-\delta$; if δ is (−), $p=90°+\delta$.
$(s-p)$=arithmetical difference, always positive.

Formula is $\cos \frac{1}{2}A=\sqrt{\frac{\cos s \cos (s-p)}{\cos \phi \cos h}}$

A=astronomic azimuth east or west of north. Cross inapplicable letter and deduce true azimuth.
For stars, no time need be recorded.

Station	G 46	Latitude (Φ)	38°42'36"N
S	Sun	Longitude (λ)	77°08'46"W
Mark	O X	Watch Fast (−) Slow (+)	50s
Date	April 23, 1941 P.M.	When tested	April 23, noon M.
Locality	Fort Belvoir, Fairfax Co., Va.	ST Zone	E
Weather	Clear, warm	Observer	S. Sgt. Scarlett
Instrument	White 1234-1"	Recorder	T. Sgt. Wood
Reliability of observation	Fair		



Diagram

Notes checked: L L S

Apparent position of sun	Tel. to	h	m	Hor. Angle °	′	V. A. °	′		h	m	Hor. Angle °	′	V. A. °	′		h	m	Hor. Angle °	′	V. A. °	′
	D Mark			0	00						0	00						0	00		
D	☌	3	20.0	105	20	40	22	☌	3	26.7	106	01	39	06	☌	3	34.8	108	06	37	47
D	☌			105	28	40	15	☌			106	09	38	59	☌			108	20	37	35
D	☌			105	36	40	07	☌			106	17	38	52	☌			108	26	37	29
R	☌			105	12	39	22	☌			107	22	37	57	☌			108	11	36	33
R	☌			105	21	39	13	☌			107	31	37	50	☌			108	18	36	27
R	☌	3	24.4	105	34	39	02	☌	3	31.3	107	37	37	44	☌	3	38.9	108	27	36	18
	D Mark			0	00.0						0	00						0	00		
	Mean	3	22.2	105	25.2	39	43.5		3	29.0	106	49.5	38	24.7		3	36.8	108	18.0	37	01.5
	W & PM	12	00.8	Parallax		+	.1		12	00.8	Parallax		+	.1		12	00.8	Parallax		+	.1
	TZC	5		Refract.		−	1.2		5		Refract.		−	1.2		5		Refract.		−	1.3
	GCT	20	23.0	Barom. ±			—		20	29.8	Barom. ±			—		20	37.6	Barom. ±			—
				Therm. ±			—				Therm. ±			—				Therm. ±			—
				h (sum)		39	42.4				h (sum)		38	23.6				h (sum)		37	00.3
δ at 0h GCT				±		+12	19.5	δ at 0h GCT			±		+12	19.5	δ at 0h GCT			±		+12	19.5
GCT x δvar. per hour				±		+	17.0	GCT x δvar. per hour			±		+	17.1	GCT x δvar. per hour			±		+	17.2
δ				±		+12	36.5	δ			±		+12	36.6	δ			±		+12	36.7
P		77	23.5					P	77	23.4					P	77	23.3				
Φ		38	42.6	colog cos		0	10773	Φ	38	42.6	colog cos		0	10773	Φ	38	42.6	colog cos		0	10773
h		39	42.4	colog cos		0	11389	h	38	23.6	colog cos		0	10581	h	37	00.3	colog cos		0	09768
2)		155	48.5					2)	154	29.6					2)	153	06.2				
S		77	54.2	log cos		9	32131	S	77	14.8	log cos		9	34391	S	76	33.1	log cos		9	36655
P		77	23.5					P	77	23.4					P	77	23.3				
S-P		0	30.7	log cos		9	99998	S-P	0	08.6	log cos		0	00000	S-P		50.2	log cos		9	99995
				(sum) 2)		9	54291				(sum) 2)		9	55745				(sum) 2)		9	57191
				log cos ½A		9	77146				log cos ½A		9	77872				log cos ½A		9	78596
	½A					53	47.1	½A					53	04.4	½A					52	20.8
	A E or W					107	34.2	A E or W					106	08.8	A E or W					104	41.6
	Azimuth of S					252	25.8	Azimuth of S					253	51.2	Azimuth of S					255	18.4
	Angle, Mark − S					105	25.2	Angle, Mark − S					− 106	49.5	Angle, Mark − S					− 108	18.0
	True Az. to Mark					147	00.6	True Az. to Mark					147	01.7	True Az. to Mark					147	00.4

Magnetic Azimuth to Mark	154	Mean Azimuth to Mark	147 00.9
Grid Azimuth to Mark	144 36	Grid Correction (S.P. No. 59)	± − 2 24.5
Magnetic Decl. from Grid N. E(−) W(+)	9½	Grid Azimuth to Mark	144 36.4

Date of computation: April 24, 1941 Computed by: J. H. B. Checked by: S. S. H. FORM · 22

FIGURE 141.—Azimuth determination by altitude of sun.

d. Exercise XXX. Azimuth determination—altitude method.—After studying paragraphs 168, 169, and 171, take observations on the sun near the prime vertical and determine the grid azimuth between two stations assigned by the instructor, and also the magnetic declination. Form 22 will be used for both field notes and the computation.

172. Ageton method as applied to azimuth problem.—This solution is both simple and ingenious, and has other important advantages. A description of the procedure is omitted here only to avoid such variety in methods as to cause confusion. A complete description, with examples showing the application of the tables to the solution of azimuth and other problems, is contained in the publication of the United States Hydrographic Office, No. 211, entitled *Dead Reckoning Altitude and Azimuth Table—Ageton.* The tables are reprinted in TM 5–236 and are obtainable from other government sources.

173. Hour angle method.—*a. General.*—(1) As stated in paragraph 163*c*, the azimuth problem consists of measuring the horizontal angle, at the observer's station, between the mark whose azimuth is desired and some celestial body whose azimuth can be computed. The simple operations of adding this horizontal angle to, or subtracting it from, the computed azimuth of the sun or star gives the desired azimuth of the mark. Practically the entire computation, therefore, is in determination of the azimuth of the celestial body. This computation involves the solution of the astronomic triangle for the azimuth (the angle at the zenith), using the elements which are known and observed.

(2) In the hour angle method the known elements of the triangle are—

(*a*) The side between the zenith and the pole, which is the complement of the latitude of the place of observation.

(*b*) The side between the pole and the celestial body, which is the complement of the declination of the celestial body at the time of observation. The observed element is the angle at the pole, which is the hour angle of the celestial body. It is observed by recording the time when the instrument is pointed toward the sun or star, using a watch of known error, and reducing this corrected time to the hour angle by the method described in paragraph 166. The desired element is the angle at the zenith which, when reduced to the proper quadrant, will give the azimuth of the celestial body.

(3) The formulas for making the computations are as follows:

$$\tan \tfrac{1}{2}(A-q)=\frac{\sin \frac{1}{2}(\phi-\delta)}{\cos \frac{1}{2}(\phi+\delta)} \cot \tfrac{1}{2}t$$

$$\tan \tfrac{1}{2}(A+q)=\frac{\cos \frac{1}{2}(\phi-\delta)}{\sin \frac{1}{2}(\phi+\delta)} \cot \tfrac{1}{2}t \quad (1)$$

in which

ϕ=the latitude of the place of observation.
δ =the declination of the celestial body.
t=the hour angle (less than 12^h) of the celestial body.
A=the azimuth of the celestial body measured east or west of the meridian.
q=the parallactic angle, which cancels in the computation.

(4) In making the computations, always take the angles $\frac{1}{2}(A-q)$ and $\frac{1}{2}(A+q)$ less than 90°. If the algebraic sign of the quantity $(\phi-\delta)$ is positive, the azimuth will be the numerical sum of the angles $\frac{1}{2}(A-q)$ and $\frac{1}{2}(A+q)$ measured east or west of north, depending on the hour angle. If the algebraic sign of $(\phi-\delta)$ is negative, the azimuth will be the numerical difference of the two angles, measured east or west of north, depending on the hour angle. If the hour angle is less than 12 hours, the azimuth is to the west; if more than 12 hours, it is to the east of the meridian. If the hour angle is greater than 12 hours, subtract its value from 24 hours for use in the formulas.

(5) Success with the hour angle method depends upon knowledge of the correct time, as an unsuspected 10-second error of the watch might prevent even reasonably accurate results. Whenever hour angles of the sun have to be used for any important azimuths, the results should be checked by other methods at the earliest opportunity.

b. Procedure in observing.—To avoid overcrowding the form, the field notes are made in the notebook (fig. 142 ①) and the mean times and horizontal angles are afterward transcribed to the computation form. Another departure from the usual practice is that only 2 D/R observations are made instead of 3 D/R, and both A and B verniers are read and recorded, in order to get full benefit of the principle of letting the sun make its own contacts.

(1) Center the instrument over the station and level carefully. Do not change the leveling during a single set of readings. Attach the prismatic eyepiece and check the parallax adjustment. Slip the dark glass over the prismatic eyepiece and point telescope at the sun. Focus the image of the sun to sharp definition. Note the apparent direction of movement of the sun through the prismatic eyepiece, and call that direction west.

(2) With telescope direct, center on the azimuth mark using the lower motion, and record the readings on both horizontal verniers. The foot screws, lower motion, and the lightly clamped horizontal axis are not altered until the set has been completed.

(3) Unclamp the upper horizontal plate and point on the sun with the vertical wire slightly ahead of the sun and with the center horizontal wire roughly bisecting the sun. Clamp the upper horizontal motion screw. If no colored glass prismatic eyepiece is available, the sun's image must be projected on a piece of white paper held 2 or 3 inches behind the eyepiece instead of sighting it direct. This is accomplished by turning the telescope on the sun and adjusting the eyepiece until the image of the cross hairs shows up clearly across the image of the sun's disk on the paper. Then move the telescope until the image of the center horizontal cross hair roughly bisects the projected image of the sun.

(4) Keeping the image of the sun roughly bisected by the center horizontal wire, traverse the transit by means of the upper slow motion screw until the vertical cross hair falls a little west of the west limb of

SUN OBSERVATION–HOUR ANGLE

Obs: Sgt H.L. Chapeau; Rec: Cpl. Mower; Inst: K&E 60606 (10"); Warm, partly cloudy

May 8, 1940; Watch 24s slow; Compared 3 p.m. May 8; Eastern St

(14)

Station O56 — Mark Sta O2

Locality: Fort Belvoir, Fairfax Co, Va — Lat 38°42'36"N Long. 77°08'46"W

Reliability of observation: Good

Objects Observed	Time p.m. West limb	Time p.m. East limb	Time p.m. Mean	Reading	Verniers A	Verniers B	Verniers Mean		Remarks
	h m s	h m s	h m s	° '	"	"	"	° ' "	
Mark D				00 00	00	00	00		
Sun D	4 37 49	4 41 37	4 39 43	123 49	50	50	50	123 49 50	
Sun R	4 43 07	4 46 54	4 45 00	304 38	40	40	40	124 38 40	
Mark R				180 00	00	00	00		
Average			4 42 22					124 14 15	
Mark R				60 00	20	20	20		
Sun R	4 47 30	4 51 18	4 49 24	185 19	50	50	50	125 19 30	
Sun D	4 51 50	4 55 38	4 53 44	5 59	20	20	20	125 59 00	
Mark D				240 00	20	20	20		
Average			4 51 34					125 39 15	
Mark D				120 00	10	10	10		
Sun D	4 57 04	5 00 51	4 58 58	246 47	00	00	00	126 46 50	
Sun R	5 01 34	5 05 21	5 03 28	67 27	40	40	40	127 27 30	
Mark R				300 00	10	10	10		
Average			5 01 13					127 07 10	
Mag. Az. of Mark 154°—(Prismatic Compass)									
									Checked: H.L.C.

① Sun observation.

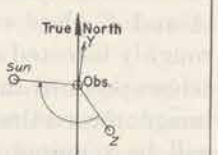
FIGURE 142.—Hour angle method of determining azimuth.

AZIMUTH COMPUTATION—HOUR ANGLE METHOD

(See par. 173c for sun, 178c(3) for star)

Station	O 54	Latitude (φ):	38° 42' 36" N
S	Sun	Longitude (λ):	77° 08' 46" W
Mark	O Z	Watch ~~Fast~~ Slow	24 s
Date	May 8, 1941 P.M.	When tested:	May 8 3 P.M.
Locality	Fort Belvoir, Fairfax Co., Va.	S T Zone:	E
Weather	Warm, partly cloudy	Observer:	Sgt. Chapeau
Instrument	K & E 60606-10"	Recorder:	Cpl. Mower

Reliability of observation: Good



Diagram

Notes checked: H. L. C.

HOUR ANGLES

			SET I			SET II			SET III		
			h	m	s	h	m	s	h	m	s
1	Time of observation		4	42	22	4	51	34	5	01	13
2	Watch, ~~Fast (−)~~ Slow (+)	±	+		24	+		24	+		24
3	E S T (add 12 hours if PM)		16	42	46	16	51	58	17	01	37
4	GCT of obs. (± diff. in time)		21	42	46	21	51	58	22	01	37
5	0^h Greenwich EQT ~~or Sid T (Sun)~~	±	+	3	33	+	3	33	+	3	33
6	GCT x var. per hour ~~or Table III (Eph.)~~	±	+		3.3	+		3.3	+		3.4
7	(5+6) Correct EQT ~~or GSid T (4+5+6)~~	±	+	3	36.3	+	3	36.3	+	3	36.4
8	(4+7) GAT ~~or − RA (Star)~~		21	46	22.3	21	55	34.3	22	05	13.4
9	GHA (time) (−12^h) ~~or (−8)~~		9	46	22.3	9	55	34.3	10	05	13.4
10	GHA (arc) (Table 20, TM 5-236)		146	35	34	148	53	34	151	18	21
11	Longitude, W(−) E(+)	±	− 77	08	46	− 77	08	46	− 77	08	46
12	LHA	±	69	26	48	71	44	48	74	09	35

To find LHA, add GHA and the longitude algebraically, subtracting 360° if necessary.
If LHA is greater than 180°, subtract from 360°, making the difference negative.
If (φ−δ) is (+), $A = \frac{1}{2}(A+q) + \frac{1}{2}(A-q)$; if (φ−δ) is (−), A = difference of same.

AZIMUTHS (A)

$\tan\frac{1}{2}(A+q) = \frac{\cos\frac{1}{2}(\phi-\delta)}{\sin\frac{1}{2}(\phi+\delta)}\cot\frac{1}{2}$ $\quad$ $\tan\frac{1}{2}(A-q) = \frac{\sin\frac{1}{2}(\phi-\delta)}{\cos\frac{1}{2}(\phi+\delta)}\cot\frac{1}{2}$

LHA	69	26	48	½LHA	34	43	24	cot	0 1 59 2445	cot	0 1 59 2445
φ	38	42	36	½(φ−δ)	10	46	22	cos	9 9 92 2778	sin	9 2 71 6432
δ	17	09	53	Numerator				log	0 1 51 5223	log	9 4 30 8877
φ−δ	21	32	43	½(φ+δ)	27	56	14	−sin	9 6 70 7132	−cos	9 9 46 1876
φ+δ	55	52	29					tan	0 4 80 8091	tan	9 4 84 7001
$A_1 = \frac{1}{2}(A+q) + \frac{1}{2}(A-q) =$					88	41	13	½(A+q)	71 42 38	½(A−q)	16 58 35
LHA	71	44	48	½LHA	35	52	24	cot	0 1 40 7595	cot	0 1 40 7595
φ	38	42	36	½(φ−δ)	10	46	18	cos	9 9 92 2794	sin	9 2 71 5990
δ	17	09	59	Numerator				log	0 1 33 0389	log	9 4 12 3585
φ−δ	21	32	37	½(φ+δ)	27	56	18	−sin	9 6 70 7291	−cos	9 9 46 1831
φ+δ	55	52	35					tan	0 4 62 3098	tan	9 4 66 1754
$A_2 = \frac{1}{2}(A+q) + \frac{1}{2}(A-q) =$					87	16	36	½(A+q)	70 58 15	½(A−q)	16 18 21
LHA	74	09	35	½LHA	37	04	48	cot	0 1 21 6245	cot	0 1 21 6245
φ	38	42	36	½(φ−δ)	10	46	15	cos	9 9 92 2806	sin	9 2 71 5658
δ	17	10	06	Numerator				log	0 1 13 9051	log	9 3 93 1903
φ−δ	21	32	30	½(φ+δ)	27	56	21	−sin	9 6 70 7410	−cos	9 9 46 1798
φ+δ	55	52	42					tan	0 4 43 1641	tan	9 4 47 0105
$A_3 = \frac{1}{2}(A+q) + \frac{1}{2}(A-q) =$					85	48	57	½(A+q)	70 10 43	½(A−q)	15 38 14

GRID AZIMUTH: A is west if LHA is (+), east if (−).

Azimuth (A) ~~East or West~~	88	41	13	87	16	36	85	48	57
True azimuth to S	271	18	47	272	43	24	274	11	03
Average angle, Mark—S	− 124	14	15	− 125	39	15	− 127	07	10
True azimuth to Mark	147	04	32	147	04	09	147	03	53
Magnetic azimuth to Mark	154	—		Mean true Az to Mark			147	04	11
Grid azimuth to Mark −	144	40		Grid correction (SP 59) ±			− 2	24	32
Mag. decl. from Grid North, ~~E (−)~~ W (+)	9¼			Grid azimuth to Mark			144	39	39

Date of computation: May 9, 1941 Computed by: J. H. B. Checked by: S. D. H. FORM 23

② Azimuth computation.

FIGURE 142.—Hour angle method of determining azimuth—Continued.

the sun's image, and call, "Ready." At the instant the image touches the vertical hair, call, "Tip." The exact time is recorded, and the *A* and *B* plate verniers are read and recorded. Keeping the image roughly bisected as before, allow the sun to move across the field of the telescope until the vertical cross hair falls on the east limb of the sun's image; record the exact time of the contact. Between 2 and 3 minutes will be required for the sun to move across the vertical cross hair. During this time the transit must not be moved in azimuth.

(5) Unclamp the upper horizontal plate, reverse the telescope, and again point on the sun, recording the readings as in (4) above.

(6) Unclamp the upper horizontal plate and sight the aximuth mark with the transit reversed.

(7) This completes one set of readings. Two additional sets should be taken with the relative positions of the horizontal plates shifted so as to have the readings on different parts of the horizontal circle. The mean of the times of pointing on the two limbs of the sun will correspond to the readings on the horizontal circle for the center of the image of the sun. The average time and angle for each set are recorded opposite the word "average."

c. Computation is accomplished on form 23, "Azimuth computation, hour angle method" (fig. 142 ②). The data are transcribed from the notebook to the spaces at the top of the form, the average times of each set to line (1) of the "hour angle" block, and average angles for each set to the third line of the "grid azimuth" block. As this form is also used for the computation of similar observations on stars, the abbreviations referring to the star computation on lines (5) to (9) are crossed out. The solution then proceeds as follows:

(1) Correct the time of observation for watch error and record true standard time of observation.

(2) Convert the standard time of observations into Greenwich apparent time (see par. 165*i*).

(3) Determine the hour angle of sun by the equation: Hour angle=apparent time minus 12 hours.

(4) Determine the declination of the sun for the time of observation by referring to part I of ephemeris for date of observation, and applying correction for hourly variation since 0^h Greenwich civil time in the same manner as the declination was determined in paragraph 166*e*.

(5) Solve for the azimuth of the sun by substituting latitude, declination, and hour angle in the formula given above.

(6) The formula (1) is printed on the form, also the following rules:

To find *LHA*, add *GHA* and longitude algebraically, subtracting 360° if necessary.

If *LHA* is greater than 180°, subtract from 360°, making the difference negative.

If $\phi-\delta$ is (+), $A=\frac{1}{2}(A+q)+\frac{1}{2}(A-q)$; if $\phi-\delta$ is (−), A=difference of same.

d. Exercise XXXI.—Azimuth determination from hour angles of the sun.—Using a watch of known error, occupy the station and observe the sun while near the prime vertical for the angle mark–sun, recording the time to the nearest second, as shown in figure 142 ①. Compute the true grid azimuth on form 23.

174. Equal altitude method.—*a.* This method is not a desirable one for military surveys, as the same station has to be occupied in both morning and afternoon. Even then the above method is not so accurate as others, as it is not well adapted to the refinement of results by repetition or by taking the mean of a series of observations.

b. If a station can be reoccupied, and it is desirable to check some hour angle observations made in the morning, for instance, a better check could be obtained from three additional sets by either the altitude or hour angle method.

c. If conditions require the use of an equal altitude method, observations should be on a star as described in paragraph 178*e*.

Section XXX

AZIMUTHS FROM OBSERVATIONS ON STARS

	Paragraph
Polaris	175
Azimuth from observations on Polaris at elongation	176
Azimuth from observation on Polaris at any hour angle	177
Azimuth from other stars	178

175. Polaris.—*a. Identification.*—Polaris, listed in the ephemeris as α Ursae Minoris, is a star of the second magnitude which rotates about the north celestial pole at a radius of about 1°. No other close circumpolar star is of comparable magnitude. Figure 143 shows Polaris and also another constellation, about 30° from the pole, called the Great Dipper (Latin name *Ursa Major*) because of the outline formed by its seven bright stars. A pair of these on the side of the bowl opposite the handle are called the "pointers" because a line through them points at Polaris, which is sometimes called the North or Pole Star. Polaris is almost on line between ζ Ursae Majoris (special name, Mizar) and δ Cassiopeiae, the latter in a group of five stars forming a large *W*. All of these stars are shown in the figure and are easily recognized during any clear night by an observer north

of the Tropics. Like all stars they seem to rotate about the celestial axis in a westward direction. The same effect may be had by rotating (fig. 143) counterclockwise about the pole as a center.

b. Location.—When the star crosses the meridian above the pole it is said to be at upper culmination; when it crosses the meridian below the pole it is said to be at lower culmination. When the star

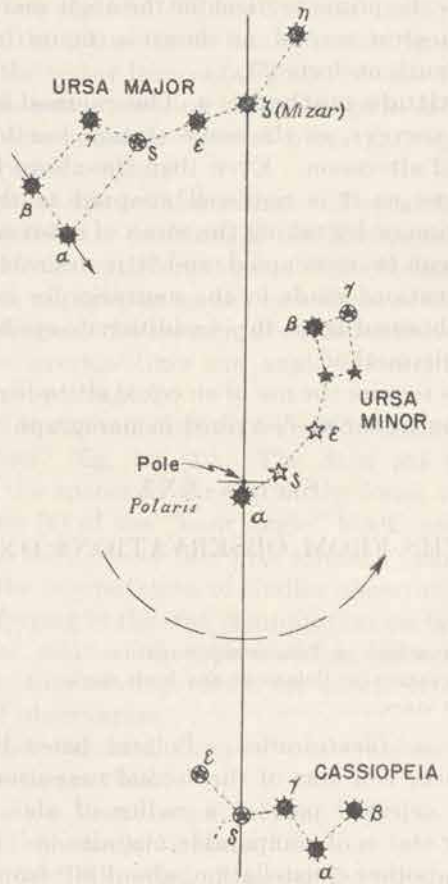


FIGURE 143.—Location of Polaris near lower culmination.

reaches the point in its apparent path where its azimuth east of geographic north is the greatest it is said to be at eastern elongation, when it reaches the point where its azimuth west of geographic north is greatest it is said to be at western elongation. The star appears to make one complete revolution every sidereal day ($23^h56^m04.1^s$ of mean solar time). Consequently, it crosses the meridian approximately 3^m56^s earlier each successive day. The apparent motion of

the star being uniform, one-quarter of its apparent path will be described in $5^h59^m01^s$, one-half in $11^h58^m02^s$, three-quarters in $17^h57^m03^s$ of mean solar time. However, only to an observer on the Equator does this star reach western or eastern elongation at one-quarter or three-quarters of its apparent path. At other points the mean time interval between upper culmination and west or east elongation is less than $5^h59^m01^s$ by an amount depending upon the latitude. Observations on Polaris for azimuth may be made at either elongation, or at any time between. Referring again to figure 143, Polaris is on the opposite side of the celestial pole from Mizar, in the Dipper handle. By remembering this relationship, it is easy to estimate how soon the star may be at elongation, or even to estimate the hour angle, within an hour or less. In the figure the hour angle is about $11^h\ 30^m$, according to the ephemeris, table VI.

c. Advantages.—Polaris is the most useful star in military surveying because of its proximity to the pole. There is no comparable star within 10° of the south celestial pole. The small radius of rotation so reduces the apparent speed of motion that accurate pointings can easily be made in any aspect, and errors due to mistakes in watch time have much less effect on results. There is a smaller star quite close to Polaris, invisible to the eye but easily seen through the telescope. If the observer will note the position of the small star as he makes the first deliberate pointing, it will serve as an identification for succeeding pointings which may be more hastily made.

d. Special tables.—(1) Precise determinations of azimuth by observations on Polaris are made and computed in exactly the same manner as observations on any other star.

Forms 22 and 23 may be used for Polaris and other stars (see par. 178), as for the sun, with some slight changes.

(2) For ordinary military purposes computations of azimuths are shortened and simplified by the use of tables. Polaris is not included in the Ephemeris general tables of "apparent places of stars," but an enlarged table, adjacent to the other special tables for Polaris, gives the right ascension and declination for every day of the year. The special tables in the Ephemeris include the following:

Table I.—*For Finding the Latitude by an Observed Altitude of Polaris.*
Table II.—For Converting *Sidereal into Mean Solar Time.*
Table III.—For Converting *Mean Solar into Sidereal Time.*
Table IV.—For Finding the *Azimuth of Polaris at All Hour Angles.*

Table V.—For Finding the *Azimuth of Polaris at Elongation.*
Table VI.—*For Finding, by Observation, when Polaris passes the Meridian.*
Table VII.—For Finding the *Apparent Place, Time of Upper Culmination, and Time Interval between Upper Culmination and Elongation, of Polaris.*

176. Azimuth from observations on Polaris at elongation.—*a. General.*—(1) Figure 144 shows in proper relation the various

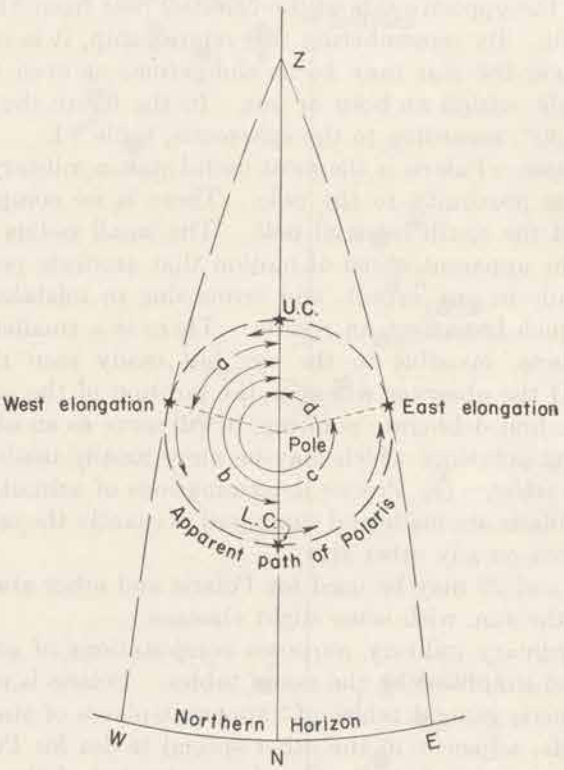


FIGURE 144.—The aspects of Polaris, including hour angles.

a=Hour angle of Polaris at western elongation.
b=Hour angle of Polaris at lower culmination.
c=Hour angle of Polaris at eastern elongation.
d=One apparent whole revolution of Polaris around the pole, taking $23^h\ 56.1^m$.
U. C.=Upper culmination.
L. C.=Lower culmination.

aspects of Polaris in its daily apparent motion around the North Pole. A careful study of this figure is necessary before the observation and computation of an azimuth from Polaris should be attempted.

In the figure the vertical line represents a portion of the meridian passing through the zenith and intersecting the northern horizon at the point N (north point) from which, for all practical purposes, the azimuths of Polaris may be reckoned east or west. At the instant Polaris crosses the meridian it is at either upper or lower culmination, depending upon whether its position is above or below the North Pole. In the figure Polaris is supposed to be on the meridian. At the instant Polaris is farthest east or west of the North Pole it is said to be at either eastern or western elongation. The corresponding hour angles are also indicated.

(2) Observations for determining azimuths may be made at any time, the exact time being carefully noted. Whenever possible observations should be made when Polaris is either at its eastern or western elongation, as this method gives somewhat more accurate results and involves very little computation besides calculating the time of the star's elongation. During certain times of the year Polaris is not easily visible (on account of daylight) at elongation; therefore, the method of determining azimuths from observations from Polaris at any hour angle, described in paragraph 177, is used to a large extent.

(3) Azimuth observations made on Polaris at elongation offer to the inexperienced observer less chance of error either in observing or in computation than any other method. When near either elongation, the star is changing very little in azimuth. With only approximate time the star may be tracked until no change in azimuth is apparent. During 10 minutes before or after either elongation, the star varies not more than $\frac{1}{10}$ minute in azimuth. (See ephemeris, table Va.)

(4) Though it is a convenience, the time of elongation need not be known in advance of observation. When Polaris is at culmination it is nearly on the same vertical line (meridian) as the stars Mizar and δ Cassiopeiae. About 6 (and 18) hours later Polaris may be observed for elongation.

b. Procedure in observing.—The observations are recorded and all computations are made on Form 24 (fig. 145).

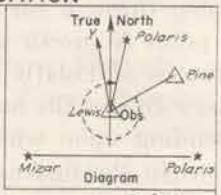
(1) Before the night of observation compute the standard time of elongation within 1 or 2 minutes.

(2) Set up the instrument one-half hour before elongation and keep the vertical crosshair exactly on Polaris until about 10 minutes before elongation, when the star no longer appears to move laterally.

(3) Measure the angle mark–star, 3 D/R, reading the A vernier of the first angle to secure a check on the sixth repetition. The vertical hair should not be set upon Polaris at its intersection with the center

AZIMUTH BY POLARIS AT ELONGATION

Station	△ Lewis	Latitude (φ)	44° 10' 49" N
POLARIS	E W elongation	Longitude (λ)	76° 18' 32" W
Mark	△ Pine	Watch Fast Slow ±	m s
Date	Oct 2, 1941 P.M.	When tested:	Oct 2, 1941 5 P.M.
Place	Lewis, King Co, N.Y.	ST Zone:	E
Weather	Cool, hazy, no wind	Observer:	T Sgt Burr
Instrument:	Berger 12803-20"	Recorder	S Sgt Alden
Reliability of observation:	Fair (very hazy)		



Notes checked J. of R. 18.

1	Oct		3	Date of observation
2	Sept		27	Next preceding date in Ephemeris Table XII
3			6	Days intervening (no fractions)
4	-	23	36	(3) X 3m 56s = Correction to GCT, Upper Culmination
5	1	23	22	GCT UC for date (2) Table XII
6		59	46	(5)-(4) GCT UC for date obs.
7	-		51	± Correction for λ (Degrees of λ X 10s/15) E(+) W(-)
8		58	55	(Sum) LCT UC
9	-5	55	04	± Mean time interval between UC and elongation at φ of Station Table VII
10	19	03	51	LCT El. at Sta. W El.(8)+(9) E El.(8)-(9)
11			76 18 32	Longitude of Station
12			-75 00 00	Meridian of standard time
13			1 18 32	(11)-(12) Difference in longitude
14	+	5	14	± (13)/15 Difference between LCT and standard time
15	PM 7	09	05	(10)±(14) Standard time of elongation (14) is (+) W of ST meridian (-) if E

	I	II	III	IV	V	VI
Time ID	6 57 45	7 06 40	7 14 00			
Time GR	7 05 20	7 12 30	7 22 10			
Set No.						
Mark	00 00 00	60 05 20	120 10 20			
D1	299 54 00	359 59 40	60 04 20			
R6	359 25 00	59 30 20	119 34 00			
Sum	1799 25 00	1799 25 00	1799 23 40			
1/6	299 54 10	299 54 10	299 53 57			
Table Vo ±						
Cor. Angle						

17. Mean of good sets: 299 54 06

18. With declination for nearest 0h from Eph., "Apparent place of Polaris" and the latitude of the Sta., from Eph. Table V "Azimuth of Polaris at elongation," interpolate the azimuth of Polaris.

Eph. Table V	Declination		
Latitude	88 59 00	88 59 06	88 59 10
44 10 00	1 25 03		1 24 49
44 10 49	1 25 04	1 24 56	1 24 50
44 20 00	1 25 17		1 25 03

19	Azimuth of Polaris E(+) ~~or W(-)~~ of North			±	+ 1 24 56
20	True azimuth to Polaris, 360° ± (19)				1 24 56
21	Magnetic azimuth to Mark	51 31	Mean azimuth to Mark (20)-(17)		61 30 50
22	Grid azimuth to Mark	- 63 49	Grid correction (S.P. 59)	±	+ 2 18 19
23	Magnetic Decl. from grid N E(-) ~~W(+)~~	-12 18	Grid Azimuth to Mark		63 49 09

Date of computation: Oct. 4, 1941 Computed by: J.H.B. Checked by: S.B.M. FORM 24

FIGURE 145.—Azimuth by Polaris at elongation.

hair, but a little above (or below). When the telescope is reversed, use the same part of the hair, which is then a little below (or above) the center hair. Record the *A* vernier on the 1 *D* and 6 *R* settings, reading the *B* vernier only as a check. Calculate the mean of the six repetitions.

(4) Continue taking observations in sets until 10 minutes after elongation time.

c. The computation consists almost entirely of calculating the time of elongation, as explained on the form.

d. Exercise XXXII.—Azimuth by observation on Polaris at elongation.—Compute the time of elongation, observe Polaris during the 20-minute bracket of the calculated time, and find the true grid azimuth and magnetic declination, making all necessary notes and computations on form 24.

177. Azimuth from observations on Polaris at any hour angle.—*a. Procedure in observing.*—Both field notes and computation are recorded on Form 25 (fig. 146). The watch error should be known within 10 seconds.

(1) Center and level the instrument carefully over the mark. Do not change the leveling or the *lightly* clamped, horizontal axis clamp until the set is finished.

(2) Set the *A* vernier at zero, sight the mark with the telescope in the direct position using only the lower motion, and read and record the magnetic azimuth.

(3) Measure by 6 repetitions (3 *D*/*R*) the angle mark–star, recording exact time of each setting on the star, read and record the *A* vernier for the 1 *D* and 6 *R* settings on the star. Subtract the initial reading from the sixth repetition and divide by 6 to get the mean value of the angle for the set after restoring the multiples of 60° which may have been dropped by passing the 360° of the circle.

(4) This set should be immediately followed by two more.

b. The computation is accomplished in the following steps:

(1) Correct the time of observation for watch error, thus obtaining the accurate standard time of observation.

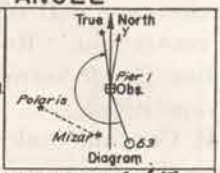
(2) Convert the standard time of observation to local sidereal time.

(3) Compute the hour angle of Polaris by the equation, hour angle equals local sidereal time minus right ascension. The right ascension is found in *Apparent Place of Polaris* (ephemeris table).

(4) Determine the azimuth of Polaris by referring to table IV of the ephemeris using the latitude of the point of observation and the hour angle computed in (3) above as arguments.

AZIMUTH BY POLARIS AT ANY HOUR ANGLE

Station: Pier 1 — Latitude: 38°42'55"N
Polaris — Longitude: 77°08'50"W
Mark: 063 — Watch Fast Slow 10s
Date: Mar. 6, 1941 P.M. — When tested Mar. 6, 1941 5 P.M.
Place: Fort Belvoir, Fairfax Co., Va. — S T Zone: E
Weather: Cool and windy. — Observer: Cpl. Mower
Instrument: Berger 12803-20" — Recorder: Sf. Sgt. Scarlett
Reliability of observation: Good



Diagram

Notes checked: J. J. M.

		A Vernier Set I	Time	A Vernier Set II	Time	A Vernier Set III	Time
2	D1	196 09 00	7 30 46	196 07 20	7 39 03	196 07 00	7 48 11
3	D2		7 32 29		7 40 06		7 49 36
4	D3		7 33 20		7 41 20		7 50 34
5	R4		7 34 25		7 42 46		7 51 51
6	R5		7 36 01		7 43 59		7 53 29
7	R6	96 48 20	7 36 58	96 41 20	7 45 18	96 37 20	7 54 48
8	Sum	1176 48 20	203 59	1176 41 20	252 32	1176 37 20	308 29
9	1/6	196 08 03	7 34 00	196 06 53	7 42 05	196 06 13	7 51 25

			Set I	Set II	Set III
10	Mean time of observation	from (9)	7 34 00	7 42 05	7 51 25
11	Watch Fast (−) ~~Slow (+)~~	±	− 10	− 10	− 10
12	E ST (add 12 hours if P.M.)	(10)+(11)	19 33 50	19 41 55	19 51 15
13	Longitude of station	77 08 50			
14	Meridian of Standard Time	− 75 00 00			
15	Difference in longitude (13−14)	2 08 50			
16	Difference between ST and LCT (15)/15	±	− 8 35	− 8 35	− 8 35
17	LCT (16) is (+)E, (−)W of ST Meridian	(12)±(16)	19 25 15	19 33 20	19 42 40
18	Eph. Table III, mean solar into Sid T		3 11	3 13	3 14
19	Sidereal interval since 0^h	(17)+(18)	19 28 26	19 36 33	19 45 54
20	Sid T 0^h GCT (Eph. of Sun)	10 53 20			
21	Corr. for λ (Degrees λ x 10/15) W(+) E(−)	+ 51			
22	Sid T Local 0^h	(20)±(21)	10 54 11	10 54 11	10 54 11
23	Sid T of observation	(19)+(22)	6 22 37	6 30 44	6 40 05
24	RA of Polaris nearest 0^h (Approximate place of Polaris)	−	1 42 54	1 42 54	1 42 54
25	HA of Polaris at time of observation	(23)−(24)	4 39 43	4 47 50	4 57 11

26	Table IV	Latitude			Table IV.a	With the HA and φ of the Sta. from Eph. Tab IV,
27	HA	38	38 42 55	40		Azimuth of Polaris at all HA, interpolate the azi-
28	4 39 43		1 13.6		—	muth of Polaris. With declination for nearest 0^h
29	4 40 00 / 4 47 50	1 12.8	1 13.6 / 1 14.4	1 14.9	—	from Eph. Apparent place of Polaris, and the φ of
30	4 50 00	1 13.8	1 14.6	1 16.0	—	the Sta. from Eph. Tab. IV.a interpolate the corr-
31	4 57 11		1 15.3		—	ection for each of the azimuths found above.
32	5 00 00	1 14.8	1 15.6	1 16.9		

33	Azimuth of Polaris E(+) or W(−) of north	±	− 1	13	36	− 1	14	24	−1	15	18
34	True azimuth of Polaris, 360 ± (33)		358	46	24	358	45	36	358	44	42
35	Mean Angle to Mark from (9)	−	196	08	03	196	06	53	196	06	13
36	True azimuth to Mark		162	38	21	162	38	43	162	38	29

37	Magnetic azimuth to Mark	166 55	Mean azimuth to Mark, from (36)		162 38 31
38	Grid azimuth to Mark	− 160 14	Grid correction (S.P. 59)	±	−2 24 31
39	Magnetic Decl. from Grid N. ~~E(−)~~ W(+)	+ 6 41	Grid azimuth to Mark (37)±(38)		160 14 00

Date of computation: March 7, 1941 — Computed by: J.H.B. — Checked by: A.B.H. — FORM 25

FIGURE 146.—Azimuth by Polaris at any hour angle.

(5) Correct this azimuth for variation in declination by referring to table IVa and using the declination and azimuth determined in (4) above as arguments. The declination of Polaris for any day in the year is found in *Apparent Place of Polaris*. The result is the azimuth of Polaris east or west of north. Convert to true azimuth.

(6) Determine the true azimuth of mark by deducting the angle mark–star from the true azimuth of Polaris.

(7) Determine the correction for grid divergence from Specification Publication No. 59, and obtain the grid azimuth.

c. Exercise XXXIII.—Azimuth by observations on Polaris at any hour angle.—Recording all notes and computations on Form 25, obtain the azimuth of a line assigned by the instructor by observations on Polaris at any hour.

178. Azimuth from other stars.—*a. General.*—(1) In making azimuth observations on a star the procedure is practically the same as for the sun, except that pointings are made on the center of the star and not on the limb. The procedure in observing and the forms of field notes are similar to those for the corresponding methods on the sun. Forms 22 and 23 may be used for the computation with but slight modification, as explained below in *b* and *c*.

(2) The above applies equally well to the computation of Polaris observations, when a more precise solution is desired than can be obtained with the special tables previously described (par. 175*d* (2)). A still more precise computation of Polaris observations may be secured by using formula (2) (par. 167*e*), which is especially adapted for close circumpolar stars.

(3) More nearly correct azimuths may be obtained from the computations of observations taken on stars which are in favorable positions. Rather than wait until some particular star is in a position favorable for observation, it is generally better to select from the Ephemeris some other star which will answer the requirements at the desired time of observation.

b. Altitude method.—(1) *Favorable positions.*—The accuracy of the solution is maximum where A is 90° or, in other words, when the celestial body is to the east or west of the observer. However, other factors must also be considered. When the altitude is less than 20° the corrections for refraction are uncertain. When the altitude is greater than 45° the inaccuracies of the oridnary instrument will introduce serious errors when the telescope is depressed to sight the azimuth mark. The instrument should be carefully leveled before each set of readings. Observations by the altitude method should not be taken when the celestial body is near the meridian, as the altitude is changing

very slowly and the solution will give poor results. Time is needed only in order to interpolate for declination at the time of observation. In observations on stars the exact time need not be known.

(2) Figure 147 ① shows the field notes and computation of observations made with a direction theodolite of the Wild type, reading directly to one second. Three sets of 1 *D/R* each were taken, setting random new initials for each set. When computed with the 1939 ephemeris, the azimuth was correct within 3 seconds.

(3) The computation differs from that of a solar observation (par. 171*c*) only in that the altitude needs no correction for parallax and the declination needs only correction to the date of the observation, the hourly change being disregarded.

c. Hour angle method.—(1) *Favorable positions.*—Examination of the formula (1) (par. 167*e*) will show that the most favorable time for observations by the hour angle method is when the hour angle is near 6 or 18 hours, and the least favorable is when the hour angle is near 0. The error introduced by observations on celestial bodies near the meridian depends on the latitude of the place and the declination of the body. Consequently, no definite limits can be given. In latitude 45° *N*, when the sun is on the meridian and its declination is zero, an error of 1 second in time will give an error of 21 seconds in azimuth. Under the same conditions with an hour angle of 6 hours, an error of 1 second in time will give an error of 11 seconds in azimuth. In observations on Polaris, in latitude 45° *N*, when the star is on the meridian an error of 2 seconds in time will give an error less than 1 second in azimuth. In the same latitude, when the star is at elongation an error of 30 seconds in time will give an error of less than 1 second in azimuth. In general, hour angle observations should not be made on a celestial body less than 2 hours from the meridian. In the case of close circumpolar stars, the most favorable time for observation is when the star is near elongation. In the case of other stars the most favorable time is when the star is to the east or west of the observer and less than 45° in altitude.

(2) The notes are recorded in a manner similar to that shown in figure 142①, but three individual pointings direct and three reversed, reading only the vernier next the eyepiece, with the exact time for each, are recorded for each set, and three sets are taken. The initials are changed to 60° and 120° and the second set may be begun in the reversed position. This method is similar to that shown on Form 22 (fig. 141), except that times are required for each horizontal angle, and vertical angles are not read. The star may be allowed to make its own contacts if it does not move too slowly. The star should not

be set on the intersection of the cross hairs, but on the same point each time, a little above the intersection direct, and the same amount below for the reversed position. An unsuspected watch error is just as harmful as with the sun (par. 173*a* (5)).

(3) The computation is made on Form 23 (fig. 147 ③). In lines (5) to (9) of the "hour angles" block, the abbreviations for *EQT* and *GAT* and other references to the sun are crossed out, as sidereal time is used with stars. The *LHA* is found as demonstrated in paragraph 166*b* (3); the time of the observation is converted to *G Sid. T*, from which the *RA* is subtracted to get *GHA*, which in turn is corrected to *LHA*. If *LHA* is less than 12^h, the star is to the west; if more than 12^h, it is to the east of the meridian. If *LHA* is greater than 12^h, subtract its value from 24^h for use in the formulas. The right ascensions and declinations of the principal stars are given in the ephemeris for the upper transit at Greenwich in 10-day intervals. The variation is so small that interpolation to the date of observation is all that is required, interpolation for longitude from the meridian of Greenwich being unnecessary.

d. Observations on east and west stars.—(1) Excellent results will be obtained by observations made on two stars, one to the east and one to the west of the meridian. These stars should be selected near the prime vertical and between limits of 20° and 45° of altitude. The average azimuth of the mark determined from the two observations should be adopted. This procedure will eliminate gross errors both in observation and computation. If the results are computed immediately at the station, two sets to each star may be sufficient. If the computations are to be made later or at another location, 3 sets should be observed on each star, in case one (or more) of the sets should prove defective.

(2) With the altitude method, this procedure should minimize errors in the assumed latitude or in the adopted refraction. If east and west stars of roughly the same altitude can be observed, the error in refraction may be balanced out.

(3) In the hour angle method, any error in latitude or an incorrect value of the watch error should be diminished by observing east and west stars.

(4) This method is not well adapted for sun observations as the east and west observations will be separated by several hours, during which the declination will change materially.

(5) Observations on east and west stars should yield more consistent accuracy than any other method, except precise computation of Polaris observations, which include correction for curvature of the star's path during each set of observations.

Station	: 0 38	Latitude (φ):	38° 41' 20" N.
S	: Regulus	Longitude (λ):	77° 08' 13" W.
Mark	: 0 40	Watch Fast (−) Slow (+)	— s
Date	: Jan 24, 1941 P.M.	Barom:	29.8
Locality	: Fort Belvoir, Fairfax Co., Va.	Therm:	48°F
Weather	: Clear, cold	Observer:	St. Sgt. Kosten
Instrument	: Wild 6388-T2	Recorder:	St. Sgt. Alden

Diagram: True North; 40; Obs.; 38; Regulus

Reliability of observation: Very good. Notes checked: L. M. K.

Tel. to		Hor. Angle σ	Hor. Angle t	V.A. σ	V.A. t		Hor. Angle σ	Hor. Angle t	V.A. σ	V.A. t		Hor. Angle σ	Hor. Angle t	V.A. σ	V.A. t
D ~~Mark~~	h m	00° 33'	00.0"			D	60° 18'	38.3"			D	120° 32'	40.8"		
D +	119 14	21.1	62°	04'	07.0"	D +	179 20	13.3	61°	41'	07.5"	D + 240 01	26.9	61°	10'	12.7"
☉ +	299 22	11.3	298	04	57.0	☉ +	359 29	25.8	298	29	26.5	☉ + 60 10	22.2	299	00	21.0
☉ +		Angles				+		Angles				+	Angles			
R +	118 41	21.1	27	55	53.0	+	119 01	35.0	28	18	52.1	+ 119 28	46.1	28	49	47.3
R +	118 49	11.3	28	04	57.0	+	119 10	47.5	28	29	26.5	+ 119 37	41.4	29	00	21.0
R +						+						+				
☉ Mark																
Mean		118° 45'	16"	28°	00' 25"		119° 06'	11"	28°	24'	09"	119° 33'	14"	28°	55'	04"
~~W-GPM~~			Parallax	+ —				Parallax	+ —				Parallax	+ —		
: ~~TZC~~			Refract	−01	49			Refract	−01	47			Refract	−01	45	
~~GCT~~			Barom. ±	−	04			Barom. ±	−	04			Barom. ±	−	04	
			Therm. ±	+	04			Therm. ±	+	04			Therm. ±	+	04	
			h(sum)	27°	58' 36"			h(sum)	28°	22' 22"			h(sum)	28°	53' 19"	

δ at 0ʰGCT ±					δ at 0ʰGCT ±					δ at 0ʰGCT ±					
GCT x δvar. per hour ±					GCT x δvar. per hour ±					GCT x δvar. per hour ±					
δ ±	+12° 15' 10"				δ ±					δ ±					
P	77° 44' 50"				P	77° 44' 50"				P	77° 44' 50"				
φ	38 41 20	colog cos	0	10	760	φ	38 41 20	colog cos	0	10	760	φ	38 41 20	colog cos	0	10	760
h	27 58 36	colog cos	0	05	397	h	28 22 22	colog cos	0	05	558	h	28 53 19	colog cos	0	05	771
2)	144 24 46					2)	144 48 32					2)	145 19 29				
S	72 12 23	log cos	9	48	514	S	72 24 16	log cos	9	48	043	S	72 39 44	log cos	9	47	422
P	77 44 50					P	77 44 50					P	77 44 50				
S−P	05 32 27	log cos	9	99	797	S−P	05 20 34	log cos	9	99	811	S−P	05 05 06	log cos	9	99	829
	(sum) 2)		9	64	468		(sum) 2)		9	64	172		(sum) 2)		9	63	782
	log cos ½A		9	82	234		log cos ½A		9	82	086		log cos ½A		9	81	891
½A		48° 22' 26"				½A		48° 32' 51"				½A		48° 46' 24"			
A E ~~or W~~		96 44 52				A E ~~or W~~		97 05 42				A E ~~or W~~		97 32 48			
Azimuth of S		96 44 52				Azimuth of S		97 05 42				Azimuth of S		97 32 48			
Angle, Mark−S	−	118 45 16				Angle, Mark−S	−	119 06 11				Angle, Mark−S	−	119 33 14			
True Az. to Mark		337 59 36				True Az. to Mark		337 59 31				True Az. to Mark		337 59 34			

Magnetic Azimuth to Mark	Local Attraction	Mean Azimuth to Mark	337 59 34
Grid Azimuth to Mark		Grid Correction (S.P. No. 59)	± 2 24 49
Magnetic Decl. from Grid N. E(−) W(+)		Grid Azimuth to Mark	335 34 45

Date of computation: Jan. 25, 1941 Computed by: J. H. B. Checked by: L. E. K. FORM 22

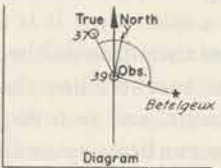
① Altitude method.

FIGURE 147.—Azimuth computation—Continued.

AZIMUTH COMPUTATION—HOUR ANGLE METHOD

(See par. 173c for sun, 178c(3) for star)

Station: O 39 | Latitude (φ): 38° 41' 36" N
S: Betelgeux | Longitude (λ): 77° 08' 13" W
Mark: O 37 | Watch: Fast Slow 12 s
Date: Dec. 13, 1941 P.M. | When tested: Dec. 13 7 P.M.
Locality: Fort Belvoir, Fairfax Co., Va. | S T Zone: E
Weather: Cool, clear, windy | Observer: 3gt. Chapeau
Instrument: Berger 11813-20" | Recorder: Cpl. Mower
Reliability of observation: Fair



Diagram

Notes checked: H.L.C.

HOUR ANGLES

		SET I			SET II			SET III		
		h	m	s	h	m	s	h	m	s
1	Time of observation	0	16	45	8	25	24	8	33	58
2	Watch, Fast (−) ~~Slow (+)~~ ±	−		12	−		12	−		12
3	E S T (add 12 hours if PM)	20	16	33	20	25	12	20	33	46
4	GCT of obs. (± diff. in time)	1	16	33	1	25	12	1	33	46
5	0^h Greenwich ~~EQT or~~ SidT (Sun) ±	+ 5	29	5.1	+ 5	29	5.1	+ 5	29	5.1
6	GCT x ~~var. per hour or~~ Table III (Eph.) ±	+		12.6	+		14.0	+		15.4
7	~~(5+6) Correct EQT or~~ GSidT (4+5+6) ±	+ 6	45	50.7	+ 6	54	31.1	+ 7	03	06.5
8	~~(4+7) GAT or~~ −RA(Star)	− 5	52	02.8	− 5	52	02.8	− 5	52	02.8
9	GHA(time)(~~−12h~~) ~~or~~ (7−8)		53	47.9	1	02	28.3	1	11	03.7
10	GHA(arc)(Table 20, TM5-236)	13	26	58	15	37	04	17	45	56
11	Longitude, W(−) E(+) ±	−77	08	13	−77	08	13	−77	08	13
12	LHA ±	−63	41	15	−61	31	09	−59	22	17

To find LHA, add GHA and the longitude algebraically, subtracting 360° if necessary.
If LHA is greater than 180°, subtract from 360°, making the difference negative.
If (φ − δ) is (+), A = ½(A+q) + ½(A−q), if (φ − δ) is (−), A = difference of same.

AZIMUTHS (A)

$\tan \frac{1}{2}(A+q) = \frac{\cos \frac{1}{2}(\phi-\delta)}{\sin \frac{1}{2}(\phi+\delta)} \cot \frac{t}{2}$ $\quad \tan \frac{1}{2}(A-q) = \frac{\sin \frac{1}{2}(\phi-\delta)}{\cos \frac{1}{2}(\phi+\delta)} \cot \frac{t}{2}$

LHA	63	41	15	½LHA	31	50	38	cot	0	206	8474	cot	0	206	8474		
φ	38	41	36	½(φ−δ)	15	38	56	cos	9	983	5960	sin	9	430	9479		
δ	7	23	44	Numerator				log	0	190	4434	log	9	637	7953		
φ−δ	31	17	52	½(φ+δ)	23	02	40	−sin	9	592	6708	−cos	9	963	8829		
φ+δ	46	05	20					tan	0	597	7726	tan	9	673	9124		
$A_1 = \frac{1}{2}(A+q) + \frac{1}{2}(A-q) =$					101	05	45	½(A+q)	75	49	48	½(A−q)	25	15	57		
LHA	61	31	09	½LHA	30	45	35	cot	0	225	3609	cot	0	225	3609		
φ				½(φ−δ)		~~		cos	9	983	5960	sin	9	430	9479		
δ			above	Numerator				log	0	208	9569	log	9	656	3088		
φ−δ				½(φ+δ)		~~		−sin	9	592	6708	−cos	9	963	8829		
φ+δ								tan	0	616	2861	tan	9	692	4259		
$A_2 = \frac{1}{2}(A+q) + \frac{1}{2}(A-q) =$					102	37	13	½(A+q)	76	23	56	½(A−q)	26	13	17		
LHA	59	22	17	½LHA	29	41	08	cot	0	244	0826	cot	0	244	0826		
φ				½(φ−δ)		~~		cos	9	983	5960	sin	9	430	9479		
δ			above	Numerator				log	0	227	6786	log	9	675	0305		
φ−δ				½(φ+δ)		~~		−sin	9	592	6708	−cos	9	963	8829		
φ+δ								tan	0	635	0078	tan	9	711	1476		
$A_3 = \frac{1}{2}(A+q) + \frac{1}{2}(A-q) =$					104	09	57	½(A+q)	76	57	10	½(A−q)	27	12	47		

GRID AZIMUTH. A is west if LHA is (+), east if (−).

Azimuth (A) East ~~or West~~	101	05	45	102	37	13	104	09	57
True azimuth to S	101	05	45	102	37	13	104	09	57
Average angle, Mark−S	−122	06	53	−123	38	05	−125	11	27
True azimuth to Mark	338	58	52	338	59	08	338	58	30
Magnetic azimuth to Mark	345	55		Mean true Az. to Mark			338	58	50
Grid azimuth to Mark −	336	34		Grid correction (SP 59) ±			− 2	24	50
Mag. decl. for Grid North, ~~E (−)~~ W (+)	9	21		Grid azimuth to Mark			336	34	00

Date of computation: Dec. 14, 1941 Computed by: J.W.B. Checked by: J.B.W. FORM 23

② Hour angle method.

FIGURE 147.—Azimuth computation—Continued.

e. (1) *Equal altitudes of star.*—The meridian (true north or south) may be determined in a very simple manner if observations are made on a star when it is at the same altitude on both sides of the meridian. The meridian is then midway between the two positions of the stars. The method has the advantage that time, use of tables, and computations are not required, but a decided disadvantage in that two observations several hours apart are necessary. The best results will be obtained by selecting a star of polar distance a few degrees less than the latitude, observing it in the lower part of its apparent path at a suitable altitude, first east of the meridian and later to the west.

(2) *Procedure in observing.*—(*a*) Center the instrument over the station and level plate bubbles carefully. Do not change the leveling during a single set of readings.

(*b*) With telescope direct, sight the star, clamp the horizontal and vertical circles, and record the readings of both verniers.

(*c*) Unclamp the upper horizontal plate and sight the azimuth mark. Record the readings on horizontal verniers.

(*d*) About 10 minutes later repeat the operations with telescope reversed.

(*e*) When the star is approaching the same altitude on the opposite side of the meridian, set the telescope in reversed position at exactly the same altitude as found in the second set. Keep the image of the star covered by the vertical wire and, at the instant it is bisected by the horizontal wire, clamp the horizontal plate and measure the angle to the mark.

(*f*) Repeat the operation with telescope direct and at exactly the same altitude as in the first set.

(*g*) Bisect the horizontal angles between corresponding equal altitude positions of the star and adopt the mean as the true meridian.

f. Exercise XXXIV.—Azimuth determination from the altitude of a star.—By this method find the azimuth of a direction assigned by the instructor, making all notes and computations on form 22, figure 147①.

SECTION XXXI

PRELIMINARY AND LOCATION SURVEYS

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179. General.—The survey operations for a road, railroad, or other route may be classified as—reconnaissance, preliminary survey, and location survey.

180. Study of existing maps.—A study of the latest available maps will generally furnish a dependable guide as to routes or section of country to be examined. Maps of the United States Geological Survey, with contours and topography shown, and maps and aerial photographs of other agencies are very helpful in planning a reconnaissance.

181. Reconnaissance.—The final location of a military road or railroad may be governed more by tactical and strategical considerations than by terrain. After studying the map (or photographs) to determine best possible lines, the engineer should go with the surveyor and observe actual field conditions, recording such information as will assist in deciding the final location.

182. Preliminary survey.—The preliminary survey is based upon results of the reconnaissance and is made with the ordinary surveying instruments. In military practice, the preliminary survey is often conducted as a part of the location survey, and in general for the following purposes:

a. To determine the shortest practicable route.

b. To fix the maximum grade for use in locating the line.

c. To provide a map upon which the "location" can properly be made.

183. Topography.—The result of the preliminary survey is a topographical map, preferably to a scale of not less than 1:20,000, and data for a profile (par. 55), showing the best practical line, with alternate lines or sections if required. Special attention should be given to difficult places such as river crossings, tunnels, and steep grades.

184. Location survey.—*a. Definition.*—The location survey is the final fitting of the line to the ground, with curves laid out to connect straight lines or "tangents," ready for construction.

b. Party.—The party consists of—

1 chief of party, who makes final selection of line.

1 transitman.
1 recorder.
1 stakeman.
2 chainmen.
2 flagmen.
} Transit party.

1 levelman.
1 rodman.
} Level party.

c. Methods.—There are two distinct methods of location. The first is to use the preliminary survey and preliminary profile to locate the line upon the ground. Experience will enable the surveyor to get excellent results in this way. As a rule it is best to lay the tangents first and then the curves. The other method is to use the preliminary line, preliminary profile, and contour lines on the preliminary map, make a paper location, and run this in on the ground.

185. Running center line.—The final line or "location" is staked out by center stakes which mark a succession of straight lines connected by curves to which the straight lines are tangent. The center line is staked, numbered, and measured as explained for profile leveling (par. 55).

186. Curves.—*a. General.*—(1) Circular curves are classed as simple, compound, reverse, and spiral. Only simple circular curves will be considered here. It is a well accepted fact that no reverse curve should be used; a tangent of at least 200 feet should be between curves, and no curve should be less than 300 feet in length.

(2) A *simple curve* is a circular arc, extending from one tangent to the next (fig. 148).

(3) The *measurements of a curve* are made—

(*a*) From *PC* by a subchord (sometimes a full chord) to the next full (100-foot) station, then

(*b*) By full chords of 100 feet each between full stations, and finally

(*c*) By a subchord (sometimes a full chord) to *PT*.

(4) The total distance form *PC* to *PT* measured in this way is the *length of curve L*.

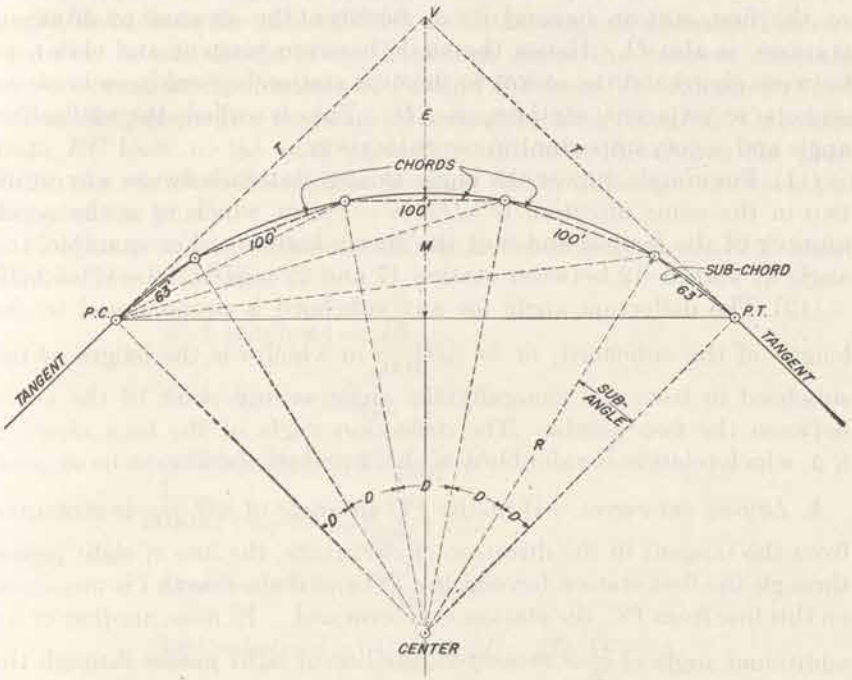
(5) The *degree of curve* is the angle subtended by a chord of 100 feet.

(6) The radius of a 1° curve=½ chord × cot 30 minutes=50 ×114.589=5,729.45 or 5,730 feet. (This should be memorized.)

(7) When a chord of 100 feet subtends an angle of 1° at the center, the curve is said to be a 1° curve. Within the range of curvature used in railroad construction the elements of the curve are approximately inversely proportional to the degree of curve, *D*, and having the elements for the curvature of 1°, those for any other curvature may be obtained by dividing the quantity for 1° by the desired value of *D*. A table in TM 5–236 gives the elements for a curvature of 1°. To obtain the elements for any other curvature, take out the quantities from this table for some central angle and divide them by *D*.

(8) It is desirable to have the number of the station at any point indicate the total length of the line to that point, and hence in pass-

ing from curve to tangent, or from tangent to curve, there should be 100 feet between the last station on the one and the first station on the other. If the point of curve, *PC*, is at a fractional station on a tangent (called a "plus") the curve should begin with a subchord equal to the difference between 100 feet and the plus. If the point of tangent, *PT*, is at a plus, or the curve ends with a subchord, the



PC=Point of curvature, where curve leaves first tangent.
PT=Point of tangency, where curve joins next tangent.
V=Vertex, point of intersection if tangents were produced.
T=Tangent distance, *PC* to *V* and *PT* to *V*.
E=External distance, *V* to *C*.
M=Middle ordinate, distance *C* to *A*.
R=Radius.
D=Degree of curve.
I=Intersection angle.

FIGURE 148.—Elements of simple curve.

first station on the tangent will be at a distance from *PT* equal to 100 feet less the length of the subchord. Only in rare cases will the *PC* fall on a station of the tangent, hence we usually find a subchord at each end of a curve.

(9) The angle between a chord (or subchord) and a tangent to the circle at one of its ends is measured by one-half of the arc corresponding

to the chord or subchord. The angle between a chord or subchord and a secant, or the angle between two secants intersecting on an arc, is measured by one-half the difference of the arcs subtended by the chords or secants.

(10) With equal chords of 100 feet, as in railroad practice, the arc corresponding to a chord is D, the degree of curvature; and the difference of the arcs corresponding to a chord or subchord and the secant to the first station beyond it, or between the secants to adjacent stations, is also D. Hence the angle between tangent and chord, or between chord and the secant to the next station beyond it, or between secants to adjacent stations, is $\frac{1}{2}D$. This is called the deflection angle and is very important in curve location.

(11) For simple curves the angle at any station between any other two in the same direction is $\frac{1}{2}D \times (n' - n)$; in which n' is the serial number of the farther and n of the nearer station. For example, the angle at station 12 between station 17 and $22 = \frac{1}{2}D \times (22 - 17) = 2\frac{1}{2}D$.

(12) The deflection angle for any subchord is proportional to the length of the subchord, or is $\frac{1}{2}D\frac{l}{100}$, in which l is the length of the subchord in feet, and generally the angle at any point of the curve between the two points. The deflection angle of the long chord is $\frac{1}{2}\Delta$, which relation is valuable as a check and should always be so used.

b. Laying out curves.—If at the PC an angle of $\frac{1}{2}D\frac{l}{100}$ is measured from the tangent in the direction of curvature, the line of sight passes through the first station beyond the PC and if the length l is measured on this line from PC, the station is determined. If, now, another or an additional angle of $\frac{D}{2}$ is measured, the line of sight passes through the next station, and if a 100-foot tape is stretched from the previous station and its forward end swung until it is on this line, the point so determined is the next station. Similarly, any station may be located. The above-described method of determining the chords and subchords of a simple curve is called "location by deflections." It is the usual method and is employed for curves of two or more stations whenever possible.

c. Practical lay-out of a curve.—(1) The general case will be that in which two tangents already located are to be connected by a curve. The steps in running in the curve follow:

(2) Run the tangents out to their intersection, if it has not already been done, and measure the external angle or difference of azimuth, which is Δ.

(3) Chain or pace over the ground on which it is desired to locate the curve, determine its approximate length in feet, and point off two places from the right, which gives the length in stations. If a map is available, this distance may be scaled. Divide Δ by this number and the quotient will be D. If this is not a convenient degree of curve to use, take the nearest one above or below it and divide it into Δ for the corrected length in stations.

(4) Take from the table (TM 5–236) the tangent distance T corresponding to Δ; divide it by D, which gives the tangent distance of the curve, or the distance of PC and PT from V. Locate PC and PT on their respective tangents by measuring back from PI. Measure from PC back to the next preceding station to determine the plus. Begin the curve with a subchord equal to the difference between 100 feet and the plus.

(5) Compute and tabulate all the deflections from the PC thus: The deflection for the—

$$\text{first subchord} = \tfrac{1}{2}D\frac{l}{100}$$

$$\text{first chord} = \tfrac{1}{2}D\frac{l}{100} + \tfrac{1}{2}D$$

$$\text{second chord} = \tfrac{1}{2}D\frac{l}{100} + D$$

$$\text{third chord} = \tfrac{1}{2}D\frac{l}{100} + 1\tfrac{1}{2}D$$

$$\text{fourth chord} = \tfrac{1}{2}D\frac{l}{100} + 2D$$

$$\text{last subchord} = \tfrac{1}{2}D\frac{l}{100} + \frac{n}{2}D + \tfrac{1}{2}D\frac{l^1}{100} = \tfrac{1}{2}\Delta$$

In the above tabulation l and l^1 represent the lengths of the first and last subchords in feet, and n the number of 100-foot chords in the curve.

(6) Set up the transit at PC, put the 0–180 line on the tangent, and turn off in the proper direction the first tabulated angle. Measure on this line the length of the first subchord and locate the first station of the curve. Turn off, from the tangent also, the second tabulated angle and locate the second station by swinging a 100-foot tape from the first station. Continue as long as the stations can be seen clearly from PC. If all cannot be seen, shift the transit forward and set on a station which has been determined, and orient by the following rule:

(7) Set the vernier at the reading in the tabulated deflections corresponding to any convenient preceding station, point to that station, and clamp the limb. Plunge the telescope and locate forward sta-

tions by using the tabulated deflections originally computed for those stations, precisely as if still working from *PC*. In using this rule remember that the deflection corresponding to *PC* is the azimuth of the back tangent from which the deflections were started.

(8) Usually the forward tangent will not have been run out much beyond *PT*. Measure off on it from *PT* a distance equal to 100 feet minus the last subchord of the curve, and locate the station next to the curve on the tangent. From this run out the tangent, setting stations 100 feet apart until another curve is reached.

(9) When the *PI* is inaccessible, it may be necessary to make several changes of direction, as in figure 150. The point *V* would be

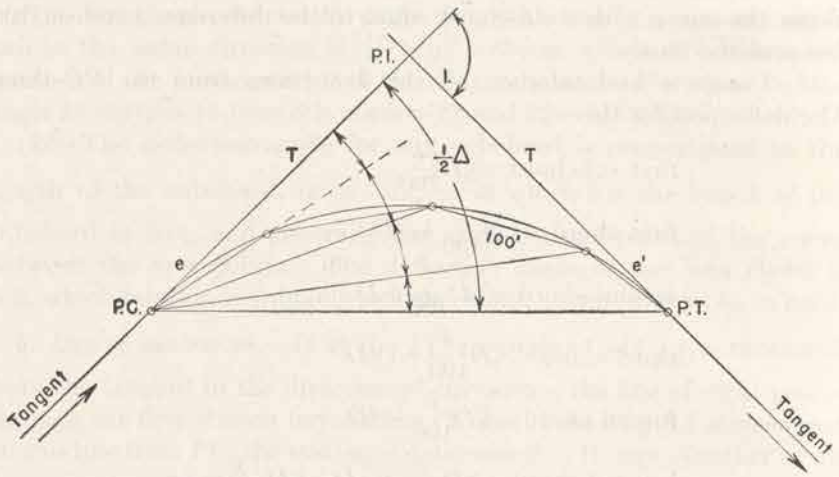


FIGURE 149.—Laying out of simple curve by deflection angles.

the *PI*, but it is inaccessible. The line is run along *APQB*, *QB* being the desired direction of the new tangent. The external angle is then equal to the sum of the deflection angles at *P* and at *Q*. In the triangle *QPV* all the angles and the side *PQ* are known. Solve the triangle for *QV* and *PV*. Find the tangent distance *VB* and *VA*, and lay off from *Q*, *QB* equal to *VB* minus *VQ*; and from *P*, a distance equal to *VA* minus *VP*. The points *B* and *A* thus located are the *PT* and *PC*, respectively. The curve is then laid out as heretofore described.

(10) As errors will creep into the location of a curve if very long lines of sight are used in laying it out, it is customary not to lay out more than four to six stations from one point; therefore, on curves longer than this, two or more settings of the instrument will be necessary.

d. Vertical curves.—At each change of grade a different vertical angle is formed. To avoid sudden bumps it is customary to introduce a vertical curve at such points. The curve commonly used is a para-

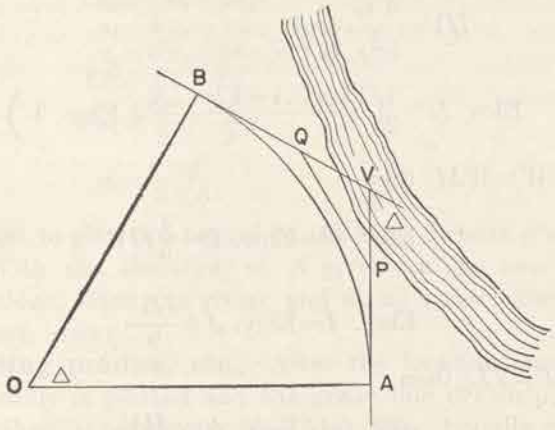


FIGURE 150.—Point of intersection inaccessible.

bola, as it effects the transition better than a circle. This curve is generally laid to extend an equal number of stations on each side of the vertex. Two cases are shown in figures 151 and 152, respectively.

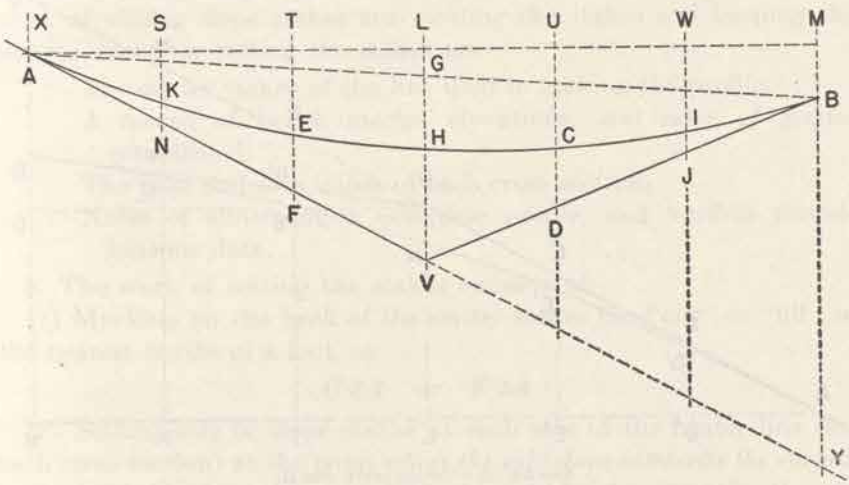


FIGURE 151.—Vertical curve, case I.

Let AVB be the known points, and AHB the parabola. Join AB and produce AV to Y. Draw vertical lines AX, LV (through H) and MY (through B). Let AV and VB be equal, also their horizontal

projection $AL=ML$. Then $AG=GB$ and $AV=VY$. VG is a diameter of the parabola, also AX is a diameter. Now

$$HV=\frac{VG}{2}, \text{ and}$$

$$\text{Elev. } H=\frac{1}{2}\left(\frac{\text{Elev. } A+\text{Elev. } B}{2}+\text{Elev. } V\right).$$

Let $LU=UW=WM$; then

$$\text{Elev. } C=\text{Elev. } D+\frac{4}{9}HV$$

$$\text{Elev. } I=\text{Elev. } J+\frac{HV}{9}.$$

Let $AS=ST=TL$; then

$$\text{Elev. } K=\text{Elev. } N+\frac{HV}{9}$$

$$\text{Elev. } E=\text{Elev. } F+\frac{4}{9}HV.$$

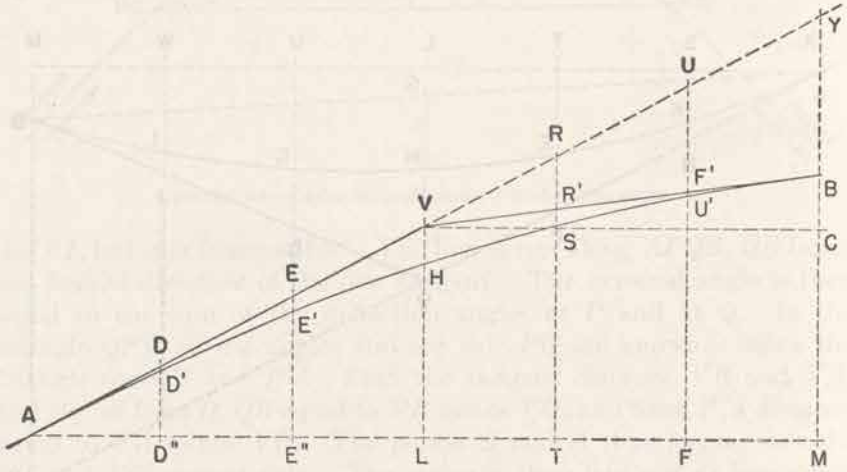


FIGURE 152.—Vertical curve, case II.

Given the rates of grade (g) of AV; (g') of VB; the number of stations (n), half on each side of the vertex, covered by the vertical curve; and elevation of A. (See fig. 152.)

To obtain the elevations of D', E', H', etc.,

Let $a_1 = DD^1$, $a_2 = EE^1$, etc.

Then
$$a_2 = 2^2 a_1 = 4a_1$$
$$a_3 = 3^2 a_1 = 9a_1$$
$$a_4 = 4^2 a_1 = 16a_1 \ldots\ldots$$
$$a_6 = 6^2 a_1 = 36a_1 = YB, \text{ and}$$
$$YB = YC - BC$$
$$6^2 a_1 = \frac{6g'}{2} - \frac{6g'}{2}$$
$$a_1 = \frac{g - g'}{2 \times 6}$$

It is important to give due regard to the signs of both g and g' in the formulas. With the elevation at A given we can now find the required elevations, since g is given, and a_1, a_2, a_3, etc., are easily computed as shown above.

187. Plotting profiles, etc.—After the location survey is completed the profile is plotted and the grade line drawn on the profile plot as described in paragraph 54, TM 5–230. Usually cross sections showing the existing ground in relation to the finished ground are required. For field work relating to cross sections, see instructions in paragraph 56. The plotting of cross sections from field notes is explained in paragraph 55, TM 5–230.

188. Setting slope stakes.—*a.* The two important parts of the work of setting slope stakes are—setting the stakes and keeping the notes. Data for setting the stakes are—

The center stakes of the line used in making the profile.
A record of bench marks, elevations, and rates of grades established.
The base and side slopes of each cross section.
Notes of alinement, a complete profile, and various miscellaneous data.

b. The work of setting the stakes consists of—

(1) Marking on the back of the center stakes the "cut" or "fill" to the nearest tenths of a foot, as

C 3.7 or F 2.4

(2) Setting side or slope stakes at each side of the center line (for each cross section) at the point *where the side slope intersects the surface of the ground* and marking upon the inner side of each stake the cut or fill at that point.

(3) To find the cut or fill at the center stake, first obtain the *rod reading* for grade which is equal to the *HI* (height of instrument) minus the elevation of grade. Next obtain the rod reading for

the ground at the center stake and subtract same from the difference between the *HI* and the finished grade. If this difference is larger than the rod reading, the result indicates a "cut"; if smaller, a "fill."

(4) To set stakes for side slope when the surface is level (see fig. 153)—

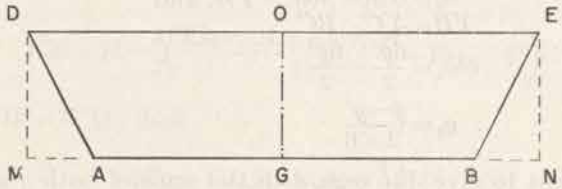


FIGURE 153.—Slope stakes on level surface.

Let $b=AB$ (base of finished section).
$c=OG$ (center height).
$s=\frac{NB}{EN}=\frac{AM}{DM}$ (side slope).
$d=OD=OE$ (distance out).

Then $d=GB+BN$
$=\frac{1}{2}\,b+s\times DM=\frac{1}{2}\,b+s\times EN$

d, of course, establishes the distance from O for E and D at which points the stakes are set.

(5) To set stakes for side slopes when the surface is not level the process is less simple. Referring to figure 154:

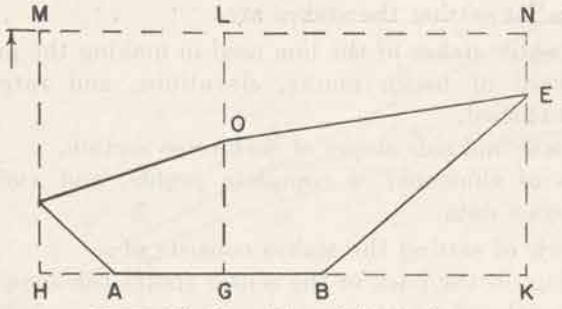


FIGURE 154.—Slope stakes on inclined surface.

(*a*) Let $b=AB$ (base of finished section).
$c=OG$ (center height).
s=side slope.
h_r=side height, right.
h_l=side height, left.
d_r=distance out, right.
d_l=distance out, left.

Then $d_r=\frac{1}{2}\,b+sh_r$
$d_l=\frac{1}{2}\,b+sh_l$

(*b*) But h_r and h_l not being known, these, together with corresponding d_r and d_l are best secured upon the ground by approximation.

(*c*) Having calculated c (center height OG, or "cut"), make an estimate as to the probable value of h_r at the point where the side slope will intersect the surface, and figure $d_r = \frac{1}{2}b + sh_r$ to correspond. Measure out the distance so found, set rod at new point, and take rod reading to surface. If the cut or fill found (from the rod reading)

CROSS-SECTIONS — Lacy to Spring Station — Road bed 20 ft in cut, 16 ft in fill. Slope 1½ : 1 — July 7, 1939 — C.C.R. Cloudy — Final location — Instrument: Sgt A.A. Borr — Recorder: Cpl J.C. Lee — Rodman: Pvt A.I. Muir — (24)

Sta.	B.S.	H.I.	F.S.	Elev.	Grade	Left	Center	Right	Remarks
⊡ L15	2.67	166.98		(164.31)					Concrete monument
15			1.9	165.1	156.00	C 8.6 / 22.9	C 9.1	C 11.2 / 26.8	
16			4.0	163.0	155.60	C 5.9 / 17.5	C 7.4	C 8.4 / 22.6	
17			4.5	162.5	155.20	C 4.6 / 16.9	C 7.3	C 8.0 / 22.0	
18			8.0	159.0	154.80	C 1.8 / 12.7	C 4.2	C 5.1 / 17.7	
+25			9.2	157.8	154.70	0.0 / 10.0	C 3.1	C 2.6 / 13.9	
T.P.	1.94	157.19	(11.73)	(155.25)					Iron pin.
+90			2.8	154.4	154.44	F 1.8 / 10.7	F 0.0	C 2.4 / 13.6	
19			3.4	153.8	154.40	F 3.2 / 12.8	F 0.6	0.0 / 4.0 — C 1.0 / 11.5	
+20			5.6	151.6	154.32	F 3.6 / 13.4	F 2.7	0.0 / 8.0	
20			11.1	146.1	154.00	F 8.2 / 20.3	F 7.9	F 6.3 / 17.5	
+40			11.2	146.0	153.84	F 8.0 / 20.0	F 7.8	F 7.8 / 19.7	Top of bank
+45			14.6	142.6	153.82	F 11.2 / 24.8	F 11.2	F 11.2 / 24.8	Waters edge (creek)
+60			14.5	142.7	153.76	F 11.1 / 24.7	F 11.1	F 11.1 / 24.7	Waters edge (creek)
+65			11.6	145.6	153.74	F 8.2 / 20.3	F 8.1	F 8.0 / 20.0	Top of bank
21			10.9	146.3	153.60	F 7.6 / 19.7	F 7.3	F 7.2 / 18.8	
22			9.7	147.5	153.20	F 7.6 / 19.4	F 5.7	F 3.1 / 12.7	
23			10.8	146.4	152.80	F 7.8 / 19.7	F 6.4	F 2.6 / 11.0	
T.P.	11.96	159.69	(9.46)	(147.73)					X in boulder
	16.57	164.31	21.19						
		159.69	16.57						
		4.62 =	4.62	check					
									Checked: J.C.L.

FIGURE 155.—Cross section notes.

yields a value of d_r equal to the value measured out, the point is correct. If not, make a new approximation using the better data just obtained, always starting with h_r and calculating d_r. If necessary, repeat the process until a point is reached that is correct. Then set the stake and mark the cut or fill corresponding to h_r upon the inner side.

(*d*) In a similar way find $d_l = \frac{1}{2}b + sh_l$, and set and mark the stake.

c. A form of keeping notes is shown in figure 155. Surface elevations being given, all other data are to be recorded as shown.

SECTION XXXII

CONSTRUCTION SURVEYS

	Paragraph
General	189
Roads	190
Railroads	191
Water reservoirs	192
Miscellaneous construction	193
Utilities maps	194

189. General.—For construction surveys the use of instruments and methods relating to plane surveying are commonly used—namely, transit, planetable, level, and tape. Stadia, as a rule, is seldom employed. The principles described for road and railroad surveys are applicable in a lesser degree for other route surveys, such as drainage ditches, canals, power transmission lines, etc.

190. Roads.—*a. Hasty locations.*—In forward areas of the combat zone where careful location is prohibited by lack of time and enemy activities, roads and trails are located largely by eye after personal reconnaissance. In rear areas the location is as careful as conditions permit, since careful location economizes material and labor. In either case, the road is located by the use of a map, if available, and aerial photographs. If neither of these is available, sketch maps may be made by the usual reconnaissance methods. The location, while primarily such as will satisfy the demands of military operations, aims at keeping the earthwork (cuts and fills) to a minimum and economizing in the haulage of surface materials. The alinement and grade of the road are indicated to the constructing troops by means of stakes on the center line at 50-foot intervals. The cut or fill is marked on the side of the stake.

b. Careful location.—When time is available, the steps in the location of a road are—

(1) The reconnaissance (par. 181) with the best available map to determine which of several possible routes should be selected.

(2) The preliminary location (par. 182), according to the accurate methods of topographic surveying, and including a belt of terrain on the general route selected. This provides the data for final location and construction. The main traverse is run on the most probable location.

(3) The final location of the center line of the road (par. 184) as determined from the map and notes of the preliminary location. This is placed on the map and then staked out on the ground.

c. Curves.—The principles for laying out curves for roads are the same as described in paragraph 186. For roads of a more or less

permanent character which have abruptly changing grades, vertical curves also (par. 186*d*) are laid out and constructed.

d. Stream crossings.—Stream crossings, bridges, ferries, or fords, should be located as far as practicable so that approaches are easy and on grade with the bridge or ferry floor, or in case of fords, gradually descend to the water's edge.

e. Earthwork computation.—Computations for earthwork, when made, are usually obtained from data gathered during profile and cross level operations (pars. 55 and 56). Volume computations for quantities of earthwork to be moved are explained in section X, TM 5-230.

191. Railroads.—*a. General.*—The methods of surveying for a standard gage railway line in accordance with the refined standards of civil practice are beyond the scope of this manual. The principles and methods described herein are used in the kind of work commonly undertaken in the combat zone, in the hasty construction of other railway facilities, and in military supply installations set up in the communications zone. The survey for roadbed comprises the following steps, in order:

(1) A preliminary reconnaissance of the terrain for the selection of the general location of the line (pars. 180 and 181).

(2) A survey of the line selected to obtain topographical data upon which to base the exact location of the line (par. 182).

(3) A construction survey for setting stakes to give line and grade to the construction party (pars. 184 to 188).

b. Preliminary survey.—(1) After reconnaissance of favorable routes and determination of the general route to be followed, a preliminary survey is begun. The officer who made the reconnaissance should be in charge of the survey.

(2) *Organization.*—The survey party is organized into three groups: A *transit party* consisting of one instrumentman, two chainmen, two flagmen, and a number of axmen; a *level party* consisting of one levelman and one or two rodmen; and a *topographical party* consisting of one topographer and two rodmen.

(3) *Procedure.*—The locating engineer's place is at the head of the transit party. The head chainman works under his direct supervision. The locating engineer selects the points where it is necessary to change the course and these points are marked by driving hubs (not less than 2 inches in diameter) in the ground. A tack is placed in each hub to indicate the center line.

(4) *Stationing.*—The chainmen measure the distance along the center with a 100-foot chain, setting stakes every 100 feet to designate

stations which are numbered serially, beginning with No. 1 (par. 55). The numbers are marked on the stakes with crayon.

(5) *Hubs.*—The head flagman is directed by signals from the transit man as to the center line of the survey. He locates the point where hubs are to be placed and the points where tacks are to be driven in the hubs. The rear flagman holds a flagpole on the tack in the first hub directly back of the transit man for the back or rear sight.

(6) *Clearing.*—The axmen clear away the brush and obstructions and work under the direction of the head chainman. One axman should be detailed to make stakes for the transit party.

(7) *Notes.*—The method of recording the transit notes is illustrated in figure 156.

Preliminary Survey (Transit notes)
Coal River Extension
PRELIMINARY SURVEY

Instrument Sgt. R. Roe
Recorder Cpl. J. Doe
Hd. Rodman: Pvt. H. Jones
Rr. Rodman: Pvt. T. Smith

Berger #11611

(9)

Stations	Deflection	Course	Remarks
46+00 P.O.T.		N 26° 30' E	
39+40 ⊙	5° 30' L	N 32° 00' E	12" oak 15' N. 15" sycamore 30' S.
38+00 ⊙	26° 00' L	N 58° 00' E	40' South Jake. Pettry d'w'g. S.E. corner.
36+60 ⊙	22° 30' L	N 80° 30' E	Opp. pt of hill 50' below falls at Pettry's ford.
30+00 P.O.T. BS			
			Date: July 15, 1939
			Weather: Cloudy
			Temp.: 75°

FIGURE 156.—Transit notes, preliminary survey.

(8) *Levels.*—The level party establishes bench marks and levels along the traverse, taking a reading and recording the elevation at each 100-foot station. The elevation of the starting point, if not definitely known, is assumed at some round figure. Two or more bench marks should be established in every mile. The method of recording level notes is illustrated in figure 157. These notes are used to assist in preparing a contoured map, and the bench marks are used as checks in making the field location.

(9) *Topography.*—The topographic party records data from each side of the center line by stadia readings, obtaining that necessary for contours and locating features of the landscape over a strip extending at least 500 feet on each side of the center line.

c. Paper location.—Upon the map and profile prepared from the notes taken in the preliminary survey, a "paper location" of the proposed railway line is projected (figs. 158 and 159). The line of the preliminary survey is shown in figure 158 as a broken line, while the located line is made solid. The grade elevation for each station

Preliminary Survey (Level notes) Coal River Extension — Instrument: Sgt. R. Roe, Rodman: Cpl. J. Doe — B&B Dumpy #12320 — (10)

PRELIMINARY		SURVEY			
Sta	Rod	R'd'g.	H.I.	Elev	Remarks
B.M.		5.50	130.50	125.00	Nail in syamore root Sta 29+90 6'N 6" beech
30+00		4.3		126.2	10' E
1		6.4		124.1	
2		6.9		123.6	Date: July 16, 1939
3		7.5		123.0	Weather: Cloudy
4		5.7		124.8	Temp 79°
T.P.	-F.S.=	2.20			
	+B.S.=	8.60	136.90		
35+00		11.8		125.1	
6		10.9		126.0	
7		1.7		135.2	
T.P.	-F.S.=	2.10			
	+B.S.=	14.60	149.40		
38+00		11.4		138.0	
9		9.9		139.5	
40+00		11.4		138.0	
1		8.9		140.5	
B.M.		6.62		142.78	Spike in river boulder.

FIGURE 157.—Level notes, preliminary survey.

on the line being located is taken from the profile (fig. 159) or is obtained by interpolating between the contours in figure 158; points having the required elevation are located opposite the corresponding stations of the preliminary survey. These points are inclosed in a circle and marked with the number of the station of the preliminary survey to which they correspond. A line joining the points thus marked is called the grade contour. The paper location is made by selecting a line that will nearly conform to the configuration of the ground, in order to avoid heavy cuts and fills. An approximate

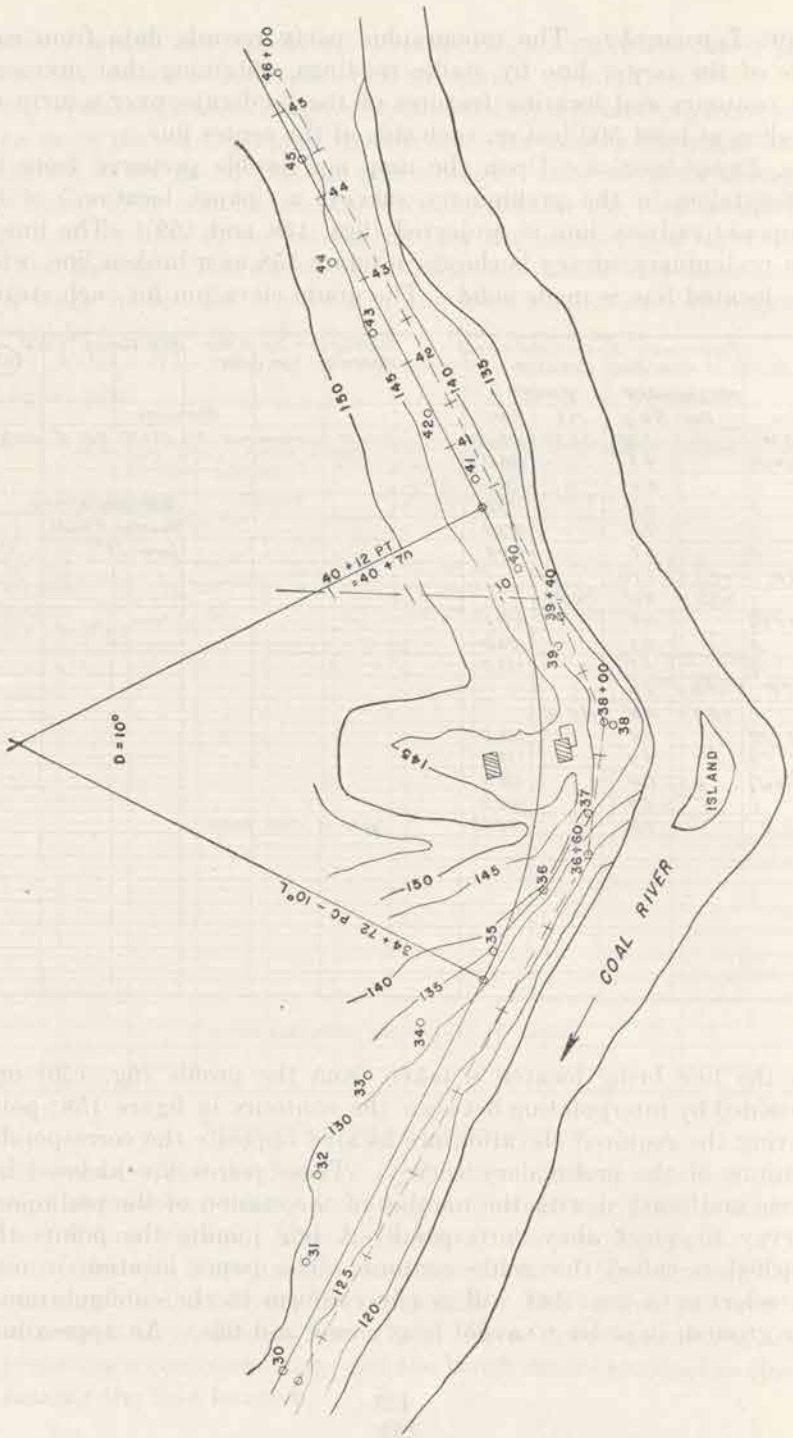


FIGURE 158.—Map from preliminary survey showing paper location.

profile of the ground along the paper location is then made and approximate grades established, with particular attention to balancing the cuts and fills. This paper location is taken into the field and used by the final locating parties.

d. Field location.—One or more field parties may be organized in accordance with the urgency of the situation. If only one party is organized, it establishes the first tangent in the field by scaling (from the paper location) distances from the original traverse and establishing base points. It then establishes the second tangent in the same manner, produces these two tangents to a common point of intersection, and measures the central angle. With this point of intersection and the degree of curvature shown on the paper location, the party then establishes the tangent distance, which is measured back from the point of intersection to the point of curve and ahead from the point of intersection to the point of tangent. The stationing is established on the tangent up to this curve, and the

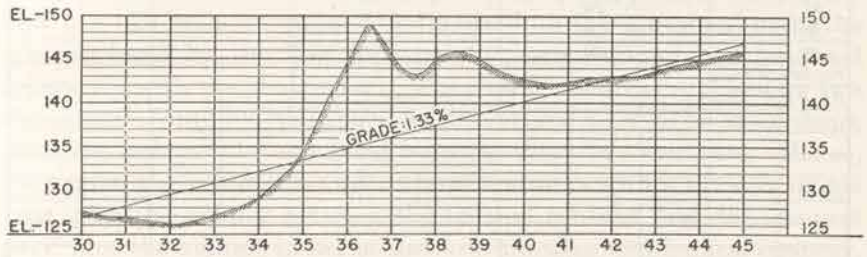


FIGURE 159.—Preliminary profile and paper location.

correct station of the point of curve is then determined; the instrument is set up on this point of curve and the curve laid out as described in paragraph 186. The party next proceeds with the stationing on the second tangent until it reaches approximately the point of curve of the next curve. The third tangent is then established in the same manner as described above, the point of intersection fixed, the degree of central angle measured, and the curve laid out again in the same manner as described for the first curve. The notes kept by this party are illustrated in figure 160. If two or more parties are sent to the field to establish the line, the line is divided into several sections according to the number of parties and each party begins with the establishing of the first tangent within its section and proceeds as described above, with this difference, that it does not establish the stationing on the line but merely determines by stakes the points of curve and points of tangent, so that the party following can proceed without interruption. In other words, the second, third, or fourth parties will only lay out

the line, chopping down trees and clearing brush where necessary so as to have a clear sight for establishing the tangents and points of curve. The chaining is carried on by party No. 1, which will then upon arriving at the section of party No. 2 or 3 lay out the curves as found necessary. Where two parties join and where revisions have been necessary in the paper location, a modification will have to be made in the curve joining the tangents of the two adjacent sections, but the degree of curvature on the paper location should be adhered to if possible. After the location has been definitely established in the field and stakes

Final Location (Transit notes)
Coal River Extension
LOCATION

Inst. Sgt. R. Roe
Recorder Cpl. J. Doe
Front Rod: Pvt. H. Jones
Rear Rod: Pvt. T. Smith

Berger #1611

(12)

Station	Def.	Chk. Def.	Course		Remarks
45+00 P.O.T.			N 27° 10' E	D = 10°	Date July 30, 1939
P.T. 40+12 = 40+70 ahead	27° 00'	54° 00'		A = 54° 00'	Weather: Clear
40	26° 24'			T = 291.94'	Temp: 90°
39	21° 24'			L = 540'	
38	16° 24'				
37	11° 24'				
36	6° 24'				
35	1° 24'				
P.C. 34+72 ○ 10° L.					
B.S. P.O.T. 30 +00			N 81° 10' E		

FIGURE 160.—Transit notes, final location.

placed every 100 feet on the line, a leveling party is organized to take levels at every one of these stakes, establishing bench marks as it goes along. This leveling party should follow immediately after party No. 1 so as not to delay the work. The level notes kept by this party are similar to those in figure 157.

e. Construction stakes.—In order to proceed with the grading, slope stakes are set out, being placed at the top of a slope when in a cut, and at the foot of a slope when on a fill. (See par. 188.) The construction party grades up to or down from these stakes. No further

instrumental work is necessary except for the definite location of culverts, abutments, or similar structures. It is advisable to place stakes at 50-foot intervals on sharp curves. For curves up to 6 it is not necessary to place stakes closer than 100 feet apart for military railroads.

f. Exercise XXXV.—With a party properly organized and equipped, lay out as instructed the "location" for a railroad bed, including necessary curves and set all grade and slope stakes as described in section XXXI and in this paragraph.

192. Water reservoirs.—*a. Areas and volumes.*—(1) For a preliminary determination of the capacity of a reservoir a study of a contoured map will ordinarily be sufficient. The position of the flow line, marking the outline of the submerged area of a proposed reservoir can be plotted, and the area of the drainage basin can readily be found with the planimeter. Two general methods used in determining the capacity of reservoirs are by contours and cross sections.

(2) The contour method gives more accurate results. After the flow line has been determined and plotted on the contoured map, the area enclosed by the flow line and by each contour is determined separately with the planimeter. The average of areas enclosed by two consecutive contours, multiplied by the vertical interval between them, gives the volume of water lying between these two contours. Hence, the sum of the volumes between all the contours within the submerged area, plus the volume between the bottom contour and the deepest part, plus the volume between the flow line area and highest contour, gives the capacity of the reservoir. The volume between the bottom contour and the deepest part is generally small and is estimated or neglected. The total volume is usually expressed in acre-feet, one acre-foot being 43,560 cubic feet, equal to 325,851 gallons.

(3) The cross section method may be used when only a fair degree of accuracy is required. After the flow line has been plotted the area is divided into sections of convenient lengths, usually 100 feet. The sections are plotted on cross section paper and the end areas determined with the planimeter. The approximate volume can then be calculated from the average end areas as described in section X, TM 5–230.

b. Flooded area and drainage basin.— The source of the water supply stored or distributed from a reservoir is the drainage basin. This must be large enough, considering all factors such as the annual yield of water from a stream, precipitation, evaporation, leakage, etc., to meet the maximum consumption at all times, especially during dry seasons of the year when water is most needed. It is best when the drainage

basin, especially near populated centers, can be made inaccessible to the general public. The flooded area should be fenced if the drainage basin cannot be completely secured against possible intrusion. Very often the drainage basin, and in all cases the flooded area, will be on property owned, leased, or otherwise controlled by the military authorities. Before acquiring control of land for the flooded area of a reservoir a careful survey of the flow line must be made to make sure

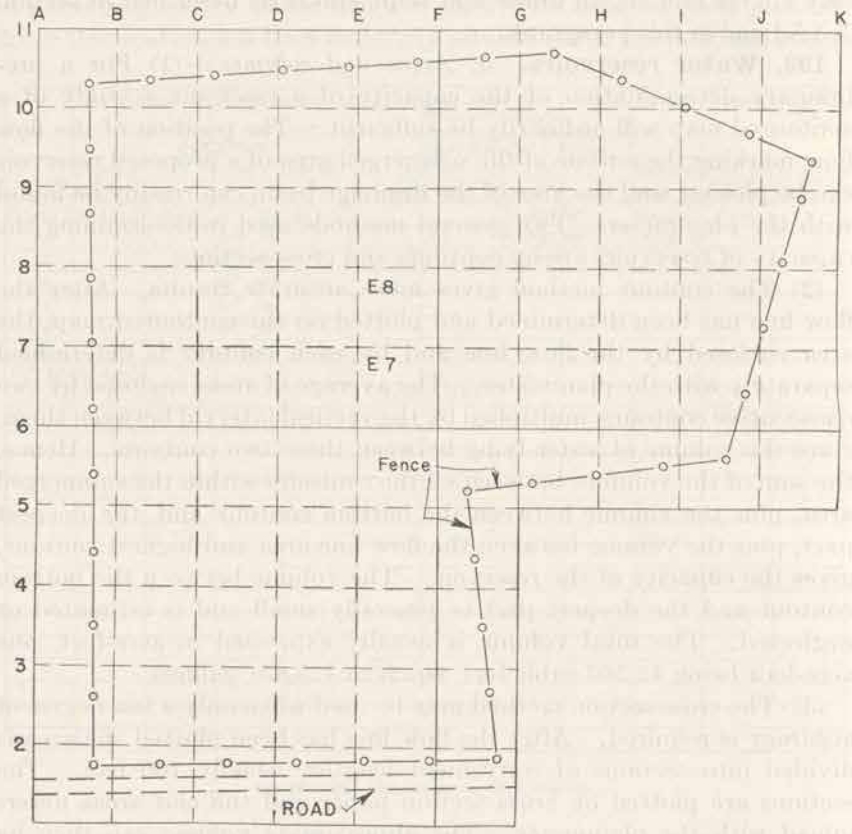


FIGURE 161.—Coordinate-point method.

that enough land is acquired so that the area after being flooded will not encroach on the property of others. This calls for precise work with the tape, transit, and level. Here the level is used first to locate the contour line representing the maximum stage of impounded water plus (the flow line). Then, with a transit and tape enough points are located (with a traverse) for boundary monuments or marks so that the area within them will embrace every part of the flooded area.

193. Miscellaneous construction.—*a. General.*—Surveys for miscellaneous construction include those for the location of structures and those for the determination of earthwork. The accuracy required in large-scale surveys is attained by either the trace-point or the coordinate-point methods. The coordinate-point method, being used more than any other, is described in *b* below. Details for large-scale maps may be located by a variety of methods and instruments.

b. Coordinate-point method.—(1) This method may be applied in the field in many ways, the principle being to establish control and to subdivide the area in question into squares with sides equal to multiples of 50 feet.

(2) The first step is to run a rectangular traverse as in figure 161 near the perimeter of the tract, setting stakes at every 100 feet (or as required) as the traverse proceeds. Next, parallel lines are laid off from each 100-foot point of the traverse. One set of parallel lines is lettered *A*, *B*, *C*, etc; the other set perpendicular thereto is numbered *1*, *2*, *3*, etc; so that each point actually is known as *A1*, *A2*, *B1*, *B2*, etc.

(3) Or coordinate axes may be established to cross near the center of the tract, and coordinate lines may be run parallel to these axes to the corners of the tract.

(4) Often, after the traverse has been run around the tract or the coordinate axes have been established, interior points are laid out with tapes only. Three chainmen and two 100-foot tapes are needed. They work as follows: With one chainman at *F7* (fig. 161) and a second at *E6*, the third chainman locates *E7* with the two 100-foot tapes stretched and meeting, and sets a stake. Proceeding, the chainman at *F7* goes to *F8* and the chainman at *E6* to *E7*, while the third chainman locates *E8*. The remaining interior points are located in a similar manner.

(5) Distances are usually not recorded. Elevations of each point are observed and recorded as for profile leveling, or are noted on the plot.

(6) The plane table is a great aid to mapping by the coordinate method if many irregular features are found. The methods of establishing the points are the same as described above. The details, however, are drawn on a plane table sheet instead of from notes or sketches in the office as is usually the case when a transit and tape are used.

c. Batter boards.—When excavation is required for structures, the first step is to set stakes that will serve as a guide in digging. Before the excavation reaches its full depth, the surveyor usually sets batter boards which are horizontal boards nailed to posts or other supports.

The upper edges of these boards are usually set at some certain elevation, as the top of a foundation wall. They are also in such position (fig. 162) that by stretching strings from one to another the contractor can get the most important lines (neat lines) of the structure. The exact points where the strings should be fastened to the boards are marked by nails or notches cut in the board.

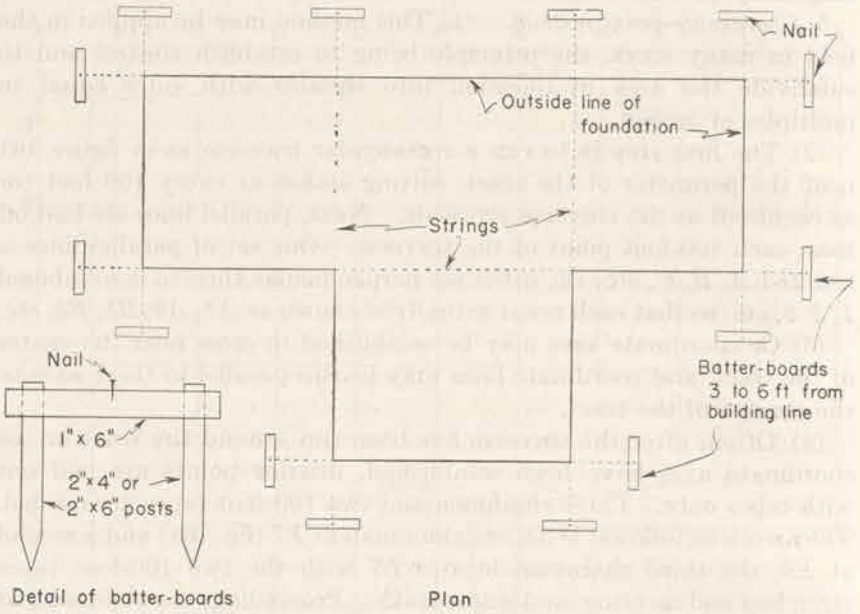


FIGURE 162.—Batter boards.

Batter boards are set a few feet out from the building line so they cannot be knocked out during the excavation. The contractor usually furnishes the surveyor with the necessary data so that the building lines he is to establish will be properly located with respect to street line, adjacent buildings, etc. No notes are kept.

194. Utilities maps.—*a. General.*—Utilities maps are made to large scales and show specified areas such as towns, military posts, cantonments, and similar places of permanent or semipermanent character. Their principal purpose is to serve as a guide for eventual extension of construction and as a permanent record of utilities installations.

b. Detail.—They show in detail all buildings, roads, sidewalks, sewer lines, water lines, gas lines, manholes, main valves, fire hydrants, electric power lines, street lights, telephone lines, and any other items

that may be useful to the construction or maintenance section serving the particular locality. Frequently they have contours, especially if additional construction is anticipated. Utilities maps may be advantageously used for planning landscape lay-outs and similar projects. Trees, bushes, hedges, etc., are usually omitted to avoid the observing of more important detail.

c. Scale and contour interval.—The scale of utilities maps is usually from 1 inch = 50 feet to 1 inch = 200 feet, and the normal interval of contours (when shown) is 1 or 2 feet.

d. Field work.—The field work consists of—

(1) Horizontal control, established by transit and tape, with perhaps some triangulation for larger areas.

(2) Vertical control, by differential leveling.

(3) Plotting detail, best done by the coordinate-point method (par. 193*b*) combined with use of the plane table. Coordinate points are established as described, to be occupied in turn by the plane table for plotting detail.

e. Symbols.—Symbols for representing detail on utilities maps are the same as prescribed in FM 21–30. Special symbols to represent objects not covered in FM 21–30 are shown in figure 163.

f. Office work.—The final map is compiled by the draftsman by methods described in TM 5–230. A careful record of all items that cannot be shown properly on the map is compiled from data gathered by the survey party and kept in a special file properly indexed. Figure 164 is part of a completed utilities map.

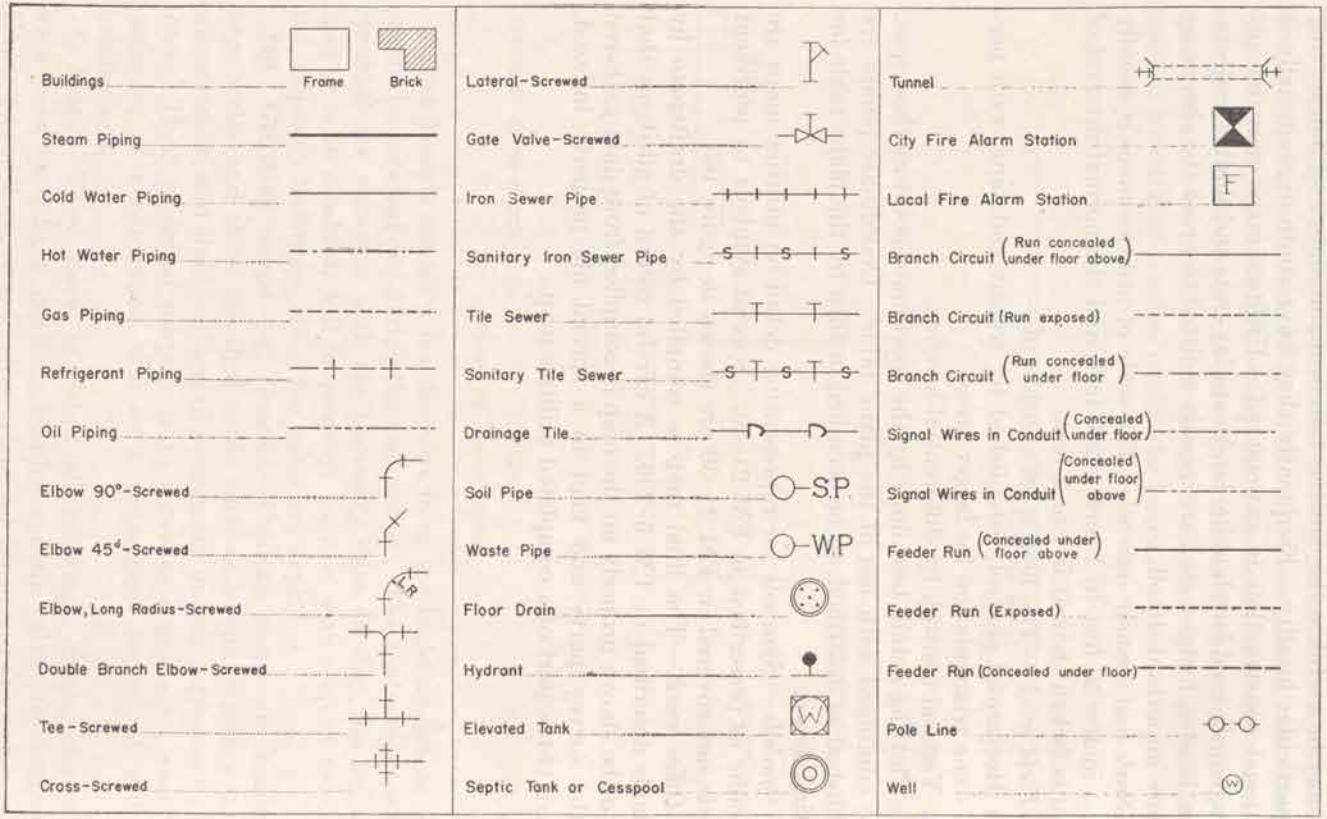


FIGURE 163.—Special symbols for utilities maps.

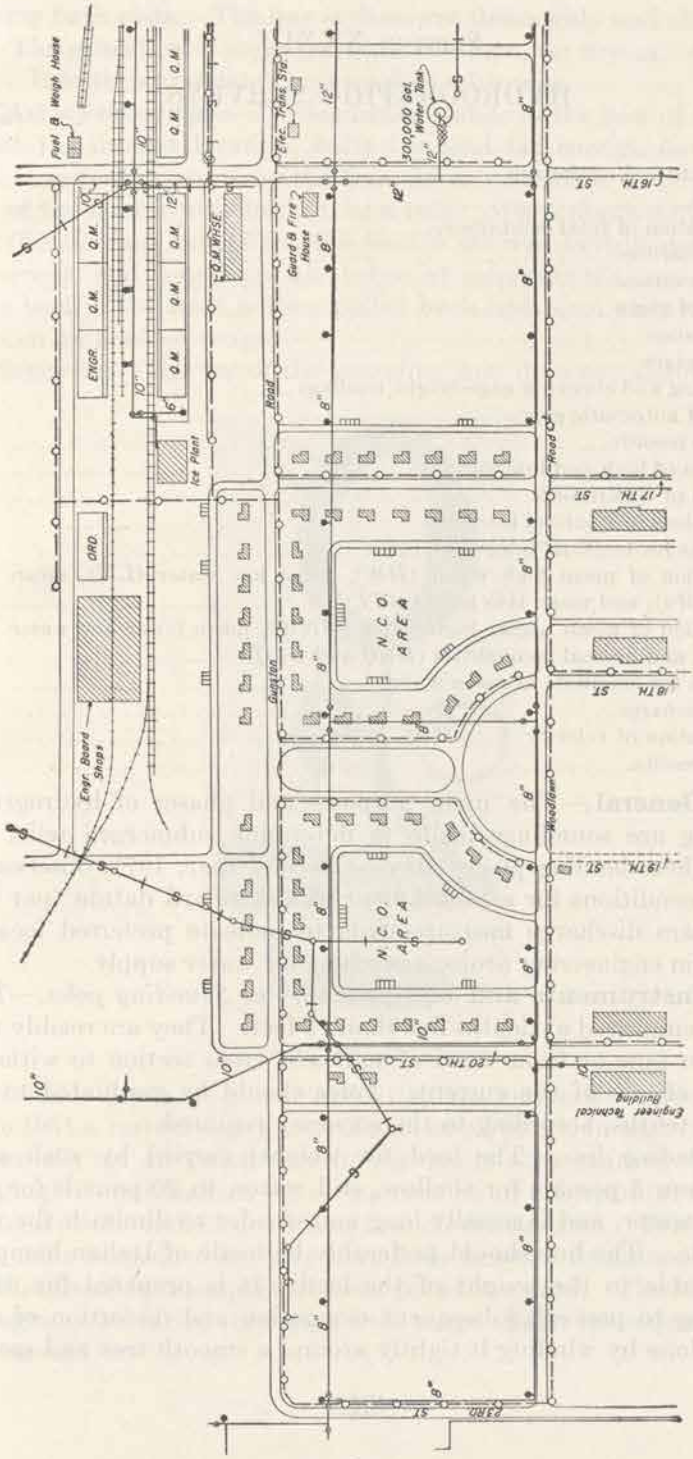


FIGURE 194.—Part of completed utilities map.

SECTION XXXIII

HYDROGRAPHIC SURVEYS

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195. General.—The main purposes and phases of hydrographic surveying are soundings made to determine submerged relief near shores where landing places may be needed (par. 197); observations of tidal conditions for establishment of a standard datum (par 198); and stream discharge measurements to indicate preferred locations for certain engineering projects, such as for water supply.

196. Instruments and equipment.—*a. Sounding poles.*—These are best employed at depths less than 15 feet. They are readily made from iron pipe or from wood of sufficient cross section to withstand bending effects of the current. Poles should be graduated to read feet and tenths, according to the accuracy required.

b. Sounding line.—The lead (or weight) carried by such a line varies from 5 pounds for shallow, still water, to 20 pounds for deep, running water, and is usually long and slender to diminish the water resistance. The line should preferably be made of Italian hemp of a size suitable to the weight of the lead. It is prepared for use by stretching to prevent subsequent elongation and distortion of scale. This is done by winding it tightly around a smooth tree and securely

fastening both ends. The line is then wet thoroughly and allowed to dry. The operation is repeated until the rope, on drying, leaves no slack. It is then graduated and marked with tags.

c. Making soundings.—The leadsman stands in the bow of the boat and, at the desired location, casts the lead far enough forward to enable it to reach bottom by the time the line becomes vertical. The speed of the boat is not checked, as a rule. When depth and current make this method impossible, the boat is allowed to drift down with the current, and soundings are taken at intervals without drawing up the lead. The boat is then pulled back upstream and soundings are taken on another range.

d. Sweeping.—The use of the sounding line does not assure a definite available depth, as pinnacle obstructions may exist. In order to be sure that a certain depth is available at all points in a given water area, usually at all points within the limits of a marked channel, "sweeping" is undertaken. A sweep consists of a horizontal member suspended at the desired depth and towed over the area. It is usually a wire cable suspended by wire lines from buoys. Sometimes heavy bars or railroad rails suspended from scows are used. The length of the drag may be varied through a wide range to suit the requirements of the survey.

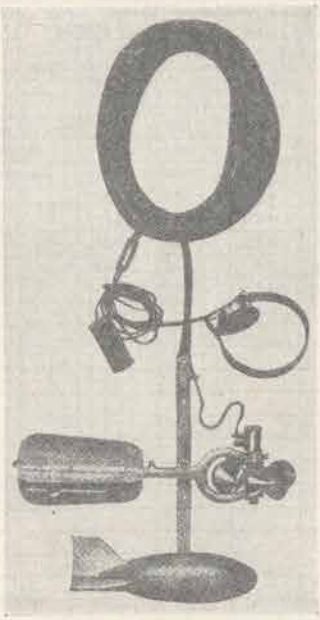


FIGURE 165.—Price current meter.

e. Current meters.—(1) A current meter consists of a propeller and a mechanism to indicate or record its revolutions. The recording mechanism of practically all meters is based upon the make and break of an electric circuit at each revolution. Current meters are mounted on poles or suspended from ropes or cables with heavy weights so that velocities may be determined at any desired depth.

(2) There are two general classes of these instruments—

(*a*) Those in which the revolving part turns about a vertical axis, as in the Price meter (fig. 165).

(*b*) Those in which it turns about a horizontal axis, as in the Haskell and Fteley meters.

Stream discharge Current meter rating

Location: Palo River Wind: SE Light Date: Nov 11, 1939 Current Meter No. A-312 (5)

Make: Price Length of course: 100 ft

Run	Time	Register Start	Register End	Revolutions	R.p.s.	Velocity (f.p.s.)			Remarks		
1	180	1,892	1,927	35	0.194	0.555	Chief of party		Sgt. Blank		
2	178	1,927	1,963	36	202	.562	Recorder		Cpl Burr		
3	60	1,987	2,023	36	.600	1.666	Assistants		Pvt. Smith		
4	58	2,023	2,065	42	.724	1.724			" Call		
5	29	2,065	2,109	44	1.517	3.448					
6	29	2,112	2,155	43	1.483	3.448					
7	19	2,155	2,199	44	2.316	5.263					
8	20	2,199	2,242	43	2.150	5.000					

FIGURE 166.—Notes for meter rating.

(3) Current meters must be rated before using; this is usually done by the manufacturer or by the Bureau of Standards. Figure 166 is an example of notes for meter rating and figure 167 shows a typical rating curve.

f. Hook gage.—The hook gage (fig. 168), consists of a hook attached to a graduated rod sliding past a vernier, or to a vernier sliding past a fixed scale. The hook is lowered below the surface of the water

and slowly raised until its point comes in contact with the surface film of the water, when the latter is distorted and a pimple appears. The instrument is then read. Variation of head to $\frac{1}{1000}$ of a foot can be readily observed in perfectly still water.

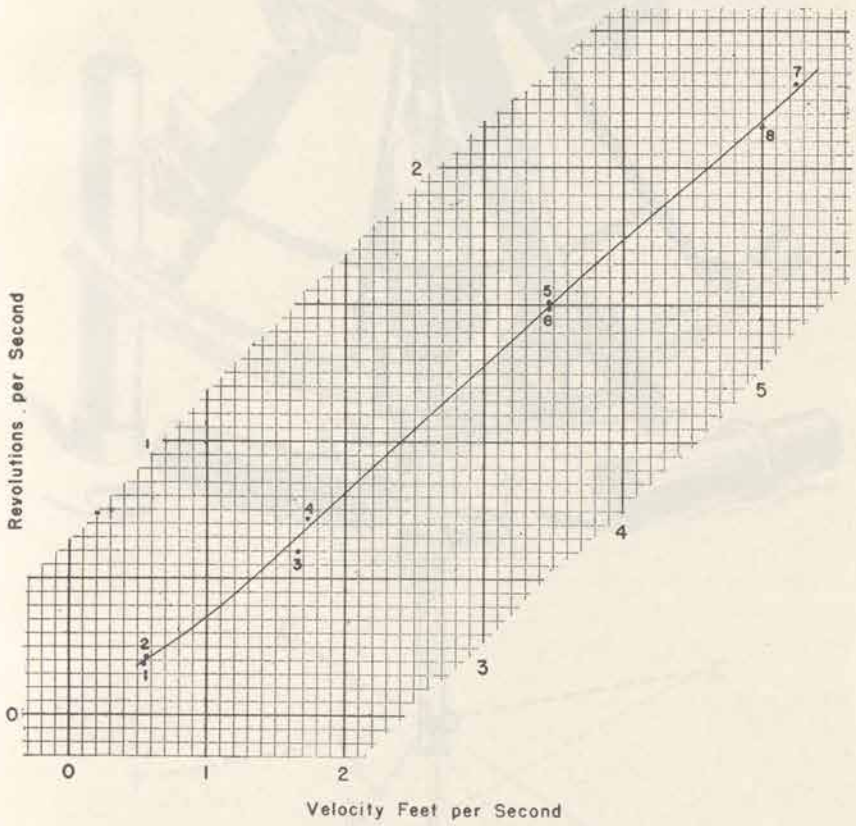


FIGURE 167.—Current meter rating curve.

g. Sextant.—(1) The sextant (fig. 169①) consists of a frame carrying a graduated arc AB (fig. 169②), about 60° long (one-sixth of a circle, hence the name); the graduations on this arc have a value in reading angles which is twice their actual value, so that the instrument measures angles up to 120°. At I, the center of the arc, is a pivot about which the arm IV rotates. This arm carries a vernier V for reading the angles on the graduated limb AB. Upon this arm is placed the index glass (a mirror) at I. H is the horizon glass which is rigidly attached to the frame of the instrument. Both of these

mirrors are set so that their planes are perpendicular to the plane of the arc AB and so that when the vernier reads 0°, they are parallel. At the point O is a low-powered telescope rigidly attached to the frame so that its line of sight is fixed in the direction OH. The upper half of H is unsilvered so that objects may be viewed with the telescope

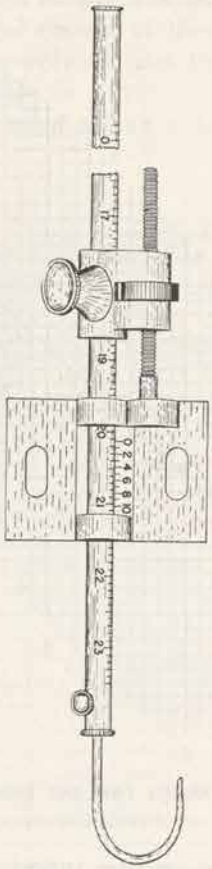
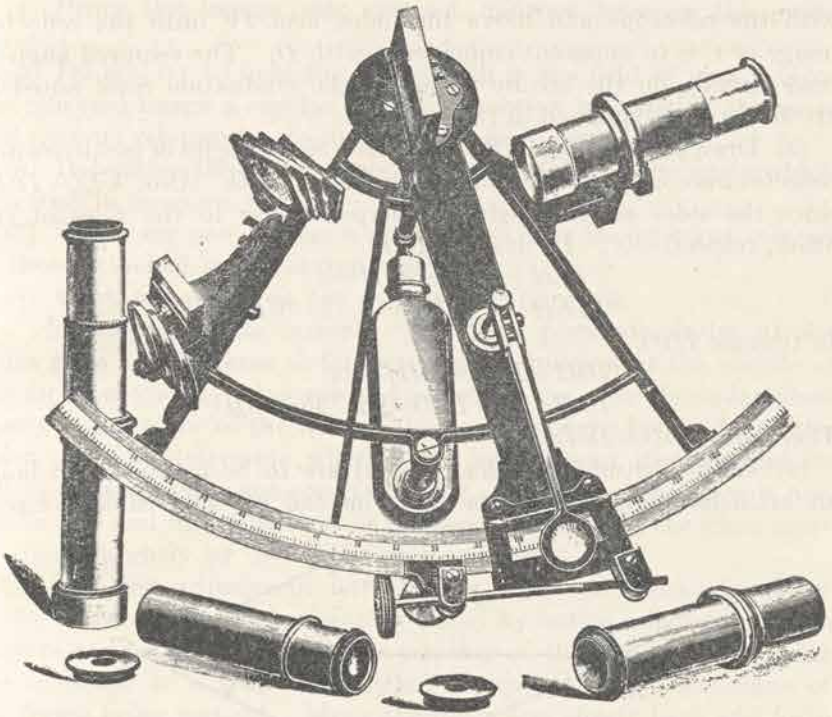


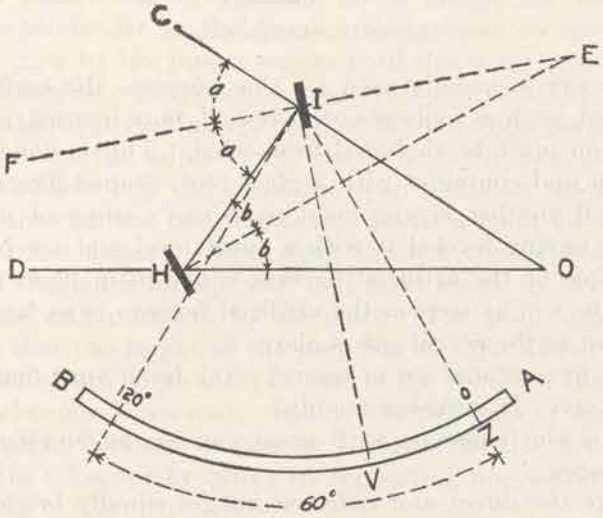
FIGURE 168.—Hook gage.

directly through the glass. In the lower silvered portion, other objects may be seen by reflection from the mirror I to H and thence to the field of the telescope at O. Movable colored glasses are placed to the left of H, and between the two mirrors, for protecting the eye of the observer when making observations on the sun.

(2) The instrument is used as follows: To measure the angle, either horizontal (par. 197*i*) or vertical, between C and D, sight upon D



① View.



② Principle.

FIGURE 169.—Sextant.

with the telescope and move the index arm *IV* until the reflected image of *C* is in apparent coincidence with *D*. The required angle is read directly on the arc by reason of the graduation ratio noted in (1) above and explained in (3) below.

(3) Draw perpendiculars *FE* and *HE*. Since angles of incidence and reflection are equal, $CIF=FIH$ and $IHE=EHO$. Also, $VIZ=IEH$, since the sides of one angle are perpendicular to the sides of the other, respectively. In triangle *IHE*

$$IEH+IEH=FIH=a$$
$$IEH=a-IHE=a-b=VIZ.$$

In triangle *IHO*,

$$IOH+IHO=CIH=2a$$
$$IOH=2a-IHO=2a-2b=COD.$$

Therefore, $COD=2VIZ$.

(4) When altitudes (vertical angles) are to be measured on land, an artificial horizon must be used instead of the visible horizon.

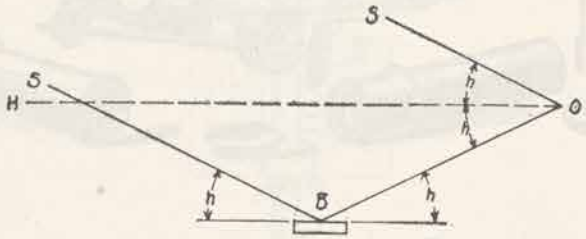


FIGURE 170.—Artificial horizon.

While mercury is usually used for this purpose, the surface of any heavy liquid, such as molasses or heavy oil, may be used. Such artificial horizon must be sheltered from wind. This is done by covering horizon and container with a glass roof, shaped like a roof of a house. Still another arrangement is to use a piece of plane black glass after having leveled it with a spirit level and leveling screws. The principle of the artificial horizon is shown in figure 170. The image of the sun as seen in the artificial horizon is as far below the true horizon as the actual sun is above it.

h. Rules for care and use of sextant.—(1) Keep your fingers off the graduated arc. It tarnishes readily.

(2) Focus the telescope with great care so as to secure sharply defined images.

(3) Make the direct and reflected images equally bright by moving the telescope to or from the plane of the sextant with the adjusting screw provided for this purpose.

(4) Bring the images into contact midway between the guide threads.

(5) Do not try to hold the images still in the field of view. Give the reflected image a regular oscillating motion by twisting the wrist and note its relation to the direct image as it swings by.

(6) Determine the index correction as carefully as the angle which you wish to measure.

(7) Whenever possible use a shade glass over the eyepiece instead of those attached to the sextant frame.

(8) Work as rapidly as you can without hurrying.

i. Adjustments of the sextant.—(1) Check perpendicularity of the index glass to the frame as follows: Set the index near the middle of the arc; remove the telescope and place the eye near the index glass nearly in the plane of the frame. Look at the right hand edge of the index glass to determine whether the arc as seen directly and its reflected image form one continuous arc. If it does, the glass is perpendicular and no adjustment is necessary; otherwise the glass must be tipped slightly by use of the proper screws.

(2) The first adjustment having been perfected, perpendicularity of the horizon glass to the frame is tested by noting whether the two mirrors are parallel for some one position of the index glass. Point the telescope to a star or a distant terrestrial point, the plane of the frame being vertical. Move the index arm slowly back and forth over the zero. This will cause the reflected image to pass through the field. If it passes exactly over the direct image, the two mirrors must be perpendicular to the frame. If it passes to one side, tip the horizon glass by the proper screws until this is corrected.

(3) In the more recent types of sextant, two peep sights are furnished with the sextant for making the axis of the telescope parallel to the frame. To make the adjustment, place the instrument on a table resting on its three legs and place the peep sights on the graduated arc. Look through the peep sights and locate a point on a wall not less than 20 feet distant. Next sight through the telescope in the direction of the mark on the wall and note where its line of sight cuts the wall with reference to the mark. The peep sights are so constructed that the height of the peep line of sight above the plane of the graduated arc is the same as that of the center of the ring into which the telescope is screwed. Hence, the two lines of sight should strike the wall at the same elevation. If this is not the case, vary the position of the telescope by tilting its supporting ring with its adjusting screws.

(4) If the sextant is not equipped with peep sights, the adjustment may be made as follows: With the sextant mounted on a stand move

the index so as to separate the direct and reflected images of some distant object (such as a star) by a distance nearly equal to the length of the parallel wires of the telescope. Turn the eyepiece until the direct image of the object coincides with one extremity of the wire at the same time that the reflected image coincides with the other extremity. The wires will then be parallel to the plane of the sextant. Select two distinct distant objects and make a coincidence of the reflected and direct images on the middle of one wire, and then on the middle of the other. If the readings agree, the adjustment is correct; if not, the adjusting screws in the collar of the up and down piece must be moved until the coincidence is exact.

(5) To make the horizon glass parallel to the index glass when the vernier reads zero, set the index at zero and point to a distant object as explained for the second adjustment. If the two images

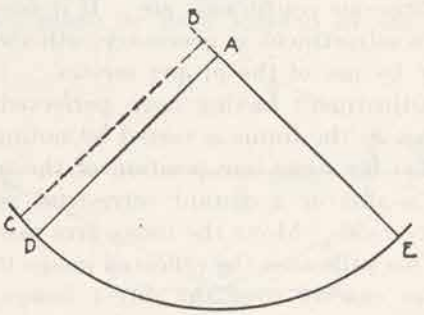


FIGURE 171.—Eccentricity error.

are exactly coincident, the adjustment is perfect. If not, turn the horizon glass about its axis perpendicular to the graduated arc, with the proper screws, until coincidence is secured. No matter how carefully this adjustment is made, it is necessary to check frequently. For this reason the adjustment is usually not made, but instead a correction is obtained by first establishing exact coincidence of images and then reading the vernier if it departs from zero; then apply this "index correction" with proper sign to all readings of the series.

(6) If the center of motion of the index arm is not coincident with the center of the graduated arc, another correction known as the "eccentricity correction" must be applied to all observations. In figure 171, B is the center of the index arm and A that of the graduated arc. When the arm is in position BE, in prolongation of the two centers, there is obviously no error in the reading due to noncoincidence of centers. But when the arm is at BC, 90° from BE, the error=

CD. Between these two points, the error is intermediate. Also beyond the point *D* the error again approaches zero. Corrections for a number of angles at suitable intervals may be obtained by measuring each angle with a good theodolite and with the inaccurate sextant. A curve may then be plotted between sextant angles (corrected for index error) and the eccentricity error; corrections for intermediate angles may be taken from this curve, which will be similar to a sine curve.

197. Soundings.—*a. Control.*—(1) Horizontal control for hydrographic surveys is effected by the establishment of known points by triangulation (fig. 172) or by the traverse methods outlined in section XII. These control points serve to locate each sounding position and are generally placed on shore.

(2) Vertical control is referred to an adopted datum plane which for navigable waters is mean low water or lower low water, as described in paragraphs 198 to 213. It is effected by the establishment of tide gages, the zero readings of which are related to the datum plane.

b. Ranges.—(1) Ranges are established by placing two or more signals on a line. These signals consist of stakes on shore or buoys anchored in the water. It is very desirable to have flags of white or colored cloth on the stakes and buoys, the selected color to contrast with the background. Whether on shore or in the water, range signals on alternate ranges should have different colored flags to prevent error and confusion.

(2) The establishment of ranges on shore is effected in varying ways, depending upon the accuracy desired. Where repeated surveys with soundings at the same points are to be made, the signals should be set on range by transit and tape or by triangulation and made to last during the period of measurements. When the accuracy of the survey permits, the signals may be set by instruments of less precision, such as the prismatic compass, and by pacing. This method is applicable to relatively small areas and where the survey is in the nature of a reconnaissance. The establishment of signals on ranges in water should be carried out with the accuracy demanded for shore points.

(3) While range signals may be located either on shore or in the water, it is best to have all of them on one side of the work so that the sounding boat may be kept on a range by the person in it. If the boat operates between two signals on a range, it is necessary to have an extra man to "line in" the boat on its run.

c. Methods.—(1) Soundings are taken at such intervals as will give the desired information. These intervals are timed or measured with

commensurate accuracy to avoid waste of time and labor in making the survey and to assure that the area is adequately and systematically covered. Without ranges a boat covers the area with unequal

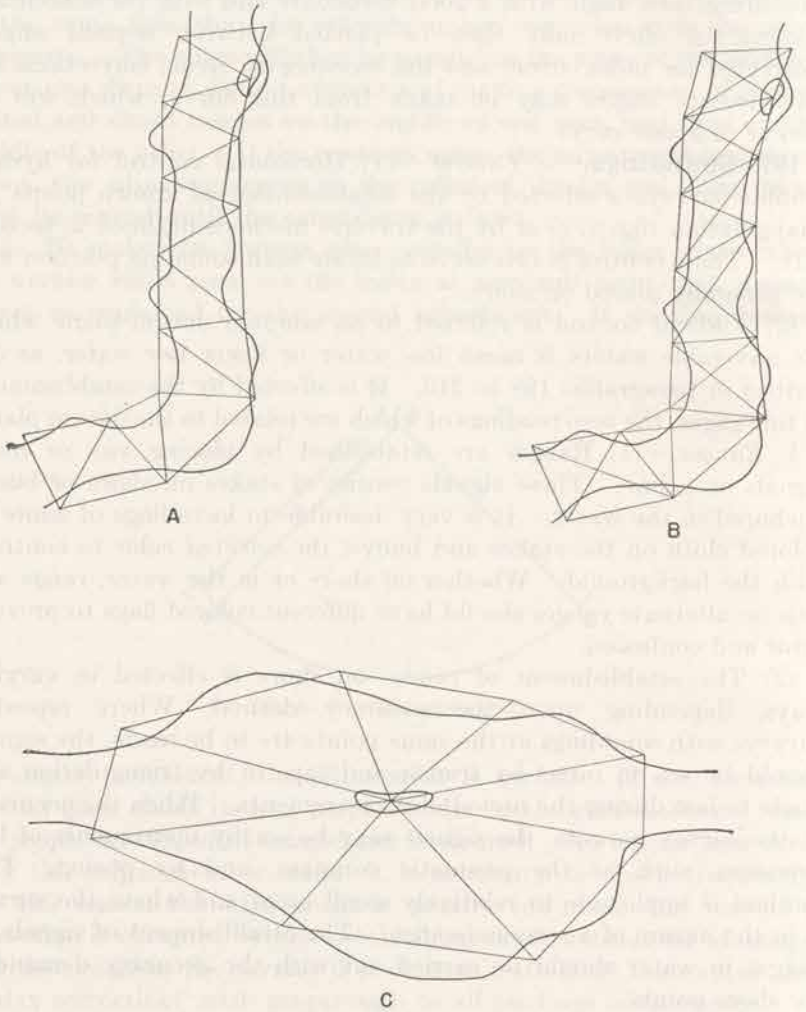


FIGURE 172.—Triangulation systems for horizontal control.

intervals; its runs are not parallel and often cross. Ranges are, therefore, placed as a guide for the boat's run, as well as for use in making locations.

(2) The following methods for determining the locations of soundings are used under varying circumstances, depending on the purpose of the survey and the area to be covered:

(*a*) Angles from establishment shore points.
(*b*) One angle from shore and one range line.
(*c*) Uniform speed and time interval on range.
(*d*) Intersection of two sets of ranges.
(*e*) Angle and stadia distance.
(*f*) Two angles from a boat to three shore points.
(*g*) Graduated line or wire.

d. Angles from established shore points (two-transit method).—(1) Each transit is set up at an established point (fig. 173) and a backsight is taken to the other transit station or to any known point. When a sounding is taken, each transitman reads the angle subtended by the backsight and the sight to the boat from which the sounding is made. The intersection of the simultaneous sights on a sounding plotted on a lay-out of the instrument points gives the position of the sounding. If a third instrument is used, the intersection of its pointing furnishes a check on the location by two transits.

(2) Before commencing work the watches of the transit recorders are synchronized with that of the boat recorder.

(3) The transit recorders record the range, number of sounding, time, and angle as read by the transitman. The boat recorder records the range, number of sounding, time, and sounding depth as given by the leadsman.

(4) A water gage is maintained during the operation, and readings of water level are recorded by the gage reader at requisite intervals of time. This gage, referred to a datum, furnishes the relation of the

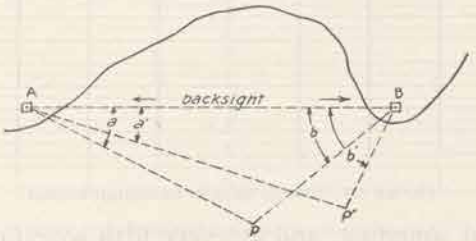


FIGURE 173.—Two-transit method.

soundings to that plane. In some cases the gage may be established close to the work so that it may be read from the boat and recorded in the sounding notes.

(5) The sounding line is checked for change in length as often as necessary and, if any change is evidenced, note is made of it and the correction applied to the soundings affected.

(6) The boat is run on ranges established before commencing operations. If transit readings are taken at each sounding, it is not necessary to maintain a uniform boat speed, as in the case of less frequent transit readings.

(7) A method often followed is to fix a time interval for soundings and wave a flag for transit reading at every fifth sounding. When ready to begin sounding on a range, a white flag is waved from the boat in a wide arc as a warning to the transitman. As the boat starts on its run the white flag is raised to a vertical position and, upon

(9)

Transit notes – Two-transit method

Range	No.	Two-transit Time	Angle to boat		Set up on ΔH / Sight on ΔL	Remarks
		8 hr. m. s	0°	00′		Date: Sept. 7, 1939
						Warm, calm
1	1	45 13	3	15		No. 1 {Transit - Sgt. A. A. Jones / Notes - Corp. J. Smith
	2	43	7	32		
	3	46 13	12	48		Watch No. 1792. This is 47 seconds slower than
	4	43	18	17		watch No. 1793 in boat.
	5	47 13	24	01		
	6	44	30	12		

FIGURE 174.—Transit notes for two-transit method.

making the first sounding, and at every fifth sounding, it is dropped to a horizontal position. If it is desired to have transit readings for each sounding, the white flag is used for each one, except that every fifth one is signaled by a red flag. When soundings are discontinued on one range the red flag is waved in a wide arc. Before commencing on another range the white flag is again waved in a wide arc.

(8) Forms for notes are shown in figures 174 (transit) and 175 (boat and gage).

(9) This method is quick and accurate for use in comparatively small areas. It requires two instrumentmen, however, and over

areas of considerable size it may be necessary to shift the transits frequently to obtain good intersections. Soundings must be suspended during these shifts.

e. One angle from shore and one range line (one-transit method) (fig. 176).—(1) Ranges are placed at desired intervals and the boat is run on them, the soundings being taken as described in *d* above. At the signal for a sounding by flag from the boat, the instrumentman reads the angle to the range subtended by the sight on the boat, and a backsight on a known point, and calls it for his recorder. The notes are similar to those for the two-transit method.

Sounding notes—Two-transit method (14)

Range	No.	Two-transit Time	Sndg	Tide Corr	Corr Sndg	Remarks			
		8 hr.				Date: Sept. 7, 1939			
		m s.				Warm, no wind; Tide ebbing			
1	1	46 00	4.7	−.8	3.9	Boat Recorder: Sgt. R. Roe		Flagman: Pvt. J. Doe	
	2	30	8.6		7.8	Leadsman: Corp. P.A. Brown		Signalman: Pvt. P. White	
	3	47	9.3		8.5	Boatman: Corp. H. Wilson			
	4	30	9.5		8.7	Watch No.	1793-8-30-14		
	5	48	9.4		8.6	" "	1792-8-29-27		
	6	30	9.8		9.0	" "	1794-8-30-23		

FIGURE 175.—Sounding notes for two-transit method.

(2) This method requires one transitman only, and if the range signals are accurately set and immovable, and the boat is kept on the line, it is quite accurate.

f. Uniform speed and time interval on range.—In this method the boat is run on the ranges between two points, the establishment of which determines the total distance of run. Starting at the initial point, the time of starting and that of reaching the second point on range determine the rate of speed which permits the location of soundings on the lines. Notes for this method are shown in figure 177.

g. Intersection of two sets of ranges.—(1) A set of ranges is placed from shore to cover the area to be sounded. On one of the ranges of this set, usually the first or the last, positions for a set of cross ranges or courses are established by anchoring buoys at desired intervals on the line. Both sets are numbered.

(2) A boat is anchored, for example, on range No. 1, course No. 1. A signalman in this boat lines in, by means of a prismatic compass, the boat carrying the sounding party and keeps it on the line during its run across the ranges of the first set. Any angle with the first set greater than 45° is used in accord with the desired lay-out of the soundings. As the sounding boat proceeds on its course, a signalman

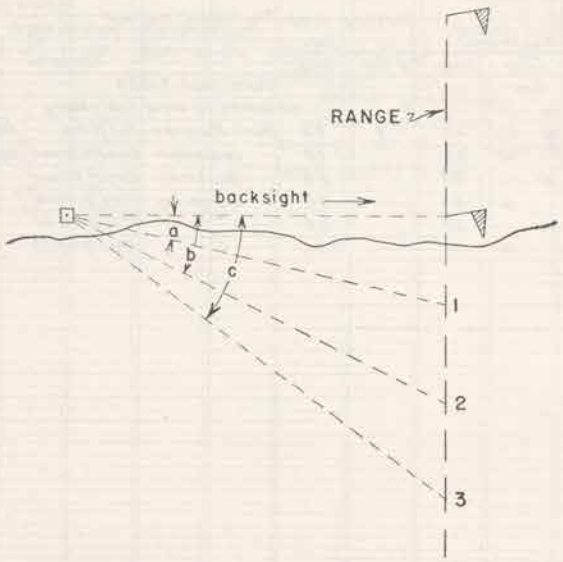


FIGURE 176.—One-transit method.

in it calls, "Sound," upon picking up a range of the first set. The resulting sounding is recorded in the notes referring to the ranges used (fig. 178). Time is also noted in order to connect the work with tide gage readings. If the position of the sounding boat on the course run is between the signals of a range, an additional man is required on shore to signal the sounding at the moment the boat crosses the range. He must thus proceed from range No. 1 to No. 2 of the first set, and so on, in order to flag the sounding party.

h. Angle and stadia distance.—(1) In this method a sounding is located by angle and stadia distance from shore. The area, if small, may be covered by one set-up, resulting in polar coordinates referred

Sounding notes - Timing method (17)

Range	No.	Timing Time	Sndg.	Tide Corr.	Corr. S	Remarks
		8 hr.				Date: Sept. 10, 1939
		m. s.				Cool, southwest winds
1	1	34 00	6.2	+0.4	6.6	250' to shore — Recorder: Sgt. A.A. Jones
	2	35 00	6.8		7.2	Tide ebbing — Leadsman: Pvt. P. White
	3	36 00	7.4		7.8	Signalman: Pvt. J. Doe
	4	37 00	10.6		11.0	Boatman: Corp. Wilson
	5	38 00	10.7		11.1	

FIGURE 177.—Sounding notes, timing method.

Sounding notes - Method of intersecting ranges (31)

Course	Range	Intersecting ranges Time	Sndg.	Tide Corr.	Corr. S	Remarks
		2				Date: Sept. 11, 1939
		m. s.				Cool, clear
1	1	45 00	4.2	+0.4	4.6	Tide flood — Recorder: Pvt. P. White
		47 10	4.0		4.4	Leadsman: Pvt. J. Doe
		49 30	4.0		4.4	Boatman No. 1, Pvt. R. Black
		51 45	3.4		3.8	Boatman No. 2, Pvt. M. Jones
		54 00	3.2		3.6	Signalman No. 1, Cpl. T. Brown
						Signalman No. 2, Cpl. J. Smith

FIGURE 178.—Sounding notes, method of intersecting ranges.

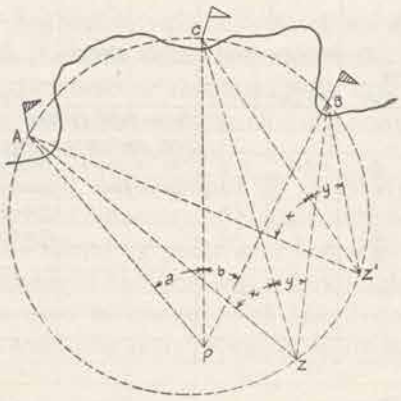


FIGURE 179.—Sextant method.

Sounding notes - Two-sextant method (37)

Range	No	Time	Two Sextant Sex. 1		Sex. 2		Sndg.	Corr. S	Remarks			
		8 hr							Date Sept. 9, 1940			
		m s	°	'	°	'			Tide flooding, warm, southwest wind			
1	1	40 00	24	44	14	08	18.0	17.8	Sextant 1, Cpl. J. Smith	Recorder Pvt. R. Black		
	2	41 15	31	18	29	14	24.3	24.1	Readings from cupola, Gunston Hall, to Signal			
	3	42 30	36	14	41	36	28.2	28.0	on Belvoir Wharf			
	4	43 45	42	27	62	40	34.6	34.4	Sextant 2, Sgt. A.A. Jones	Recorder Pvt. P. White		
	5	45 00	48	33	56	33	22.1	21.9	Readings from Signal on Belvoir Wharf to			
2	1	46 15	55	37	54	14	23.4	23.2	Fairfax Monument			
									Leadsman Pvt. J. Doe	Boatman Cpl. Wilson		

FIGURE 180.—Sounding notes, two-sextant method.

to a backsight, or the transit may be carried successively from range to range and quickly set up for distance readings.

(2) Time is recorded by the shore and boat party, watches being synchronized before operations are commenced. The transit recorder also records range, number of observation, distance, and the angle, if

only one set-up is made, ranges being set radially for assistance in keeping the boat on a line. Soundings and signals are executed as in other methods.

i. Two angles to three shore points (sextant method).—(1) In this method (fig. 179) two angles are read from a boat to three shore points. This is a form of the three-point problem discussed in paragraph 124.

(2) Three well defined, known points on shore are used for sextant readings. The angles between each of two adjacent shore points are read by the sextants at each sounding, the right and left angles being uniformly read by the same sextant.

(3) The boat is moved on ranges, signals made, and the soundings taken as in the method by angles from established shore points.

(4) The soundings are plotted by means of a 3-arm protractor and notes are kept as shown in figure 180.

(5) The advantages of this method are that it requires only a small party and that the members of the party are all together, avoiding possible misunderstanding of signals.

j. Graduated line.—Fixed and narrow channels having irregular

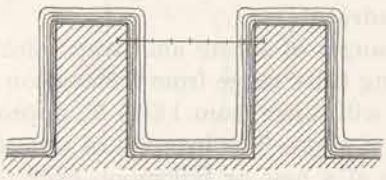


FIGURE 181.—Graduated line method.

bottoms or undergoing improvement by dredging often require accurate soundings to determine the extent and nature of necessary improvements (fig. 181). Such soundings can be located very accurately by setting and locating stakes or buoys on opposite sides of the channel and stretching a graduated line of cord or wire between these points.

198. Determination of tidal conditions.—*a. General.*—(1) It is often necessary to establish a datum from tidal observation by means of various types of gages. To obtain most accurate results the observations must extend over a period of several years. However, observations extending over 1 lunar month will give results satisfactory for all but the more precise surveys. The procedure is as follows:

(*a*) The gage, protected from rough wave action, is set in sufficient depth of water to give a definite reading at low tide.

(*b*) The zero of the gage is referred to a permanent bench mark, and elevations of high and low water are read at the proper times daily for the period desired.

(*c*) The mean of an equal number of desired readings gives the approximate value used to determine the datum.

(*d*) After the datum is obtained the proper elevation for a bench mark on shore is calculated.

(2) Observation shows that certain relations exist between tides and the moon's phases. Existence of the following conditions has long been recognized:

(*a*) There are normally two high and two low tides daily.

(*b*) Tides are highest when the moon is new or full.

(*c*) Increase and decrease in tidal heights follow regularly the varying phases of the moon.

(*d*) Time of high water is later day after day.

(*e*) Tides are greatest at equinoxes and least in summer.

(*f*) Time of high water differs at various places.

b. Definitions.—(1) *Flood tide.*—The coming in of the tide; applied also to high tide.

(2) *Ebb tide.*—The recession of the tide; applied also to low tide.

(3) *Spring tides.*—Tides of greatest range from high to low (occurring near time of new and full moon).

(4) *Neap tides.*—Tides of smallest range from high to low (occurring when moon is at quadrature).

(5) The relative ranges of spring and neap tides are about as 7 to 4; that is, if the spring tides range from 0 elevation at low tide to 7 at high, the neap tides will range from 1½ to 5½ approximately.

(6) *Establishment of port.*—The interval of time (lunitidal interval) between the passage of a new or full moon over the port's meridian and the next succeeding high water.

(7) *Corrected establishment of port.*—The mean interval of time over a lunar month between the passage of the moon over the port's meridian and the next succeeding high water. As this interval is nearly constant, fairly accurate tide prediction can be made if the establishment is known. A list of these intervals for places all over the globe is given in the *American Practical Navigator*, published by the Navy Department. Knowing the establishment of the port and utilizing the fact that the meridian passage of the moon occurs about 6 hours after moonrise, the state of the tide for any time may be predicted from the time of moonrise.

c. Tide-raising force.—The moon attracts every particle of the earth and ocean. Other forces acting on such particles are the attraction of the earth which tends to draw each one toward its center, and a centrifugal force caused by the earth's rotation. At any point these forces and the average attraction of the moon may be assumed to be in equilibrium. But the attraction of the moon for any particle of the earth at less than the average distance is appreciably greater

than its attraction for a corresponding particle at more than the average distance. At any part of the earth other than the center, these variations from the average have a disturbing effect on the equilibrium assumed to exist. In that part of the earth at less than average distance from the moon the resultant is a force directed toward the moon, while in the remainder of the earth the decreased lunar attraction permits the centrifugal force caused by the earth's rotation to overbalance it. The only part of the earth which is free to move under these distorting forces is the water in the oceans. It is drawn up in two immense mounds (flood tide or high tide) with summits at opposite ends of the diameter of the earth which, prolonged, would pass through the center of the moon, and is depressed by terrestrial gravity at a great circle 90° from these points (fig. 182). A similar tide-raising effect produced by the sun is, on account of that body's greater distance from the earth, less than half that of

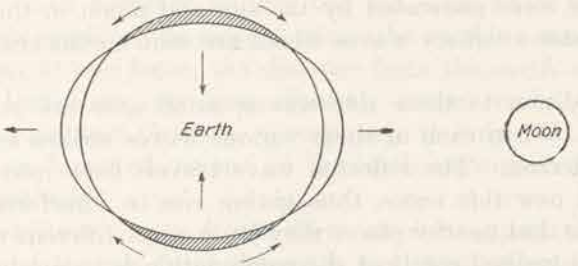


FIGURE 182.—Tide-raising force.

the moon. The rotation of the earth causes the mounds raised by each of the tide-raising bodies to travel around the earth, producing the periodic variation in the level of the water in the ocean which is known as the tide. The fact that there are two mounds and two intervening depressions or troughs accounts in general for the daily occurrence of two high and two low tides.

d. Range of tides.—At new and at full moon, the sun, earth, and moon are almost in the same line, so that the solar and lunar tides are superimposed, thus causing the spring tides. High tide is higher and low tide is lower than at any other time during the month. At new moon the sun and moon are on the same side of the earth and exercise their combined forces in such manner that, theoretically at least, the range of tides is slightly greater than at full moon. At the first and last quarters of the moon, when it is 90° from the sun, the crest formed by the lunar wave is superimposed upon the trough formed by the solar wave, resulting in a period of least fluctuation.

The tides of this period are known as neap tides. Between these two sets of events the range of tides is intermediate.

e. Propagation of tides.—(1) Theoretically, high tide should occur at any point when the moon is on the meridian at that point. Actually, however, the tide wave lags, due to irregularities of the bottom and of the coast line, inertia of the water, etc. Moreover, the aggregate tide wave at any point is the accumulation of a series of actual waves or, since the lunar wave is always predominant, may be said to be the lunar wave as affected by the solar wave and other influences.

(2) The primary tide wave is propagated in the continuous belt of water 2,000 miles wide which encircles the earth between 40° and 70° south latitude. Since the interposition of the continents prevents the formation of any larger tides in the other oceans, this primary wave is the parent wave of practically all tides.

(3) Extending north of this belt of water and at right angles to it are the three oceans—the Pacific, Atlantic, and Indian. The great primary tide wave generated by the sun and moon in this southern belt propagates auxiliary waves which are sent northward into these three seas.

(4) In addition to these tide waves, small ones are generated in each ocean. When each of these various waves strikes a shore it is partially reflected. The reflected wave travels back into the ocean and meets a new tide wave, thus giving rise to interferences. The local result is that nearby places may have quite different tides, while the final and general result of the combination is to produce a quite considerable time lag before the effect of the main parent wave is felt north of the Equator. For some places this time lag is as much as 40 hours.

f. Retardation in daily time of high and low water.—Since the motion of the moon in its orbit around the earth is such that its passage over any terrestrial meridian occurs about 51 minutes later (on the average) day after day, the times of high and low tides are also later day after day by a similar interval.

g. Prediction of tidal effects.—(1) The methods of tide prediction are largely empirical and are based upon records taken at various places over a period of many years. The preceding brief outline of the many diverse elements entering into the problem is indicative of the complexity of the subject. However, the United States Coast and Geodetic Survey, as well as similar departments of foreign governments, accurately predict the heights and corresponding time of tides for many places of the earth and for several years in advance.

(2) Several governments, including ours, have tide-predicting machines which mechanically combine the elements entering into final tidal effect at a given port. The machine itself is essentially based upon correct theory of the tides while the deviations therefrom are provided for through the use of empirical constants derived from long-continued series of observations.

h. Diurnal inequality of tides.—Morning and evening tides of the same day at any point may vary appreciably, sometimes one and sometimes the other being higher. This daily inequality is a maximum at the two periods in each lunar month when the moon's declination is greatest. When declination is near zero, twice a month, morning and evening tides of the same day are practically equal in height.

i. Annual variation in height of tides.—In addition to the cycle of heights through which tides pass each lunar month, there is also an annual cycle, due to the following:

(1) The earth revolves around the sun once each year, accompanied by its satellite, the moon. Since the earth's orbit is an ellipse with the sun at one focus, the distance from the earth to the sun is variable. Perihelion, or least distance occurs early in January; aphelion, or greatest distance, occurs early in July. At these times the attractive forces of the sun on the earth are maximum and minimum, respectively.

(2) In addition to its direct effect on solar tides, the variation in distance of the sun from the earth and moon affects the path of the moon in its orbit and thus affects the lunar tides.

j. Priming and lagging of tides.—(1) The time interval between corresponding tides of successive days varies. During spring tides, the intervals are about 13 minutes less than the average of 24 hours and 51 minutes, and the tides are said to "prime." During neap tides, this interval is greater by about 13 minutes, and the tides are said to "lag." The reason for this variation is found in the combination of the solar and lunar tides.

(2) On the days of new and full moon the two tides coincide and the tide has its crest at the normal interval after the moon's transit, which corresponds to the "establishment" of the port of observation.

(3) When the moon is at first quarter or at last quarter, the crest of the solar tide is 90° from the crest of the lunar wave. but it leaves the summit of the combined wave just where it would be if there were no solar wave at all. For intermediate relations of the sun and moon, the combination of the solar and lunar waves will make the crest of the combined wave take position at a point between the two

crests. For half of the month this crest will be on one side of the mean position and for the other half on the opposite side. The result is the total variation of about 26 minutes in the time between sucessive high waters.

k. Effect of atmospheric pressure and wind on tides.—(1) When the barometer at a given port is lower than usual, the level of the water is generally higher than the average. The reverse is true when the barometer is higher than usual. The water level is affected at the rate of about 1 foot for each inch variation of the barometer.

(2) When the wind blows into the mouth of a harbor or river, it drives in the water of the ocean by surface friction and may raise the water level several feet. In such a case the time of high water is delayed, sometimes as much as 15 or 20 minutes. The depth of the water is greater and the maximum depth is attained later than it would be if there were no wind. When the wind blows out of the harbor or river, the time of high water comes earlier and the depth is less than it otherwise would be.

l. Kinds of tides.—Tide waves may be classified as follows:

(1) The great primary wave of the southern ocean, which is directly due to the influence of the sun and moon. This wave does not undulate in the ordinary sense of the word, for its crest always moves forward in one direction. Its rise and fall may be taken as 2 feet at spring tides and 9 inches at neap tides.

(2) The ocean tide waves, propagated from this primary wave into the Atlantic and Pacific, and receiving their impulse from it. These waves undulate alternately backward and forward as the crest or the trough of the primary wave moves forward. They move at right angles to the course of the primary wave. In the open part of the ocean the rise and fall of this wave is the same as that from which it is generated.

(3) The coast tide, which is derived from that of the ocean but is altered in character through checking of its motion by contact with the coast. It is raised above the normal level by the momentum of the wave. The height of this tide varies at different parts of the coast.

(4) *River tides.*—The tide wave ascends a river at a rate which depends upon the depth of the water, the amount of friction, and the swiftness of the stream. It may, and generally does, ascend until it comes to a rapid, where the velocity of the water is greater than that of the wave. In shallow streams, however, it dies out earlier. It often ascends to an elevation far above that of the highest crest of the tide wave at the river's mouth. In the La Plata and Amazon Rivers

it goes up to an elevation of at least 100 feet above the sea level. The velocity of the tide in a river may reach 10 or even 20 miles an hour, but is usually less. The action that takes place in tidal rivers is a compound one due to a combination of—

(*a*) The ebb current, due to the water supplied from the area which the river drains, which moves down the river.

(*b*) The tidal wave, coming from the sea, which is propagated up the river with a velocity proportionate to the depth of water in the channel.

(*c*) The tidal current, which is attained when the tide has risen sufficiently in the lower part of the river to reverse the inclination.

Tidal wave and tidal current should be differentiated. The tidal wave in a river, as in the ocean, consists of a change of form in the water, with the addition of a damming back of the ebb current. It may be propagated up a river at a rate as high as 10 to 20 miles per hour. In this wave there is very little translation. On the other hand, a tidal current rarely exceeds 4 miles per hour and is capable of transporting material.

(5) *Estuary tides.*—When an estuary contracts considerably inside the entrance, the range of the tide is correspondingly magnified, due to the concentration of a large mass of water in a narrow space. At the head of Bay of Fundy tides of 70 feet are not uncommon.

(6) *Bores.*—When the tidal wave is propagated into a shallow river where there is not room for its development, the wave will break when its height above the surface of the water becomes equal to the depth below the surface. The first wave being impeded in its course by the friction of the bottom, the succeeding wave overtakes it and the tidal wave flows up the channel as a breaking wave with a crest several feet in height.

199. Tide peculiarities.—*a.* The observed tide is compounded of several elementary undulations related to the same common plane. Two of these waves can be distinguished and separated by a grouping of the data. These waves are known as the semidiurnal and the diurnal tides, because the first alone would give two high and two low waters a day, while the second would give but one high and one low water in the same time. These two tides coexist in nearly every port, but the proportion between them varies greatly for different places. The effect of the combination of these two types of tides is to produce a diurnal inequality, both in height of two consecutive high or low waters, and in the intervals of time between their occurrence. The type forms described are illustrated in figure 183, where the waves are plotted as curves.

b. Among the West Indies and in the Caribbean Sea the tidal range is only from 1½ to 2 feet, whereas in the Pacific, across the narrow neck of land which divides it from the Caribbean Sea, the range is from 12 to 18 feet. In the Gulf of Mexico between Cape St. George and Cape Florida, where the two tides are subject to large diurnal inequality, the range is nearly double that in the other parts of the Gulf. In the Gulf proper there is only one tide in the day, the rise and fall increasing or decreasing with the moon's declination. This is probably due to the fact that the flood tide, which is driven into the

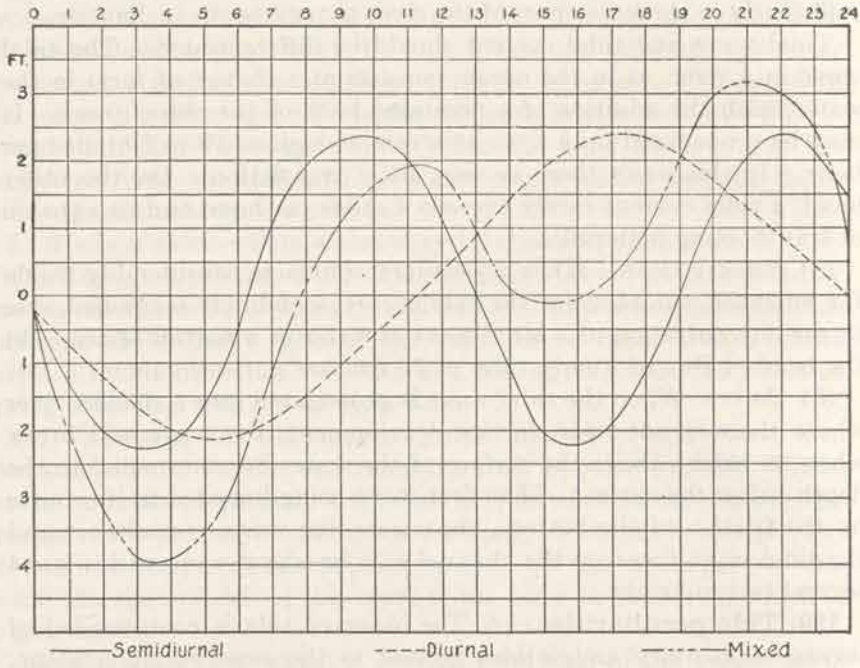


FIGURE 183.—Tidal peculiarities.

Gulf with the momentum of the ocean wave, has a long run up the Mississippi and other large rivers which pour a great quantity of water into the Gulf. The ebb has not time to get out before it is met by the succeeding flood, consequently only every alternate wave reaches the shores of the Gulf. In the same way, in some rivers into which the tide enters with a great head and has a long run, the low water of spring tide does not ebb out as low as that of neap tides.

200. Tide observations.—*a.* The height of tides at certain places and at definite times is the information usually required for hydrographic surveys. This information is necessary in order to furnish

data for computing the plane of reference and for reducing the soundings to that datum. Tide observations also furnish determinations of mean sea level for use in connection with precise leveling and give information valuable for other engineering and scientific purposes.

b. It was formerly thought sufficient for engineering data merely to note the height of the water at low and high tides, together with the corresponding times, and even now such observations will have to suffice for military work where time considerations are urgent. For general engineering practice, however, such observations are not sufficient. Observations must be made at frequent intervals throughout the day, and, when possible, continuous observations are desirable. Furthermore, such observations should extend over a long period of time. Observations extending over 1 lunar month are sufficient for ordinary military mapping operations; yearly variations and cyclic variations over a 19-year period should be considered for the more precise work. A master gage should be established at important points and its records kept. Then observations taken at various places and connecting tidal streams can be compared and corrected to coincide with the mean over a long period as given by the master gage. If mean low water is to be determined at some neighboring place, a gage is established, observations are taken over 1 lunation, and the mean determined. The mean at the master gage for this same period is determined and compared with the mean over the entire period of operation of the master gage. The difference between these results is then applied as a correction to the mean of the observations at the new gage and the result thus found is the true mean low water at that gage.

201. Location of gages.—*a.* In connection with hydrographic work it is generally desirable to set up an automatic tide gage at some central point and continue it there throughout the season. Such subsidiary staff or box gages as may be necessary should be established in the immediate vicinity of the work.

b. In selecting sites for these gages, existing facilities and the accessibility of the location to the observer should be taken into account. It is most important that all gages be located so that there is free communication with the sea, shelter from storm waves, and deep water at low tide close to the position. It is essential to place the gage in a place where its readings will give a fair representation of the oscillation of the surrounding water surface and at the same time be free from the effect of waves. It is usually desirable to sink a tube or box below the surface, into which the water will enter through one or more small holes, thus reducing the effects of

wind waves. In some cases a well or tank is built on shore communicating by a pipe with the surrounding sea. When this is done, care must be taken to be sure that the full effects of the tide reach the site of the gage.

202. Datum planes.—The word datum is used to indicate any plane of reference. Hydrographic surveys are referred to datum planes established as a result of tide or water gage records. The datum planes in general use are described below:

a. Mean sea level (*MW*) is the mean level of the sea during its continuous change of level over the period of observation. *MW* varies for different localities but this variation is not great.

b. Mean tide level (*MTL*) is the midplane between mean high and mean low water.

c. Mean high water (*HW*) is the average of all high waters for the observation period.

d. Mean low water (*LW*) is the average of all low waters for the observation period.

e. Higher high water is the higher of the two high waters of a day.

f. Mean higher high water (*HHW*) is the average of all higher high waters.

g. Lower low water is the lower of the two low waters of a day.

h. Mean lower low water (*LLW*) is the average of all lower low waters.

i. Mean range (*Mn*) is the difference in height between *HW* and *LW*.

j. Diurnal inequality (*DHQ* and *DLQ*) is the difference between morning and afternoon tides, due principally to the declination of the moon.

203. Types of gages.—There are three types of tide gages in general use—the staff, the box, and the automatic recording gage. The staff gage is used in connection with the establishment of datum planes, but mainly for the reduction of soundings to a plane already determined. The box gage is a refinement of the staff gage. The automatic gage is the one universally used for the important determination of datum planes and for continuous tidal records.

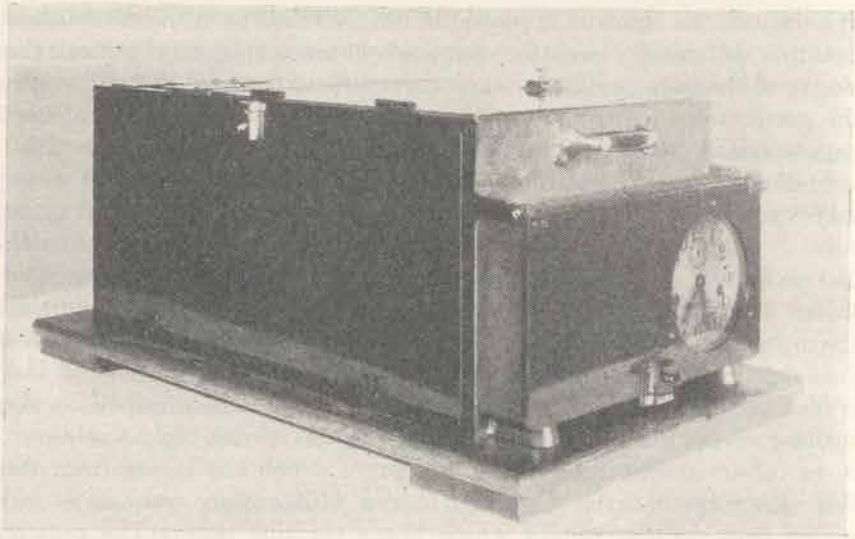
a. Staff gage.—This is made from a board about 5 inches wide and 1 inch thick of the proper length to cover the fluctuation of water levels where used. It is graduated to feet and tenths of a foot with the numbers reading upward from zero. When used to determine planes of reference, the zero end is placed well below the lowest water level and the other end extends well above the highest water level. When used to read water levels, at any time after the establishment

of a datum, the zero end is placed in proper relation to the established datum. In some cases the range of water levels may exceed the length of the gage. Extension is then made with an additional board, the graduations of which are in continuation of the first gage. Staff gages distant from the observer may be read with field glasses.

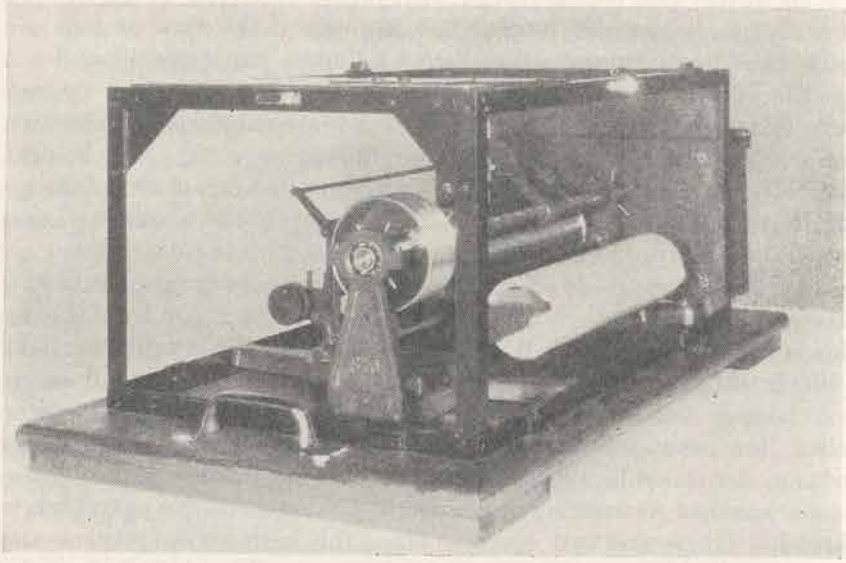
b. Box gage.—When more accurate readings are desired, or when waves render the use of the staff gage impossible, a box gage may be substituted. It consists of a vertical box closed at both ends, with one or more holes drilled at the lower end to permit the passage of water without violent fluctuation of the level inside, and a slit in the other end. Inside the box is a float to which is connected a graduated rod, which passes easily through the slit. When the box is fastened in a vertical position, the water level is determined by the reading of the graduated rod against the top of the box as an index. A variation of this type is the box gage, which has a line from the float running up over a pulley, thence horizontally over a second pulley to a counter-weight which keeps the line taut. The pulleys are separated by a distance greater than the range of water levels, and the line with an index mark between the pulleys travels against a scale of feet and tenths placed with proper reference to the datum.

c. Automatic recording gages.—(1) Records of this type of gage are made by a pen or pencil which traces a line on paper passed under it by the action of clockwork, or by means of printing the figures. Several types of automatic gages are made commercially, of which the United States Coast and Geodetic Survey gage (fig. 184) is most generally used. One type depends on the compression of air by changing heads of water for its record; another prints the water elevation to one-hundredths of a foot on a tape at 15-minute intervals.

(2) The United States Coast and Geodetic Survey gage carries a float in a vertical well extending into the water to a point well below the level of low water. A wire connects the float with the float pulley; this pulley, the counter-weight pulley, and the pencil screw are clamped together, causing the screw to turn as the tide rises or falls. The pencil travels over the paper perpendicularly to the paper's motion, the travel being limited within the width of the paper by the proper ratio of float-pulley diameter to the pitch of the pencil screw thread. There are two clocks. One, the motor clock, turns the cylinder which draws the paper under the pencil, and the other, the time clock, produces a short mark hourly on the record by tripping the pencil. The United States Lake Survey uses the same form of gage except that the time clock causes the hour mark to be made at the edge of the paper instead of on the curve of water level. A



① Front view.



② Rear view, exposed.

FIGURE 184.—United States Coast and Geodetic Survey gage.

datum pencil held in a frame traces a horizontal line on the paper. This line may be set by adjustment for any desired elevation on the sheet.

204. Establishing and checking gage-height readings.—*a.* Bench marks should be set in the immediate vicinity of each gage and connected with the survey by duplicate lines of levels. The zero of the gage is then set and referred to the bench mark. Thereafter, the zero of the gage should be checked against the bench mark frequently enough to insure that the elevation of the gage does not change.

b. The elevation of a staff gage can be determined or checked directly by differential levels between the gage and the bench mark. To establish or check the elevation of a box gage, set up a staff gage alongside the box gage. Determine the elevation of the staff gage. Take simultaneous readings of the two gages. The elevation of the box gage is then the elevation of the staff gage corrected by the difference between the readings of the two gages.

c. Automatic gages should be provided with a check gage of the staff or box type set in a separate well close to the well of the automatic gage and connected with it as well as with the outside water as shown in figure 185. The elevation of the check gage must be accurately determined as described above. The elevation of the automatic gage can be established or checked by the following methods:

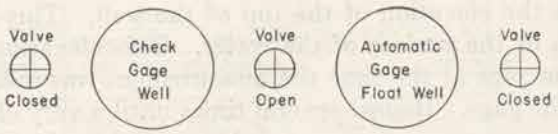


FIGURE 185.—Checking gage heights in water level.

(1) When two wells are used and the valves are in good condition, close the outside valves and open the valve connecting the two wells. Allow the water surface to come to rest and adjust the recorder of the automatic gage to read the same as the check gage. This method is accurate but cannot be used unless the valves and wells are arranged for it.

(2) The elevation of the automatic gage can be determined or checked by measuring the elevation of the water surface in the well. Close the valves of the well and pour enough water into the well to raise the water surface close to the top of the well. From some point on the top of the well measure the distance to the water surface in the well with a tape. Determine the elevation of the point. The elevation of the point minus the measurement taken to the water surface is the elevation of the water surface in the well. The elevation of the gage is then the elevation of the water surface in the well

less the gage reading. This method is accurate and eliminates the use of a check gage.

(3) If the well is in poor condition and will not retain water, the above method cannot be used. The following method is very satisfactory:

(*a*) The following equipment is required:

A battery of 4 to 6 dry cells.
A pair of telephone receivers with head band.
A small steel tape weighted at the zero end to prevent kinking and to hold tape vertical.
Five or six feet of insulated wire.
A small switch. This is advisable to prevent waste of current.
An induction coil may be put in the circuit to increase the audibility of the receivers.

(*b*) Connect these units in series, as shown in figure 186. To operate, lay a smooth stick across the top of the well, the elevation of which is found by leveling. Throw the switch into the "on" position and lower the tape into the well. The instant the zero of the tape touches the surface of the water, the circuit is completed and a very audible click is produced in the phones. At this instant, note the reading of the tape opposite the lower face of the stick. Subtract this reading from the elevation of the top of the well. This result gives the elevation of the surface of the water. This elevation minus the reading of the gage at the time the measurement was made gives the elevation of the gage. Repeat several times until a very close check is obtained.

205. Records of automatic gage.—*a*. Inspect the gages at least daily. Make the following notations on the margin of the marigram (the graphical record of the gage), indicating position of the pencil at time of inspection by an arrow:

Place.
Date.
Correct time.
Direction and intensity of wind.
Check gage reading.
Error of time clock.
Initials of observer.

Correct the time clock to standard time by comparing it with a high-grade watch which, in turn, has been compared with a master solar chronometer.

b. If the marigram is provided with hour marks recorded automatically, and the time clock has an error not greater than 3 minutes, these data may be accepted; otherwise, the time between the daily marks set by the observer must be divided into hours by dots near the datum line, numbered consecutively from 0 to 23 hours. Lines should then be drawn through these points at right angles to the datum line.

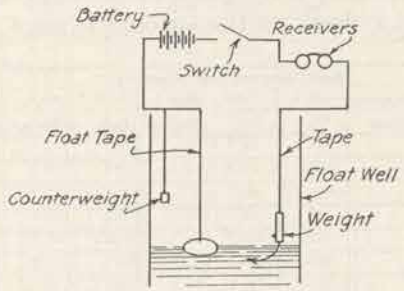


FIGURE 186.—Checking gage heights by electric contact.

c. For marigrams, assume a height for the datum. Record, on form indicated in figure 187, the height of the tide given by the check

Station _____ Observer _____ Tides - Comparative Readings Year _____
Automatic gage _____ Scale _____ Tabulator _____

Date	Time of reading	Staff G	Automatic Gage	Difference	State of tide			Remarks			

FIGURE 187.—Tides—comparative readings.

gage with reference to this datum line at the time of daily reading. Record the height of the tide given by automatic-gage reading at this time on the same form. Record the differences of the two readings. The mean of these differences over a 30-day period will give the height of the reference line of the marigram over the assumed datum. Plot this corrected datum on the marigram. The check gage and marigram readings should now correspond at the same time and the check gage can be used as a check on the automatic gage.

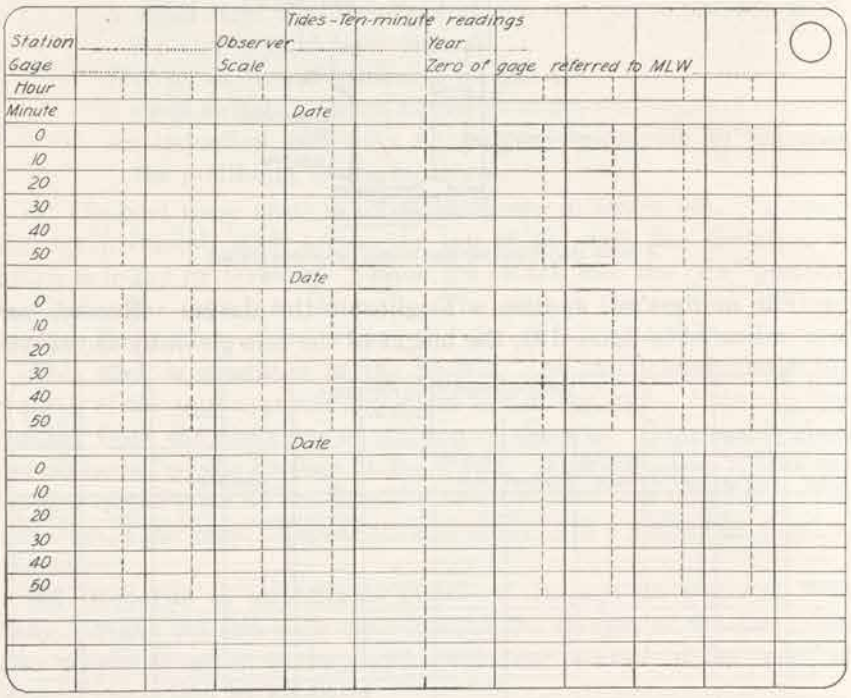


FIGURE 188.—Tides—10-minute readings.

206. Staff gage records.—It is often necessary to determine mean low and high waters when no automatic gages are available. To do this, establish a staff gage at the desired location. Beginning at about 30 minutes prior to predicted time of high and low water, take readings on the staff gage at 10-minute intervals over an hour period, or until the records show that the tide is changing direction. These records are reduced in the same manner as automatic-gage records. When time is available and more detailed information is desired, the records can be continued throughout the day at the desired interval. The form indicated in figure 188 may be used to record results.

Tides High and low waters

Station Fort Hamilton
Station No: 19-A Location: Government Wharf, Fort Hamilton
Observations begin, March 31, 1939 Observations end,

Date: May, 1939
Series No. 47
Tide Gage No. 3
Type, Gurley Printing
Correction feet

21

Date Year 1939	MOON'S TRANSITS (GREENWICH MEAN CIVIL)	Time of - High Water	Time of - Low Water	Lunitidal interval High Water	Lunitidal interval Low Water	Height of - High Water	Height of - Low Water	Remarks
	14.77							
Mo. d	Hr. dec.	Hr. dec.	Hr. dec.	Hr. dec.	Hr. dec.	Feet	Feet	
May 1	(3.22)	10.77	4.57	(7.55)	(1.30)	6.547	1.925	
	15.67	22.77	16.40	7.10	0.73	7.040	2.310	
2	(4.12)	11.41	5.53	(7.29)	(1.41)	6.507	1.966 √	
	16.58	23.78	17.53	7.20	0.95	6.904 √	2.314	
3	(5.03)	——	5.91	——	(0.88)	——	2.016 √	
	17.48	12.53	18.28	(7.50)	0.80	6.611	2.484	
4	(5.93)	0.66	7.53	7.18	(1.60)	7.489 √	2.733 √	
	18.38	13.78	19.53	(7.85)	1.15	7.201	3.345	
5	(6.83)	1.78	8.78	7.40	(1.95)	8.173 √	3.360	
	19.28	14.53	21.03	(7.70)	1.75	7.888	3.227 √	
6	(7.72)	2.78	9.53	7.50	(1.81)	7.740	2.743 √	
	20.17	15.53	22.28	(7.81)	2.11	8.090 √	2.897	
7	(8.61)	3.66	10.66	7.49	(2.05)	7.860	2.440	
	21.05	16.41	23.41	(7.80)	2.36	8.100 √	2.136 √	
8	(9.49)	4.66	11.78	7.61	(2.29)	7.442	1.580	
	21.94	16.53	——	(7.04)	——	8.003 √	——	
9	(10.39)	5.91	0.03	7.97	2.09	7.523	1.510	
	22.85	18.50	12.50	(8.11)	(2.11)	8.396 √	1.420 √	
				17	17	17	17	
Sums, carried forward				128.10	27.34	127.514	40.406	

①

Tides High and low waters - continued

Station Fort Hamilton
Station No: 19-A Location: Government Wharf, Fort Hamilton
Observations begin, March 31, 1939 Observations end,

Date May, 1939
Series No. 47
Tide Gage No. 3
Type, Gurley Printing
Correction feet

22

Date year 1939	MOON'S TRANSITS (GREENWICH MEAN CIVIL)	Time of - High water	Time of - Low water	Lunitidal interval High water	Lunitidal interval Low water	Height of - High water	Height of - Low water	Remarks
	14.77							
Mo. d	Hr. dec.	Hr. dec.	Hr. dec.	Hr. dec.	Hr. dec.	Feet	Feet	
Brot'f'w'd				128.10	27.34	127.514	40.406	
May 10	(11.30)	6.88	1.00	8.03	2.15	7.803	1.562 √	
	23.77	19.50	13.25	(8.20)	(1.95)	8.875 √	1.688	
11	——	7.77	1.77	8.00	2.00	7.966	1.704 √	
	(12.23)	20.02	14.15	(7.79)	(1.92)	8.961 √	2.018	
12	0.69	8.40	2.52	7.71	1.83	8.062	2.100 √	
	(13.16)	21.00	14.75	(7.84)	(1.59)	8.924 √	2.495	
13	1.62	9.25	3.25	7.63	1.63	7.875	2.138 √	
	(14.07)	22.00	15.38	(7.93)	(1.31)	8.425 √	2.513	
14	2.52	10.00	4.25	7.48	1.73	7.610	2.250 √	
	(14.97)	22.65	16.25	(7.68)	(1.28)	8.094 √	2.699	
15	3.40	11.15	4.90	7.75	1.50	7.338	2.488 √	
	(15.82)	23.52	17.02	(7.70)	(1.20)	7.794 √	3.120	
16	4.23	——	5.52	——	1.29	——	2.663 √	
	(16.63)	12.02	17.77	7.79	(1.14)	7.190	3.533	
17	5.03	0.02	6.27	(7.39)	1.24	7.650 √	3.053 √	
	(17.41)	12.52	18.77	7.49	(1.36)	7.351	4.352	
				16	17	16	17	
Sums, carried forward				244.51	52.46	247.432	80.782	

②

FIGURE 189.—Tides—high and low waters.

Tides: High and low waters - continued

Highest tide: Date, May 27; height, 9.030 feet
Lowest tide: Date, May 9; height, 1.420 feet
$I = (K_1+O_1) - M_2$ or $2(DHQ+DLQ) - Mn = .202$ $F(Mn) = 0.97$ $1.02\,F_1 = 1.10$

Date: May, 1939
Series No. 48
Lat 40°-36'-30" N.
Long 74°-02'-07" W.
Time meridian, 75° W.

(23)

Date	Moon's transits (Greenwich mean civil)	Time of - High water	Time of - Low water	Lunitidal interval - High water	Lunitidal interval - Low water	Height of - High water	Height of - Low water	Remarks
1939								
	17.41							
Mo. d	Hr. dec.	Hr. dec.	Hr. dec.	Hr. dec.	Hr. dec.	Feet	Feet	
Bro't f'w'd				244.51	52.46	247.432	80.782	
May 18	5.78	0.77	7.40	(7.36)	1.62	7.990 √	3.686 √	
	(18.16)	14.02	20.52	8.24	(2.36)	7.641	4.940	
19	6.52	1.27	9.03	(7.11)	2.51	8.075 √	4.453	
	(18.88)	14.41	21.03	7.89	(2.15)	7.498	3.731 √	
20	7.23	2.28	9.53	(7.40)	2.30	6.897	3.268 √	
	(19.58)	15.28	21.91	8.05	(2.33)	7.179 √	3.463	
21	7.93	3.78	9.78	(8.20)	1.85	6.813	2.890 √	
	(20.29)	16.28	23.03	8.35	(2.74)	7.158 √	3.215	
22	8.65	4.03	10.80	(7.74)	2.15	6.823	2.892 √	
	(21.01)	16.78	23.53	8.13	(2.52)	7.372 √	2.962	
23	9.38	5.03	11.30	(8.02)	1.92	7.000	2.883 √	
	(21.76)	17.55		8.17		7.650 √		
24	10.14	5.93	0.05	(8.17)	(2.29)	6.884	2.693	
	(22.53)	18.30	12.05	8.16	1.91	7.630 √	2.502 √	
25	10.94	6.55	1.05	(8.02)	(2.52)	7.057	2.398 √	
	(23.36)	18.75	12.68	7.81	1.74	7.940 √	2.500	
				17	16	17	16	
Sums; carried forward				371.33	85.37	365.039	129.258	

③

Tides High and low waters - continued

Highest tide: Date, May 27; height, 9.030 feet
Lowest tide: Date, May 9; height, 1.420 feet
$I = (K_1+O_1) - M_2$ or $2(DHQ+DLQ) - Mn = .202$ $F(Mn) = 0.97$ $1.02\,F_1 = 1.10$

Date: May, 1939
Series No. 48
Lat 40°-36'-30" N.
Long 74°-02'-07" W.
Time meridian, 75° W.

(24)

Date	Moon's transits (Greenwich mean civil)	Time of - High water	Time of - Low water	Lunitidal interval - High water	Lunitidal interval - Low water	Height of - High water	Height of - Low water	Remarks	
1939									
	17.41							27 days: May 2-28 (Give first and	
Mo. d	Hr. dec.	Hr. dec.	Hr. dec.	Hr. dec.	Hr. dec.	Feet	Feet	last date)	
Bro't f'w'd				371.33	85.37	365.039	129.258	HHW 25	LLW. 26
May 26	11.78	7.50	1.63	(8.14)	(2.27)	7.040	2.010 √	Sums 199.950	65.148
		19.63	13.50	7.85	1.72	8.068 √	2.235	Means 7.998	2.506
27	(0.22)	7.88	2.25	(7.66)	(2.03)	8.354	2.324 √		
	12.66	20.25	14.25	7.59	1.59	9.030 √	3.793	DHQ = .358	
28	(1.11)	9.00	3.25	(7.89)	(2.14)	7.410	2.490	DLQ = .147	
	13.57	20.75	14.98	7.18	1.41	7.980 √	2.365 √	Corrected Mn = 4.987 X 0.97 = 4.837	
29	(2.03)	9.73	3.61	(7.70)	(1.58)	7.105	1.917	Corrected DHQ = .358 X 1.10 = .394	
	14.49	22.11	15.73	7.62	1.24	7.836	2.165	Corrected DLQ = .147 X 1.10 = .162	
Sums, 29 days: May 1-29 (Give first				9	9	9	9		
and last date)				432.96	99.35	427.862	148.557	Datum is 5 feet below MSL	
Means				7.731	1.774	7.640	2.653		
Correction to interval				- 0.108	- 0.108				
				7.623	1.666	4.987	Mn.		
Duration of rise				5.957		5.146	MTL.		
Mean rise interval				4.644				Tabulated by C.S.	
30	(2.96)	10.48	4.48	(7.52)	(1.52)	7.234	2.039	Date 6-23-39	
	15.42	22.73	16.23	7.31	0.81	7.972	2.594	Reduced by: C.S.	
31	(3.88)	11.48	5.36	(7.60)	(1.48)	7.420	2.310	Date 6-23-39	
	16.33	23.61	17.36	7.28	1.03	7.758	2.700	Checked by: J.M.B.	

④

FIGURE 189.—Tides—high and low waters—Continued.

207. Tabulation of high and low waters.—*a.* The form shown in figure 189 is used for the tabulation of high and low waters, which may be either read from the tide curve made by an automatic gage, or taken from plain staff readings as recorded in a tide book, or from the printed record of a printing gage. The times are to be expressed in hours and decimals instead of hours and minutes; when great accuracy is desired, the times are carried to hundredths of an hour. One-tenth of an hour, or 6 minutes, is as close as an observed high or low water can usually be determined. The following table gives the equivalents of the minutes in tenths of an hour:

Minutes	Tenths of hour	Minutes	Tenths of hour	Minutes	Tenths of hour	Examples
						h m hrs.
0 to 3	0.0	21 to 27	0.4	45 to 51	0.8	4 02=4.0
4 to 8	.1	28 to 32	.5	52 to 56	.9	4 31=4.5
9 to 15	.2	33 to 39	.6	57 to 59	1.0	4 50=4.8
16 to 20	.3	40 to 44	.7			4 58=5.0

b. The heights are usually referred to the zero of the tide staff, and are given in feet and decimals of a foot. If the position of the tide staff is changed during the observations, the heights should all be referred to the zero of one staff, and a full explanation should be given in the column "Remarks." Any point of an automatic tide gage curve is readily referred to the zero of staff by using the true or corrected scale setting as calculated from the comparative readings on the datum. (See par. 205.)

c. When the period of observations is less than 6 months, the high and low waters should be tabulated in groups of 29 days each, beginning each group on the first line of the front side of a sheet. Allow two lines for each day, which will enable 17 days of record to be tabulated on the front page. The remaining 12 days of the group will be tabulated on the back of the form. If any part of the record is lost, leave vacant line for missing tide. If the series is longer than 6 months, the high and low waters should be tabulated by calendar months. Begin each sheet with the first of the month, and after 29 days have been tabulated, place the remaining days of the month below the long black horizontal line near the bottom of the back of the form. For February of common years insert March 1 after February 28 in order to complete the 29-day group. The high and low waters for March 1 should be repeated at the beginning of the sheet for March.

d. The method of tabulating the year, month, and days is shown on the specimen in figure 189. Generally, the morning tides are entered

on the first line and the afternoon tides on the second line for each day. A tide occurring at midnight (0^h) is taken as belonging to the morning of the day just beginning. While tabulating the times and height of the high and low waters, the columns "Moon's transits" and "Lunitidal interval" are left blank. These are to be filled in afterward in case the reductions described in paragraphs 208 to 213 are made.

e. After the times and heights have been tabulated, the highest and lowest tide occurring during the entire month, or during the period represented by the sheet should be noted and entered in the heading on the back of the sheet. If during this time the observer was unable to obtain a complete record because of some abnormal weather conditions, an estimation of the height of an extreme high water or extreme low water referred to the tide staff may be made from the evidence at hand and an explanation entered in the column "Remarks."

208. Reduction of tide records.—To obtain tidal constants and tidal datum for any station, the tabulated tides together with intervals and ranges depending upon them must be reduced to their mean values. In order to secure uniformity, the spaces for the sums are indicated in the forms, and the number of items included in each sum should be written in small figures just above the sum, as indicated in the specimen, figure 189. In the tabulations the individual times are given to two decimal places. The mean values are carried to three decimal places, and heights to three decimal places. Where great accuracy is not essential the number of decimal places may be made one less. When this is done, the last decimal figure of the mean value should be taken to the nearest hundredth, but if the remainder should be exactly one-half of the divisor, the second decimal should be made even, if not already even, by adding one.

209. Computation of lunitidal intervals.—This computation is made directly on the form shown in figure 189.

a. The mean solar (civil) times of the moon's transit for the meridian of Greenwich are obtained from ephemeris tables and centered in the proper column; minutes are converted into decimals of an hour. The times inclosed by parentheses are for the lower transits of the moon; the unmarked ones are for the upper transits.

b. Subtract from the time of each high and low water the time of the last preceding moon's transit and write the difference in the appropriate column on the same line as the tide from which it was obtained. In case the time of high or low water runs nearly the same as that of the moon's transit, take the transit which precedes the tide by about 12 hours; but in no case must the same transit be used for two consecutive high waters or for two consecutive low waters. When the time of the moon's transit is on one day and the following high or low water is

on the next day, add 24 hours to the time of the tide, then subtract the time of the transit. The high water intervals will usually be approximately 6 hours greater or less than the low water intervals, but the intervals for each phase of tide will rarely vary among themselves more than a few hours. Intervals from the lower transit of the moon are indicated by parentheses.

c. Sum both columns of the intervals for 29 days, placing the results in the spaces provided on the back of the form.

d. Compute the means by dividing each sum by the number of intervals combined to obtain it, carrying the results to an additional decimal place, and enter the means on next line.

e. Apply the correction to intervals, as obtained from tables I and II, and enter the results in the spaces provided below the second horizontal black line near the bottom of the form. The corrected high water interval thus obtained is known as the corrected establishment of the port.

210. Corrections for lunitidal intervals.—*a.* The true lunitidal interval is the difference between the mean local time of the tide and the mean local time of the moon's transit over the local meridian. But on account of the use of standard (instead of local) time and the inconvenience of changing the times of moon's transits from the Greenwich to the local meridian, it is customery to tabulate the intervals between times of Greenwich transit and standard times of tides, as shown in figure 189. From these the mean intervals are computed, then corrected to local meridian and time.

Let L=west longitude of station in degrees and decimals.

S=west longitude of time meridian used for tides.

S'=west longitude of time meridian used for transits.

X=correction to lunitidal intervals in hours.

Then, the correction for lag of the moon is

$$X_1=\frac{24.8412-24.0000}{360}(S'-L)=0.00233667\ (S'-L) \qquad (1)$$

The correction for reduction of standard time to local time is

$$X_2=\frac{24}{360}(S-L)=0.06666667\ (S-L) \qquad (2)$$

Combining (1) and (2) gives:

$$X=0.06666667\ (S-L)+0.00233667\ (S'-L) \qquad (3)$$

When Greenwich transits are used $S'=0°$ and

$$X=0.06666667\ S-0.06900334\ L \qquad (4)$$

When Greenwich transits and mean local time are used $L=S$, and (4) becomes:

$$X=-0.00233667\ L \qquad (5)$$

b. Table I has been computed from formula (4) for west longitude. For east longitude, reverse the signs in this table.

c. It is directly applicable when standard time has been used for the tides. Take the correction for the degrees of local longitude from the column headed by the time meridian used and add to this the correction for the minutes of local longitude given in table II. The latter part of the correction is independent of the time meridian.

d. The table may be adapted for any other time meridian, S_1, by using the nearest standard time meridian (S) of the table, proceeding as before, and then applying the following correction to the result from the table: $X = \pm 0.06667\ (S_1 - S)$, + for west longitude, − for east longitude.

TABLE I.—*Correction for lunitidal intervals (in hours and decimals)*

[For west longitude use sign given; for east longitude reverse sign]

Time meridian, 0°		Time meridian, 15°		Time meridian, 30°		Time meridian, 45°	
Longitude	Correction	Longitude	Correction	Longitude	Correction	Longitude	Correction
°	*Hour*	°	*Hour*	°	*Hour*	°	*Hour*
		5	+0.655	20	+0.620	35	+0.585
		6	+.586	21	+.551	36	+.516
		7	+.517	22	+.482	37	+.447
		8	+.448	23	+.413	38	+.378
		9	+.379	24	+.344	39	+.309
		10	+.310	25	+.275	40	+.240
		11	+.241	26	+.206	41	+.171
		12	+.172	27	+.137	42	+.102
		13	+.103	28	+.068	43	+.033
		14	+.034	29	−.001	44	−.036
0	0.000	15	−.035	30	−.070	45	−.105
1	−.069	16	−.104	31	−.139	46	−.174
2	−.138	17	−.173	32	−.208	47	−.243
3	−.207	18	−.242	33	−.277	48	−.312
4	−.276	19	−.311	34	−.346	49	−.381
5	−.345	20	−.380	35	−.415	50	−.450
6	−.414	21	−.449	36	−.484	51	−.519
7	−.483	22	−.518	37	−.553	52	−.588
8	−.552	23	−.587	38	−.622	53	−.657
9	−.621	24	−.656	39	−.691	54	−.726
10	−.690	25	−.725	40	−.760	55	−.795

TABLE I.—*Correction for lunitidal intervals (in hours and decimals)*—Continued

Time meridian, 60°		Time meridian, 75°		Time meridian, 90°		Time meridian, 105°	
Longitude	Correction	Longitude	Correction	Longitude	Correction	Longitude	Correction
°	*Hour*	°	*Hour*	°	*Hour*	°	*Hour*
50	+0.550	65	+0.515	80	+0.480	95	+0.445
51	+.481	66	+.446	81	+.411	96	+.376
52	+.412	67	+.377	82	+.342	97	+.307
53	+.343	68	+.308	83	+.273	98	+.238
54	+.274	69	+.239	84	+.204	99	+.169
55	+.205	70	+.170	85	+.135	100	+.100
56	+.136	71	+.101	86	+.066	101	+.031
57	+.067	72	+.032	87	−.003	102	−.038
58	−.002	73	−.037	88	−.072	103	−.107
59	−.071	74	−.106	89	−.141	104	−.176
60	−.140	75	−.175	90	−.210	105	−.245
61	−.209	76	−.244	91	−.279	106	−.314
62	−.278	77	−.313	92	−.348	107	−.383
63	−.347	78	−.382	93	−.417	108	−.452
64	−.416	79	−.451	94	−.486	109	−.521
65	−.485	80	−.520	95	−.555	110	−.590
66	−.554	81	−.589	96	−.624	111	−.659
67	−.623	82	−.658	97	−.693	112	−.728
68	−.692	83	−.727	98	−.762	113	−.797
69	−.761	84	−.796	99	−.831	114	−.866
70	−.830	85	−.865	100	−.900	115	−.935

TABLE I.—*Correction for lunitidal intervals (in hours and decimals)*—Continued

Time meridian, 120°		Time meridian, 135°		Time meridian, 150°	
Longitude	Correction	Longitude	Correction	Longitude	Correction
°	*Hour*	°	*Hour*	°	*Hour*
110	+0.410	125	+0.375	140	+0.340
111	+.341	126	+.306	141	+.271
112	+.272	127	+.237	142	+.202
113	+.203	128	+.168	143	+.133
114	+.134	129	+.099	144	+.064
115	+.065	130	+.030	145	−.005
116	−.004	131	−.039	146	−.074
117	−.073	132	−.108	147	−.143
118	−.142	133	−.177	148	−.212
119	−.211	134	−.246	149	−.281
120	−.280	135	−.315	150	−.351
121	−.349	136	−.384	151	−.420
122	−.418	137	−.453	152	−.489
123	−.487	138	−.522	153	−.558
124	−.556	139	−.591	154	−.627
125	−.625	140	−.660	155	−.696
126	−.694	141	−.729	156	−.765
127	−.763	142	−.798	157	−.834
128	−.832	143	−.867	158	−.903
129	−.901	144	−.936	159	−.972
130	−.970	145	−1.005	160	−1.041

TABLE I.—*Correction for lunitidal intervals (in hours and decimals)*—Continued

Time meridian, 157°30′		Time meridian, 165°		Time meridian, 180°	
Longitude	Correction	Longitude	Correction	Longitude	Correction
°	*Hour*	°	*Hour*	°	*Hour*
148	+0.288	155	+0.304	170	+0.269
149	+.219	156	+.235	171	+.200
150	+.149	157	+.166	172	+.131
151	+.080	158	+.097	173	+.062
152	+.011	159	+.028	174	−.007
153	−.058	160	−.041	175	−.076
154	−.127	161	−.110	176	−.145
155	−.196	162	−.179	177	−.214
156	−.265	163	−.248	178	−.283
157	−.334	164	−.317	179	−.352
158	−.403	165	−.386	180	−.421
159	−.472	166	−.455	181	−.490
160	−.541	167	−.524	182	−.559
161	−.610	168	−.593	183	−.628
162	−.679	169	−.662	184	−.697
163	−.748	170	−.731	185	−.766
164	−.817	171	−.800	186	−.835
165	−.886	172	−.869	187	−.904
166	−.955	173	−.938	188	−.973
167	−1.024	174	−1.007	189	−1.042
168	−1.093	175	−1.076	190	−1.111

TABLE II.—*Correction for minutes of longitude*

Longitude	Correction	Longitude	Correction	Longitude	Correction
′	*Hour*	′	*Hour*	′	*Hour*
1	−0.001	21	−0.024	41	−0.047
2	−.002	22	−.025	42	−.048
3	−.003	23	−.026	43	−.049
4	−.005	24	−.028	44	−.051
5	−.006	25	−.029	45	−.052
6	−.007	26	−.030	46	−.053
7	−.008	27	−.031	47	−.054
8	−.009	28	−.032	48	−.055
9	−.010	29	−.033	49	−.056
10	−.012	30	−.034	50	−.058
11	−.013	31	−.036	51	−.059
12	−.014	32	−.037	52	−.060
13	−.015	33	−.038	53	−.061
14	−.016	34	−.039	54	−.062
15	−.017	35	−.040	55	−.063
16	−.018	36	−.041	56	−.064
17	−.020	37	−.043	57	−.066
18	−.021	38	−.044	58	−.067
19	−.022	39	−.045	59	−.068
20	−.023	40	−.046	60	−.069

e. When transits for the meridian of L' are used, the table may be adapted by proceeding as before and applying the following correction to the result: $X=\pm 0.00234L'$, + for west longitude, − for east longitude.

f. When local civil time and Greenwich transits are used, it is more convenient to use formula (5) independently of the table.

211. Computation of mean high water (*HW*), mean low water (*LW*), mean range (*Mn*), and mean tide level (*MTL*) (fig. 189).—First, add the high and low water heights for 29 days; second, obtain the means by dividing each sum by the number of high or low waters included; third, obtain the mean range (*Mn*) by subtracting the mean of the low waters from the mean of the high waters, and enter the result in the space before the symbol (*Mn*); fourth, obtain the mean tide level (*MTL*) by taking one-half of the sum of the mean high water and the mean low water, and enter in the space before the symbol (*MTL*); fifth, the mean range should be corrected for longitude of moon's node in accordance with paragraph 213.

212. Computation of mean higher high water (*HHW*), mean lower low water (*LLW*), and diurnal inequalities (*DHQ* and *DLQ*) (fig. 189).—First check off the higher of the two high waters and the lower of the two low waters of each day for 27 days, omitting the first and last days of the 29-day group. When only one high or one low water occurs on a calendar day, by reason of one of the tides having occurred after midnight and, therefore, on the next calendar day, the single tide should be checked if the tide just above it is unchecked; otherwise, it should not be checked. If, however, the tide has become diurnal and only one high and one low water occur during the tidal day, each of these should be checked. Second, the higher high waters and lower low waters thus checked should be summed and the results entered in the spaces provided in the column of "Remarks." Third, obtain the means and enter the results in the line below the sums. Fourth, subtract the mean of the high waters from the mean of the higher high waters and enter the difference after the symbol *DHQ* on the back of the form. Fifth, subtract the mean of the lower low waters from the mean of the low waters and enter the difference after the symbol *DLQ*. Sixth, correct *DHQ* and *DLQ* in accordance with paragraph 213.

213. Correction for longitude of moon's node.—*a*. The moon's nodes are the two points in which the moon's path on the celestial sphere intersects the ecliptic. The nodes travel west along the ecliptic, making a complete revolution in about 19 years. This travel affects the moon's declination; when the ascending node coincides with the vernal equinox the moon's declination has a monthly range of about 57°; when the descending node reaches the same point (about 9½ years later) the moon's declination has a monthly range of about 37°. Since the moon's declination affects mean range and diurnal inequalities of tides, the values found for these quantities for any month or year of observations require correction to obtain true mean values.

b. To correct the mean range for the longitude of the moon's node, apply the factor $F(Mn)$, or "factor for mean range," which is given in table III for the years 1940 to 1944. The factors have been computed for the middle of each year, but as they change very slowly, the same value may be taken for any month of the year. The line in the table from which the value is to be taken is determined by the argument $\frac{2(DHQ+DLQ)}{Mn}$, which may be computed to one decimal place from the uncorrected values of *DHQ*, *DLQ*, and *Mn* as obtained from the computation (fig. 189).

For stations on the Atlantic coast of the United States from Maine to Florida, $\frac{2(DHQ+DLQ)}{Mn}$ is usually small, and if the values of DH and DLQ have not been obtained, the top line of the table may generally be taken without material error. For stations on the coast of the Gulf of Mexico from Key West to the Rio Grande, the mean range of the tide is very small and the factor $F(Mn)$ need not be applied. For other localities, where DHQ and DLQ are usually computed, the value of $\frac{2(DHQ+DLQ)}{Mn}$ should be obtained. If it is larger than 1.8, no correction need be applied to the mean range (Mn).

TABLE III.—*Factor $F(Mn)$, for reducing the observed range of tide to its mean value*

$\frac{2(DHQ+DLQ)}{Mn}$	1940	1941	1942	1943	1944
0.0 to 0.2	0.971	0.971	0.972	0.977	0.985
0.3 to 0.4	.972	.972	.973	.978	.985
0.5 to 0.6	.974	.974	.975	.979	.986
0.7 to 0.8	.976	.976	.977	.981	.987
0.9 to 1.0	.979	.979	.980	.984	.990
1.1 to 1.2	.983	.983	.984	.987	.992
1.3 to 1.4	.987	.987	.988	.991	.994
1.5 to 1.6	.993	.992	.993	.994	.997
1.7 to 1.8	.999	.999	.999	.999	.999
1.9 to 2.0	1.006	1.006	1.006	1.005	1.003

c. The diurnal inequalities, DHQ and DLQ, are corrected by the factor 1.02 F_1, which is given in table IV for the years 1940 to 1944. These factors have been computed for calendar months and may be used without modification for series of 29 days beginning on the first day of the month. If a 29-day series begins on any other day of the month, the factor may be obtained from the table by interpolation.

TABLE IV.—*Factor F_1, for correcting DHQ and DLQ*

Month	1940	1941	1942	1943	1944
January	1.00	1.01	1.01	0.98	0.95
February	1.21	1.22	1.21	1.18	1.13
March	1.44	1.46	1.45	1.39	1.31
April	1.33	1.34	1.33	1.28	1.21
May	1.07	1.07	1.07	1.03	.99
June	.96	.96	.96	.93	.89
July	1.00	1.00	.99	.96	.92
August	1.22	1.22	1.20	1.15	1.09
September	1.47	1.47	1.43	1.36	1.29
October	1.33	1.33	1.30	1.24	1.17
November	1.07	1.07	1.04	1.00	.96
December	.96	.96	.94	.91	.87
Mean	1.172	1.176	1.161	1.118	1.065

214. Stream discharge.—*a. General.*—The measurement of stream discharge is often necessary prior to engineer construction, and is essential to a complete knowledge of the physical characteristics of a river.

b. Definitions.—(1) Discharge (Q) is the volume of water passing in the stream at a given point in a given unit of time. It is usually expressed in cubic feet per second (*cfs* or sec.-ft.).

(2) Slope or fall (S) is the inclination of the water surface, usually expressed as the ratio of fall to length, $h/l=S$.

(3) Sectional area (A) is the area of the cross section of the channel taken at right angles to the current and usually expressed in square feet.

(4) Wetted perimeter (p) is that portion of the cross section of the channel below the water line.

(5) Hydraulic mean depth or hydraulic radius (R) is the ratio of the sectional area to the wetted perimeter, $A/p=R$.

(6) Mean velocity (V) is the average velocity of all the elements of the current.

c. Methods of measuring discharge.—(1) Stream discharge is usually measured by one of the following methods:

(*a*) By taking measurements of slope and cross section and computing the discharge by the Chezy formula, slope method.

(*b*) By means of a dam or weir of known dimensions, together with the measurement of the head thereon, weir method.

(*c*) By measuring velocity and cross-sectional area, their product being the discharge, velocity method.

(2) When the stream bed is fairly permanent, comparatively few stream measurements are required to enable an approximate estimate of the daily flow, provided a gage is established and read daily. A discharge curve, using gage readings as ordinates, should be plotted. To plot these curves, make enough careful stream measurements at various water stages and plot the corresponding gage heights. To use the curve, enter with the daily gage readings as ordinates and read the corresponding discharges. The daily discharges may then be used as ordinates with the days of the month as abscissas and a hydrograph of the stream plotted.

d. Determination of discharge.—(1) Chezy's formula is

$$Q=CA\sqrt{RS}$$

where Q=discharge in cubic feet per second (second-feet).
A=cross-sectional area of the stream in square feet.
S=slope of water surface (ratio of fall to length, or h/l).
R=hydraulic radius (A divided by wetted perimeter, p).
C=coefficient depending upon roughness, slope, and hydraulic radius.

(2) Kutter and Ganguillet determined the following expression for C as the result of studies involving use of Mississippi River data:

$$C=\frac{41.65+\frac{0.00281}{S}+\frac{1.811}{n}}{1+\left(41.65+\frac{0.00281}{S}\right)\frac{n}{\sqrt{R}}}$$

in which n=a coefficient of roughness depending upon the general character the stream, including the material its bed, snags, plant growth on banks, sinuosity, etc.

(3) Since the complete Kutter formula is difficult of solution, it is frequently desirable to use the Manning formula, which is written

$$Q=A\left(\frac{1.486}{n}R^{\frac{2}{3}}S^{\frac{1}{2}}\right)$$

In this formula, n has the same values as in Kutter's, and the results obtained by it are as generally satisfactory.

(4) To determine the necessary data for these formulas, select and measure a straight stretch of stream 200 to 1,000 feet long having as uniform a slope and cross section as possible. The following data must be determined:

(*a*) *Slope of surface of the stream.*—Determine the slope between gages established at each end of the stretch selected. At least two, and preferably three, gages should be placed in position at the ends of the course, one on each side of the stream, and one in the center, if three gages are used. The slope should be determined for both sides of the river (for the center also if three gages are used at each end) and the mean value taken as the slope of the river. Because the height of fall is small compared with the length of the course, it must be very acccurately determined by the proper placing of gages, reading them simultaneously to hundredths of a foot, and by carefully setting the gages to read from the same datum.

(*b*) *Mean area of cross section.*—Sufficient cross sections must be taken between the gages to give an accurate mean of the sectional area and hydraulic radius.

(*c*) *Data in regard to roughness of bed.*—Special care should be taken in collecting data to determine the value of n, the choice of which coefficient requires careful judgment. It includes not only the effect of roughness of the material of which the bed is composed, but also that of all obstructions or irregularities which retard the flow by causing eddies. It is larger for the overflow portion of the stream and that portion should be treated as a separate problem in computing the discharge. This factor varies for the same channel at different river stages and even for different velocities, n having a tendency to decrease as the hydraulic radius increases. When selecting the proper value of n, the data given below may be used. In the usual case it is found that conditions of the stream do not agree with those assigned to any one value of n, in which case select what could be considered maximum and minimum values for n under conditions similar to those existing. Compute the discharge separately for each of these factors and take the mean as the discharge of the river. Following are values of n assigned to various conditions:

$n=0.0225$ for canals in earth in excellent order and regimen.

$n=0.025$ for canals and rivers in earth of fairly uniform cross section, slope, and direction, in fairly good condition, and free from stones and weeds.

n=0.0275 for canals and rivers in earth below average in order and regimen.

n=0.30 for canals and rivers in earth, in rather bad order and regimen, having stones and weeds occasionally and obstructed by detritus.

n=0.035 for canals and rivers with earthen beds, in bad order and regimen, having stones and weeds and obstructed by detritus.

n=0.050 for torrents encumbered with detritus.

n=0.040 to 0.055 for overflow banks with vegetation.

(5) The use of this method gives only an approximate result under most favorable conditions. Results are particularly unreliable for large streams. It is difficult to determine the slope with sufficient accuracy and, in addition, to select the proper value for n. However,

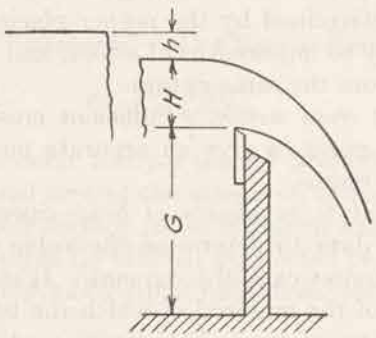


FIGURE 190.—Standard weir.

it is sometimes the only method that can be used at times of extreme flood.

e. Discharge by means of dam or weir of known dimensions, together with measurement of head thereon.—(1) *Weir method.*—A weir in its simplest form is a notch in the top of the vertical side of a reservoir through which water flows. The discharge can be determined by multiplying the cross-sectional area of the prism of water flowing through the notch by the velocity as expressed in terms of the head on the weir.

(2) The standard or sharp-edged weir (fig. 190) consists of a vertical partition across the channel to be measured, with its top edge horizontal, sharp-cornered, and narrow enough so that at the heads used the overflowing sheet jumps from the upstream edge and entirely clears the weir in passing over it. A weir with end contractions is one the crest of which extend only part way across the channel and is terminated by partitions in its plane, with their vertical edges rising

above the level of the water on the upstream side. A weir without end contractions is one having its crest extending entirely across the channel. If the level of the water below the weir is higher than its crests, there being, however, a certain definite fall in passing the weir, the weir is said to be submerged (fig. 191).

(3) A weir should be located as nearly as possible with its face at right angles to the stream axis. The upstream face should be vertical, and the crest truly horizontal. The weir should be strongly built with a solid foundation and substantial bracing. Planking must be sufficiently rigid to prevent vibration. Sides and bottoms should be

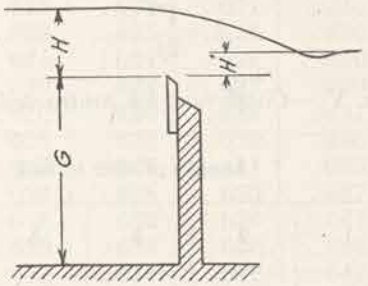


FIGURE 191.—Submerged weir.

sealed to prevent leakage, and an apron of flat stones or planks should be placed on the downstream side to prevent undermining.

(4) The formula for discharge over a rectangular weir is

$$Q=c\frac{2}{3}\sqrt{2g}\ b(H+nh)^{3/2}$$

$$Q=5.35\ cb(H+nh)^{3/2}$$

in which H=upstream head on the crest.

h=head due to velocity of approach=$\frac{V^2}{2g}$ where V is the velocity of approach.

b=length of weir crest.

c=coefficient varying with type of weir and head thereon; values are given in tables V and VI.

n=1.4 for weirs with end contraction and 1.33 for those without.

g=acceleration due to gravity=32.2 feet per second per second.

(5) For heads between 0.5 foot and 2 feet the above formula may be modified as follows for weirs with end contractions:

$$Q=3.33b(H+1.4h)^{3/2}$$

(6) *Determination of values for formula.*—The determination of H, the head of water above the crest of the weir, must be accurately made. Establish a hook gage in a well which connects with the channel at about mid-depth; the connection must be above beginning of the surface curve to the weir, usually at a distance of two and one-half times the height of the weir above the channel bottom. Determine the elevation of the zero of this gage above the crest of the weir by level reading. The head H can then be accurately determined. The value of b is obtained by direct measurement. Estimate V from float observation and determine h from the equation

$$h=\frac{V^2}{2g}$$

TABLE V.—*Coefficient for contracted weirs*

Effective head in feet	Length of weir in feet						
	0.66	1	2	3	5	10	19
0.1	0.632	0.639	0.646	0.652	0.653	0.655	0.656
.15	.619	.625	.634	.638	.640	.641	.642
.2	.611	.618	.626	.630	.631	.633	.634
.25	.605	.612	.621	.624	.626	.628	.629
.3	.601	.608	.616	.619	.621	.624	.625
.4	.595	.601	.609	.613	.615	.618	.620
.5	.590	.596	.605	.608	.611	.615	.617
.6	.587	.593	.601	.605	.608	.613	.615
.7	-----	.590	.598	.603	.606	.612	.614
.8	-----	-----	.595	.600	.604	.611	.613
.9	-----	-----	.592	.598	.603	.609	.612
1.0	-----	-----	.590	.595	.601	.608	.611
1.2	-----	-----	.585	.591	.597	.605	.610
1.4	-----	-----	.580	.587	.594	.602	.609
1.6	-----	-----	-----	.582	.591	.600	.607

TABLE VI.—*Coefficient for weirs without end contractions*

Effective head in feet	Length of weir in feet						
	19	10	7	5	4	3	2
0.1	0.657	0.658	0.658	0.659	-----	-----	-----
.15	.643	.644	.645	.643	0.647	0.649	0.652
.2	.635	.637	.637	.638	.641	.642	.645
.25	.630	.632	.633	.634	.636	.638	.641
.3	.626	.628	.629	.631	.633	.636	.639
.4	.621	.623	.625	.628	.630	.633	.636
.5	.619	.621	.624	.627	.630	.633	.637
.6	.618	.620	.623	.627	.630	.634	.638
.7	.618	.620	.624	.628	.631	.635	.640
.8	.618	.621	.625	.629	.633	.637	.643
.9	.619	.622	.627	.631	.635	.639	.645
1.0	.619	.624	.628	.633	.637	.641	.648
1.2	.620	.626	.632	.636	.641	.646	-----
1.4	.622	.629	.634	.640	.644	-----	-----
1.6	.623	.631	.637	.642	.647	-----	-----

Take c from table V or VI. Substitute these values in the formula and solve for Q. Or the value of h may be determined by the following method: Compute an approximate value for Q from the formula $Q=3.33bH^{3/2}$, where H=head of water on the weir. Divide this value of Q by the sectional area of the water in the channel leading to the wier. Use this approximate value of V to find h in the equation $h=\frac{V^2}{2g}$. With this value of h determine a second approximate value of Q using formula in d above. Determine a value of V as before. With this value of V determine a new value for h and a correct value of Q.

(7) The Bazin formula is applicable to heads from 0.2 foot to 6 feet. It accounts for the velocity of approach and is appropriate for weirs without end contractions. It is

$$Q=8.02\ mbH^{3/2}$$

where

$$m=\left(.405+\frac{.00984}{H}\right)\left(1+.55\left(\frac{H}{G+H}\right)^2\right)$$

and G is the height of weir crest above the bottom of the approach channel. Table VII gives values of Q for a weir 1 foot long and for various values of H and G. This table can be used for any length of weir by multiplying the tabular values by the length of crest in feet.

(8) For submerged or semisubmerged weirs, $Q=3.33L(nH)^{3/2}$,

where n is a number depending on the ratio of H', the downstream head, to H. Varying values of n are given in table VIII. (See fig. 191.)

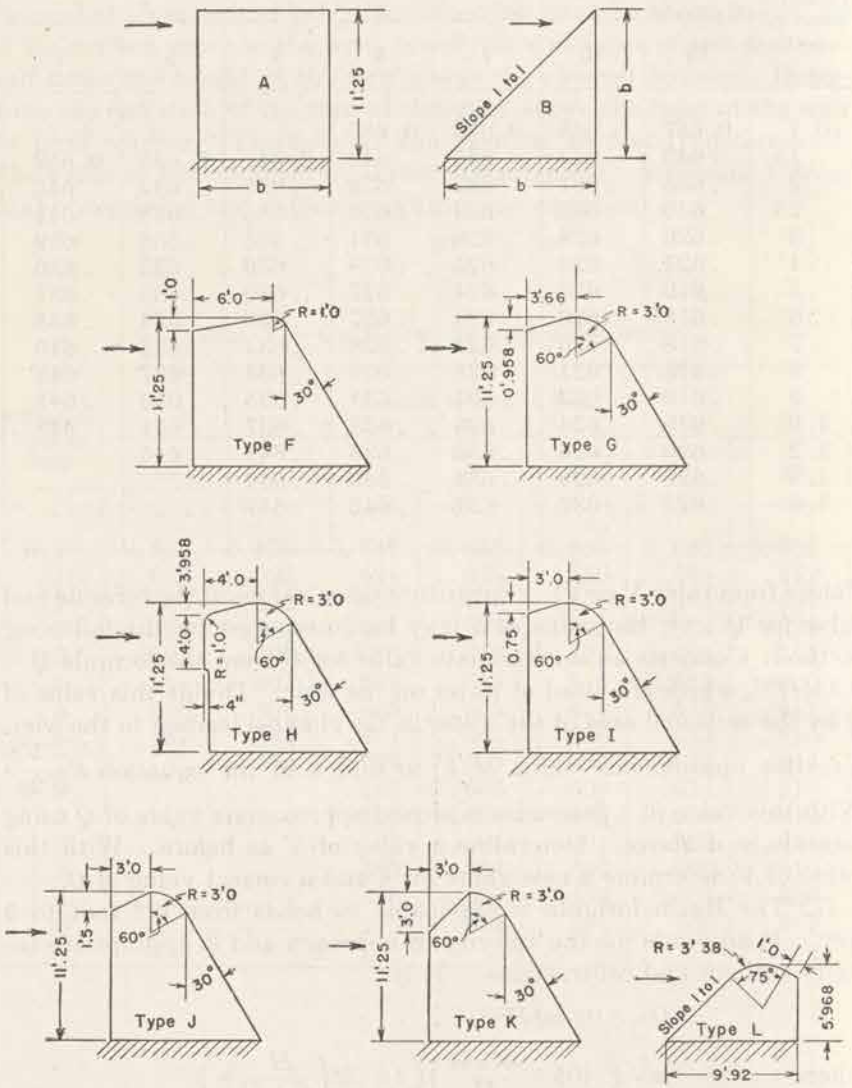


FIGURE 192.—Forms of weirs and dams.

(9) The flow over irregular crests may be computed by multiplying the discharge of a standard weir of the same height and length and at the same head by a factor depending on the form of the crests. Tables IX, X, and XI give the multipliers for various forms of weirs, as de-

termined from experiments upon full-sized models at the Hydraulic Laboratory of Cornell University. (See fig. 192.)

TABLE VII.—*Discharge in cubic feet per second per foot of length over sharp-edged vertical weirs without end contractions*

[Computed by Bazin's formula]

Head (H) feet	Height in feet of crest of weir above bottom of channel of approach						
	G=2	G=3	G=4	G=5	G=6	G=7	G=8
0.2	0.33	0.33	0.33	0.33	0.33	0.33	0.33
.3	0.58	0.58	0.58	0.58	0.58	0.58	0.58
.4	0.88	0.88	0.88	0.87	0.87	0.87	0.87
.5	1.23	1.21	1.21	1.21	1.21	1.21	1.21
.6	1.62	1.59	1.59	1.58	1.58	1.58	1.58
.7	2.04	2.01	1.99	1.98	1.98	1.98	1.98
.8	2.50	2.45	2.43	2.42	2.41	2.41	2.41
.9	3.00	2.93	2.90	2.88	2.88	2.87	2.86
1.0	3.53	3.44	3.40	3.38	3.36	3.36	3.35
1.2	4.68	4.55	4.48	4.47	4.42	4.41	4.40
1.4	5.99	5.78	5.68	5.62	5.58	5.56	5.54
1.5	6.68	6.44	6.30	6.23	6.20	6.18	6.16
1.6	7.40	7.12	6.97	6.89	6.84	6.80	6.78
1.8	8.93	8.56	8.37	8.25	8.18	8.13	8.09
2.0	10.58	10.12	9.87	9.72	9.62	9.55	9.51
2.2	12.34	11.77	11.46	11.27	11.14	11.06	10.99
2.4	14.20	13.53	13.15	12.91	12.75	12.64	12.56
2.5	15.17	14.45	14.03	13.76	13.59	13.47	13.38
2.6	16.16	15.38	14.92	14.63	14.44	14.30	14.20
2.8	18.23	17.23	16.79	16.44	16.21	16.04	15.92
3.0	20.39	19.36	18.74	18.33	18.06	17.86	17.71
3.2	22.64	21.48	20.77	20.31	19.98	19.75	19.58
3.4	24.98	23.70	22.89	22.36	21.99	21.72	21.52
3.5	26.20	24.83	24.00	23.43	23.01	22.73	22.48
3.6	27.41	25.99	25.09	24.49	24.06	23.75	23.52
3.8	29.94	28.38	27.38	26.70	26.22	25.87	25.60
4.0	32.54	30.84	29.74	28.99	28.45	28.05	27.74
4.2	35.22	33.39	32.18	31.35	30.75	30.30	29.96
4.4	37.99	36.01	34.70	33.78	33.12	32.62	32.24
4.6	40.83	38.71	37.29	36.29	35.56	35.01	34.58
4.8	43.75	41.49	39.96	38.87	38.07	37.45	37.00
5.0	46.71	44.31	42.67	41.49	40.62	39.96	39.44
5.2	49.81	47.27	45.50	44.23	43.29	42.57	42.01
5.4	52.94	50.23	48.38	47.02	46.00	45.22	44.60
5.6	56.15	53.33	51.34	49.88	48.79	47.94	47.28
5.8	59.42	56.45	54.34	52.79	51.62	50.71	49.99
6.0	62.77	59.65	57.43	55.78	54.53	53.55	52.78

TABLE VII.—*Discharge in cubic feet per second per foot of length over sharp-edged vertical weirs without end contractions*—Continued

Head (H) feet	Height in feet of crest of weir above bottom of channel of approach						
	G=9	G=10	G=12	G=16	G=20	G=25	G=30
0.2	0.33	0.33	0.33	0.33	0.33	0.33	0.33
0.3	0.58	0.58	0.58	0.58	0.58	0.58	0.58
0.4	0.87	0.87	0.87	0.87	0.87	0.87	0.87
0.5	1.21	1.21	1.21	1.21	1.20	1.20	1.20
0.6	1.57	1.57	1.57	1.57	1.57	1.57	1.57
0.7	1.97	1.97	1.97	1.97	1.97	1.97	1.97
0.8	2.40	2.40	2.40	2.40	2.40	2.40	2.40
0.9	2.86	2.86	2.86	2.86	2.85	2.85	2.85
1.0	3.35	3.34	3.34	3.33	3.33	3.33	3.33
1.2	4.39	4.38	4.38	4.37	4.36	4.36	4.36
1.4	5.53	5.52	5.51	5.49	5.49	5.48	5.48
1.5	6.14	6.13	6.12	6.11	6.10	6.09	6.09
1.6	6.76	6.74	6.73	6.71	6.69	6.69	6.69
1.8	8.07	8.05	8.02	7.99	7.98	7.97	7.96
2.0	9.47	9.44	9.40	9.36	9.34	9.33	9.32
2.2	10.95	10.91	10.86	10.81	10.78	10.76	10.75
2.4	12.50	12.45	12.39	12.32	12.28	12.25	12.24
2.5	13.31	13.26	13.18	13.10	13.06	13.03	13.01
2.6	14.13	14.07	13.99	13.90	13.85	13.82	13.80
2.8	15.83	15.76	15.66	15.54	15.48	15.44	15.42
3.0	17.60	17.52	17.39	17.25	17.18	17.13	17.10
3.2	19.45	19.34	19.19	19.02	18.93	18.87	18.83
3.4	21.36	21.24	21.06	20.86	20.75	20.68	20.63
3.5	22.38	22.22	22.00	21.83	21.69	21.62	21.60
3.6	23.34	23.20	22.99	22.75	22.62	22.53	22.48
3.8	25.39	25.23	24.99	24.71	24.56	24.45	24.39
4.0	27.51	27.32	27.05	26.72	26.55	26.42	26.35
4.2	29.69	29.48	29.17	28.79	28.59	28.45	28.36
4.4	31.94	31.70	31.34	30.92	30.66	30.52	30.42
4.6	34.25	33.98	33.58	33.10	32.84	32.65	32.53
4.8	36.62	36.33	35.88	35.35	35.05	34.83	34.70
5.0	39.03	38.70	38.21	37.61	37.28	37.03	36.88
5.2	41.56	41.20	40.65	39.99	39.61	39.33	39.17
5.4	44.11	43.71	43.12	42.38	41.96	41.66	41.47
5.6	46.74	46.31	45.65	44.84	44.38	44.04	43.83
5.8	49.41	48.94	48.22	47.33	46.83	46.45	46.22
6.0	52.15	51.64	50.86	49.90	49.34	48.92	48.67

TABLE VIII.—*Values of n for various values of* $\frac{H'}{H}$ *(fig. 191)*

H′/H	n	H′/H	n	H′/H	n	H′/H	n
0.00	1.000	0.18	0.989	0.38	0.935	0.58	0.856
.01	1.004	.20	.985	.40	.929	.60	.846
.02	1.006	.22	.980	.42	.922	.62	.836
.04	1.007	.24	.975	.44	.915	.64	.824
.06	1.007	.26	.970	.46	.908	.66	.813
.08	1.006	.28	.964	.48	.900	.70	.787
.10	1.005	.30	.959	.50	.892	.75	.750
.12	1.002	.32	.953	.52	.884	.80	.703
.14	.998	.34	.947	.54	.875	.90	.574
.16	.994	.36	.941	.56	.866	1.00	.000

TABLE IX.—*Multipliers for flat-topped weirs (A, fig. 192)*

Head (H) feet	Width of flat crest in feet							
	$b=0.48$	$b=0.93$	$b=1.65$	$b=3.17$	$b=5.89$	$b=8.98$	$b=12.24$	$b=16.30$
0.5	0.902	0.830	0.819	0.797	0.785	0.783	0.783	0.783
1.0	.972	.904	.879	.812	.800	.798	.795	.792
1.5	1.000	.957	.910	.821	.807	.803	.802	.797
2.0	1.000	.989	.925	.821	.805	.800	.798	.795
2.5	1.000	1.000	.932	.816	.800	.795	.792	.789
3.0	1.000	1.000	.938	.813	.796	.791	.787	.784
3.5	1.000	1.000	.942	.810	.793	.787	.783	.780
4.0	1.000	1.000	.947	.808	.790	.783	.780	.777

TABLE X.—*Multipliers for triangular weirs (B, fig. 192)*

Head (H) feet	$b=6.65$	$b=11.25$
0.5	1.060	1.060
1.0	1.079	1.079
1.5	1.091	1.092
2.0	1.086	1.097
2.5	1.076	1.096
3.0	1.067	1.095
3.5	1.060	1.094
4.0	1.054	1.093

TABLE XI.—*Multipliers for compound weirs (fig. 192)*

Head (H) feet	Type F	Type G	Type H	Type I	Type J	Type K	Type L
0.5	0.964	0.932	0.934	0.968	0.971	0.971	0.971
1.0	1.026	.982	1.000	1.008	1.040	1.040	.983
1.5	1.064	1.015	1.040	1.032	1.083	1.092	1.022
2.0	1.066	1.031	1.061	1.041	1.105	1.126	1.040
2.5	1.025	1.038	1.073	1.043	1.118	1.146	1.057
3.0	.992	1.044	1.082	1.044	1.128	1.163	1.072
3.5	.966	1.049	1.090	1.045	1.136	1.177	1.085
4.0	.944	1.053	1.097	1.046	1.144	1.190	1.097

f. Discharge by measuring velocity and cross-sectional area, their product being the discharge.—This method involves the accurate finding of the stream cross section and mean velocity.

(1) A study of many streams has developed the following facts:

(*a*) Velocities are least near the bottom and sides of the channel.

(*b*) The surface velocity is not the greatest velocity.

(*c*) The greatest velocity is near the center of the channel, at about 13 percent of the depth below the surface.

(*d*) The mean velocity in any vertical section is very nearly that found at 60 percent of the depth. It is given still more accurately by taking the average of the velocities at 20 percent and 80 percent of the depth.

(*e*) Velocity is affected by the presence of bridge piers, dams, etc.

(2) Determine the cross section by making soundings at measured intervals across the stream. Divide the cross section into a number of sections, the number depending upon the accuracy with which the measurement is to be made. Determine the mean velocity in each section. From the mean velocity and the area, determine the partial discharge through each section. The sum of all such partial discharges gives the discharge of the stream.

215. Determination of velocity.—*a.* Select a gaging site in a straight stretch—500 feet both above and below station is desirable—where the river bed is permanent but not stony, the banks are of sufficient height to contain floodwater, and no disturbing influences such as piers are present.

b. Velocities may be determined by use of floats or current meters. To determine velocity by floats, note the time required for floats of various kinds to traverse a given distance. This given distance is

usually a carefully determined base line from 100 feet to 300 feet in length.

c. Floats are of three kinds—surface, submerged, and rod or tube floats.

(1) Surface floats are made of any buoyant material. They may be used in shallow streams. They are placed so as to travel over the various sections of the cross sections. The time of travel of each float divided by the length of the base line gives the surface velocity. The mean velocity in each section is taken as 0.8 of the surface velocity shown for that section. Results may be accurate within about 10 percent provided the stream is shallow and no wind is blowing.

(2) A submerged "float" is connected by a slender cord to a surface float having sufficient buoyancy to hold the submerged body suspended at any desired depth.

(3) Rod floats consist of hollow tin cylinders or wooden rods weighted to float vertically with a short section exposed. They should be long enough to pass as close to the bottom as possible without striking. Such floats integrate the velocities in a vertical filament and give reliable results. Place the floats so as to give determinations of the mean velocity in each section. The total discharge of the stream is the sum of the discharges of the several sections.

d. The velocity may also be determined by the use of a current meter. See paragraph 196*e* for description of meters and their rating.

(1) To find the mean velocity of the stream, divide the cross section into a number of narrow sections. Find the mean velocity in each section by one of the following three methods:

(*a*) Observe velocity at 0.2 and 0.8 depth below the surface and average the two observations.

(*b*) Observe velocity at 0.6 depth.

(*c*) Observe the velocity at several points in the vertical by placing the meter at these points successively to determine the vertical velocity curve. From a plot of these curves, determine the mean ordinate which is the desired mean velocity for the section.

(2) The discharge for each section is obtained by multiplying the mean velocity (obtained as a result of the observations from the rating table) by the section area. The sum of the discharges for all the sections is the total stream discharge.

(3) Figure 193 is an example of notes for the observation and reduction of current meter readings for velocities.

216. Value of results.—The value of the results depends largely on the accuracy of the methods pursued. If the discharge of the

stream over a long period is desired, measurements must be made at various seasons and a mean discharge found. If only one measurement is to be made, it should when possible be made under normal conditions. Trees, landmarks, etc., can be examined to determine high water marks, and reports examined to find low water records.

Current meter observations (29)

Current Location	Meter Current	No. 3211 Meter	Section,	Indian	River					
Depth meter(ft)	Time (seconds)	Register Start	Register End	Revolutions	R.p.s.	Velocity (f.p.s)			Remarks	
19	100	1,328	1,382	54	0.54	1.35	Date: May 3, 1939			
17	100	1,382	1,462	80	.80	1.93	Wind: NE Light			
15	100	1,462	1,560	98	.98	2.30	Depth water: 20 ft, 10:30 A.M.			
13	100	1,563	1,673	110	1.10	2.60	Chief of party: Sgt R. Roe			
11	100	1,673	1,789	116	1.16	2.75	Assistant: Cpl Burr			
9	100	1,789	1,917	128	1.28	3.01				
7	100	1,917	2,054	137	1.37	3.19				
5	100	2,055	2,194	139	1.39	3.26				
3	100	2,199	2,338	139	1.39	3.26				
1	100	2,338	2,473	135	1.35	3.15				
						26.81				

FIGURE 193.—Notes for observation and reduction of current meter readings.

This information should be carefully analyzed and an attempt made to determine the average conditions.

SECTION XXXIV

OUTLINE OF INSTRUCTION COURSES

Paragraph
General 217
Basic instructions, surveyors 218
Advanced instructions, surveyors 219
Instructions, computers 220

217. General.—*a.* Special training courses for surveyors and computers, details of which are given in the preceding sections of this manual, include 36 exercises.

b. The basic instructions for surveyors, intended to cover the work usually performed in the average engineer organizations are included

in 16 exercises (see par. 218). These together with the eight exercises in paragraph 219 should be sufficient to train any suitable man for the higher branches of surveying, including geodetic work and astronomical observations.

c. The sixteen exercises listed in paragraph 220 should be given to those men who are to perform the duties of computers. It is understood that all men detailed to take any part of the surveyors or computers course must be familiar with, or be instructed in, mathematics up to and including plane trigonometry.

d. The training courses here given are to be considered as guides only, to assist instructors in planning exercises for a course based upon local conditions and specific requirements, which will vary to some extent with the type of available instructional material.

e. The time allotted for each exercise, based on experience with Army student classes over a period of some 20 years, will be found necessary to give the student showing aptitude and willingness to learn just enough practice to be useful when detailed for work with a survey party after completing the course.

f. Exercises enumerated in paragraph 218 should be given first before any attempt is made to cover the more advanced subjects in paragraph 219. Instructions for computers may be given concurrently with the field work preceding it, or, if desired, as a separate course. Exercises may be rearranged within practical limits, if desired.

218. Basic instructions, surveyors.

Exercise	Paragraph	Figure number	Hours, field	Hours, classroom	Total hours
1. Measuring distances	47*i*	7	12	6	18
2. Adjustment of dumpy level	53*e*		3	1	4
3. Differential leveling	54*f*	15, 15A	15	3	18
4. Profile leveling	55*f*	16	7	2	9
5. Cross section levels	56*c*	17, 18	7	5	12
6. Determination of stadia constant	70*e*	35	3	1	4
7. Adjustment of transit	71*h*		3	1	4
9. Conversion of azimuths and bearings	76*c*			2	2
10. Horizontal angles by repetition	79*g*	40	12	1	13
11. Azimuth traverse with stadia	80*f*	42	18	3	21
12. Compass (needle) traverse	81*f*		6	1	7
26. Adjustment of plane table	139*j*		2	1	3
27. Plotting detail on photograph	158*e*		7	1	8
28. Completing skeleton plane table sheet	159*b*	130	12	1	13
29. Locating control points on photographs	162*b*		4	1	5
35. Location survey	191*f*		18	3	21
			129	33	162

219. Advanced instructions, surveyors.

Exercise	Paragraph	Figure number	Hours, field	Hours, classroom	Total hours
8. Adjustment of theodolite	74*g*		3	1	4
16. Measuring base line with steel tape	106*f*	68	18	3	21
17. Grid triangulation	114*d*	71, 82, 83	13	4	17
30. Azimuth observation—altitude method (sun)	171*d*	141	4	2	6
31. Azimuth observation—hour angles of sun	173*d*	142*a*	2	2	4
32. Azimuth observation — Polaris at elongation	176*d*	145	3	4	7
33. Azimuth observation — Polaris at any hour angle	177*c*	146	3	1	4
34. Azimuth observation—altitude method (star)	178*f*	147	6	2	8
			52	19	71

220. Instructions, computers.

Exercise	Paragraph	Figure number	Hours
13. Calculating and adjusting azimuths	84*c*	46	3
14. Computation and adjustment of rectangular coordinates	88*b*	47	10
15. Area computation by *DMD* method	89*b*	48	6
18. Calculation of azimuths	118*j*	71, 82	5
19. Solution of triangles	118*l*	75, 82	3
20. Coordinates from azimuth and distance	118*m*	85	6
21. Trigonometric elevation	118*o*	79	6
22. Control data	118*q*	87	2
23. Semigraphic intersection	122*e*	98	5
24. Semigraphic resection	125*h*	105	6
25. The inaccessible base	130*d*	112	3
30. Azimuth computation—altitude method (sun)	171*d*	141	3
31. Azimuth computation—hour angle of sun	173*d*	142*b*	4
32. Azimuth computation—Polaris at elongation	176*d*	145	2
33. Azimuth computation—Polaris at any hour angle	177*c*	146	4
34. Azimuth computation—altitude method (star)	178*f*	147	3
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